Expectation-Driven Term Structure of Equity and Bond Yields*

Ming Zeng† Guihai Zhao‡

First draft: December 2020

This draft: October 2022

*We thank Jules van Binsbergen, Niels Gormsen, Ralph Koijen, Serhiy Kozak, Indrajit Mitra, Stijn Van Nieuwerburgh, Heungju Park, Michael Weber, Paul Whelan, and seminar participants at the 18th Annual Conference of Asia-Pacific Association of Derivatives, Nordic Finance Network Young Scholars Finance Webinar, North American Meeting of the Econometric Society, Stanford SITE Conference 2021, WFA 2022, Bank of Canada, Singapore Management University, and the University of Gothenburg for helpful comments and discussions. We are grateful to Ricardo De la O, Andrei Gonçalves, and Serhiy Kozak for making their data available online. The previous draft is titled "Term Structure of Equity and Bond Yields over Business Cycles." The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Bank of Canada.

†Department of Economics & the Centre for Finance, University of Gothenburg, and Swedish House of Finance. Email: ming.zeng@cff.gu.se.

‡Bank of Canada, 234 Wellington Street, Ottawa, ON K1A 0G9. Email: GZhao@bankofcanada.ca

Abstract

Recent findings on the term structure of equity and bond yields pose severe challenges to existing equilibrium asset pricing models. This paper presents a new equilibrium model of subjective expectations to explain the joint historical dynamics of equity and bond yields (and their yield spreads). Equity/bond yields movements are mainly driven by subjective dividend/GDP growth expectations. Yields on short-term dividend claims are more volatile because short-term dividend growth expectation mean-reverts to its less volatile long-run counterpart. Procyclical slope of equity yields is due to the counter-cyclical slope of dividend growth expectations. The correlation between equity returns/yields and nominal bond returns/yields switched from positive to negative after the late 1990s, mainly owing to a shift in correlation between real GDP growth and real dividend growth expectations from negative to positive, and only partially due to procyclical inflation. Dividend strip returns are predictable and the strength of predictability decreases with maturity due to underreaction to dividend news and hence predictable dividend forecast revisions. The model is also consistent with the data in generating persistent and volatile price-dividend ratios, excess return volatility, and momentum.

Keywords: Subjective expectations, under-reaction, term structure, equity yields, stock-bond correlation, return predictability, forecast revision, momentum, stock market puzzles

JEL Classification: G00, G12, E43.

1 Introduction

The fundamental question in asset pricing research is what drives the equity and bond price movements. To see the link between them, researchers can express equity price at time t as its discounted cash flows:

$$P_t = \sum_{n\geq 0} \frac{E_t(D_{t+n})}{R_{t:t+n}} = \sum_{n\geq 0} \frac{E_t(D_{t+n})}{\exp\left(n\left(y_t^{(n)} + \theta_t^{(n)}\right)\right)},$$

where $E_t(D_{t+n})$ is the expected nominal dividend and $R_{t:t+n}$ is the required return. $R_{t:t+n}$ can be decomposed further into nominal bond yield $y_t^{(n)}$ and dividend risk premium $\theta_t^{(n)}$. This equation holds under marginal investors' expectations, which could be rational or irrational. Motivated by the finding that prices are too volatile than expected dividend and the finding that future returns are predictable by the price-dividend ratio under the rational expectation (Shiller, 1981; Campbell and Shiller, 1988; Cochrane, 2008, 2011), researchers have proposed several asset pricing models based on time-varying dividend risk premium $\theta_t^{(n)}$. Leading examples include the habit formation model (Campbell and Cochrane, 1999), the long-run risk model (Bansal and Yaron, 2004), and the disaster risk model (Barro, 2006; Gabaix, 2012; Gourio, 2012).

Recent empirical findings pose new challenges to existing equilibrium asset pricing models from different dimensions. (1) Survey-based evidence indicates weak time-variations in expected returns (Greenwood and Shleifer, 2014; Nagel and Xu, 2022b) instead of the strong variations implied by many existing models. (2) De La O and Myers (2021) show that most aggregate stock price movements are caused by cash flow growth expectations rather than by subjective return expectations, and Bordalo et al. (2020b) show that long-term earnings growth expectations over-react to news, leading to stock return predictability and excess price volatility. (3) Short-term equity yields are more volatile than long-term equity yields and both are driven mainly by dividend growth expectations (Van Binsbergen et al., 2013) rather than by dividend risk premium. (4) The slope of equity yields (long-term minus short-term yields) is procyclical. (5) Dividend strip returns are predictable, but the strength of predictability decreases with maturity (Van Binsbergen et al., 2012). (6) The correlation between aggregate stock returns and long-term nominal bond returns has switched from positive to negative since the late 1990s (Li, 2002; Campbell et al., 2017). Although the change in inflation cyclicality is the standard explanation in the literature, Duffee (2021) provides empirical evidence that the correlation between stock returns and *real* bond returns, i.e., a real channel, plays a significant role in explaining this fact.

This paper contributes to the literature by proposing an equilibrium model that explains the joint dynamics of the term structure of equity and bond yields and is consistent with the above empirical findings. In our model, variations in equity (bond) yields are due to subjective dividend growth (GDP growth) expectations instead of dividend (bond) risk premium. We show that the model can match the historical dynamics of the term structure of equity and bond yields and their comovements. Yields on short-term dividend claims are more volatile because the short-term dividend growth expectation mean-reverts to the less volatile levered long-run GDP growth expectations. The negative slope of equity term structure during recessions reflects the countercyclical slope of dividend growth expectations. Long-term Treasury bonds have switched from risky assets to safe assets since the late 1990s, which is mainly driven by a stronger correlation between real GDP and real dividend growth expectations and only partially by procyclical inflation. Finally, underreaction to dividend news (or predictable dividend forecast revisions) implies (1) ex-post predictability of strip/market returns, (2) decreasing in predictability with maturity, and (3) return momentum. Prices are volatile because of volatile subjective beliefs.

To model subjective beliefs, we depart from rational expectation by assuming that the agent has the "belief in the law of small numbers" as labelled by Tversky and Kahneman (1971). Under this assumption, the agent perceives small samples to represent their population equally well as large samples. Such cognitive bias implies that the agent, in his human nature, produces forecasts for the future by extrapolating fundamentals; that is, the agent overreacts to the news. Meanwhile, variations of macro variables are affected by both underlying structural shocks and central bank mandates. Misperception of the central bank's reaction to inflation and output growth shocks can switch overreaction to the observed under-reaction to the news. In addition to the sticky-information model of Mankiw and Reis (2002), noisy-information model of Sims (2003), or learning about the long-run in Farmer et al. (2021), we provide an alternative channel that leads to agent's underreaction to news.

We assume that the data generating processes (DGPs) for real dividend level, real GDP growth, and inflation all contain latent states. The agent forms subjective expectations of these latent states using the learning framework discussed above. Specifically, the aggregate log dividend is decomposed into two components: (1) long-duration dividend d_t^l and (2) share of long-duration dividend in total dividend d_t^s. The aggregate endowment risk is embedded in the long-duration dividend d_t^l, which is assumed to be levered on log real GDP. The share of long-duration dividend d_t^s carries no aggregate risks and follows a stationary process.[1] Meanwhile, the real GDP growth (as endowment growth) and inflation are each decomposed into one stable and one transitory/volatile component. The stable component varies less with the business cycle and is assumed to contain a random-walk state variable (capturing trend growth and inflation), while the transitory component varies significantly with the business cycle and is assumed to contain a stationary state variable (capturing short-run deviations from the trend).

We estimate the subjective learning gains by matching model-implied expectations with consensus forecasts in the survey. The model implies that the agent under-reacts to news when forming expectations on real dividend, real GDP growth and inflation. Furthermore, we find that the model-implied subjective expectation of dividend growth closely tracks the full time-series of consensus forecasts for the aggregate dividend growth, with the correlation of 0.8 for both 1- and 2-year forecasts.

To derive asset pricing implications, our model assumes that the representative agent has a con-

[1]Empirically, we find that long-duration dividends indeed are more related to the aggregate endowment. We also find that separating long-duration dividend from total dividend can help the model best match dividend growth expectations in survey data. The two-component model is also consistent with the recent finding that the cross-section of cash-flow duration contains useful information for the aggregate market (see e.g., Kelly and Pruitt, 2013; Li and Wang, 2018; Gormsen and Lazarus, 2022).

stant relative risk aversion (CRRA) utility when pricing assets, which implies a constant subjective risk premium. We show that our model can match the entire time-series dynamics of the term structure of equity yield over the past three decades. Using equity yield data from Giglio et al. (2021), Figure 2 displays some salient features: (1) more volatile short-term equity yields, (2) a secular decline in equity yields since the late 1980s followed by an upward trend after 2000, (3) a sharp increase in yields during recessions, and (4) a procyclical equity yield slope. In our model the constant risk premium suggests that the equity yields are driven by subjective dividend growth expectations. The short-term expectation is more volatile and mean-reverts to the less volatile long-run growth expectation, thus the long-term equity yields are more stable. The subjective dividend growth expectations experienced an upward trend starting from the late 1980s and decreased steadily after 2000, which caused the equity yields to have the opposite trend movements. During recessions, growth expectations are low, with the short-term expectation being much lower than its long-run counterpart. Therefore, we observe sharp increases in equity yields and the procyclical equity yield slope. We also show that the ex-post realized returns generated from the model align well with their empirical counterparts. The implied 2-year (10-year) dividend strip returns have a correlation of 0.6 (0.5) with the data. Regarding the bond market, since our model block for bond pricing closely follows the setup of Zhao (2020), it matches several essential facts in the US bond market. Within our sample, we find that the model-implied 1- (10-) year nominal bond yields have a correlation of 0.92 (0.95) with the data, suggesting that the model is also successful in pricing nominal bonds.

We next investigate the comovements between equity and bond markets. A well-known stylized fact is that long-term nominal bonds switched from risky to safe assets after the late 1990s; that is, the correlation between bond and stock returns changed from positive to negative. The same pattern is observed for the correlation between nominal bond yields and real equity yields: the "Fed model". Change in inflation cyclicality (from countercyclical to procyclical) can potentially explain these facts since it switches the sign of inflation risk premium in equity returns (from negative to positive).[2]

[2]In equilibrium models, inflation risk premium in equity returns can arise from, for example, "money illu-

However, Duffee (2021) finds that changes in the *real* bond and stock return correlation (i.e., a real channel) play a significant role in explaining this fact. Our model reconciles these empirical findings using subjective expectations of real GDP growth, inflation, and real dividend growth (rather than inflation risk premium). While the real effect of inflation, defined as the covariance between subjective inflation and subjective real growth, explains approximately 20% of the total changes in bond-stock covariance, we find that changes in the *nominal* bond-stock covariance indeed are mainly driven by changes in the *real* bond-stock covariance. More explicitly, such changes are due to a shift in correlation between *real* GDP and *real* dividend growth expectations from negative to positive after 2000, which accounts for around 90% of total changes in nominal bond-stock covariance. That is, a new channel driving the bond-stock correlation is that real bonds provide a better hedge to stocks after 2000. In the data, we confirm the relevance of this channel.

Our model also sheds new light on why equity returns are predictable by equity yields (e.g., Campbell and Shiller, 1988) and why the strength of predictability declines from short- to long-maturity dividend claims (e.g., Van Binsbergen et al., 2012). With constant risk premium, strip returns can be decomposed to bond excess returns matched by maturity and dividend forecast revisions. We find that (1) bond return predictability has a small contribution to the total strip return predictability for short-maturity dividend strips; (2) dividend forecast revision is more predictable in the short maturity, translating to a downward-sloping term structure of return predictability. The reason is that the agent underreacts to dividend news. A positive shock to the current dividend (higher equity yield) does not push up too much the agent's dividend expectation today, leaving space for the increase in subsequent dividend forecast revision. As the maturity increases, since the current news has a smaller impact on longer-term dividend expectations, the predictability of dividend forecast revision will become weaker. It should be noted that the return predictability caused by underreaction to cash-flow news is also consistent with the momentum literature (see e.g., Chan et al.,

sion" (David and Veronesi, 2013), time-varying risk aversion (Campbell et al., 2020), long-run risk (Piazzesi and Schneider, 2007; Bansal and Shaliastovich, 2013; Song, 2017), or time-varying ambiguity Zhao (2017).

1996; Chordia and Shivakumar, 2006). In our equilibrium model, underreaction to dividend news leads to positively autocorrelated dividend forecast revision, which translates to momentum.

Finally, the model reconciles major aggregate stock market puzzles. The model implies a time-series of aggregate dividend yields close to and as persistent as the data. The unconditional mean and volatility of model-implied dividend yields and market returns are comparable to the data. The model also replicates the positive (negative) correlation between the market returns and the long-term Treasury bond returns before (after) 2000.

In Appendix E, we extend the model to further take into account the agent's fear over model misspecification. We show that when the agent is ambiguous about real GDP and dividend growth, the extended model implies time-series of equity yields and returns that are closer to the data.

Related literature

This paper is motivated by some new evidence in the empirical asset pricing literature: for example, the importance of subjective expectation in equity markets (Barberis et al., 2015; Cassella and Gulen, 2018; Bordalo et al., 2020b; De La O and Myers, 2021; Guo and Wachter, 2021) and in bond markets (Froot, 1989; Piazzesi et al., 2015; Cieslak, 2018; Duffee, 2018), the term structure of equity yields (Van Binsbergen et al., 2012, 2013; Van Binsbergen and Koijen, 2017; Van Binsbergen, 2021; Giglio et al., 2021), and the relationship between stock and bond markets (Li, 2002; Campbell et al., 2017; Duffee, 2021). To the best of our knowledge, this is the first article that proposes an asset pricing model with subjective beliefs to jointly explain the historical dynamics of the term structure of equity and bond yields and is consistent with the above empirical findings. In fact, echoing Brunnermeier et al. (2021), who suggest that "research focus should be on motivating, building, calibrating, and estimating models with non-RE beliefs ... we need structural models of belief dynamics that can compete with RE models in explaining asset prices and empirically observed beliefs," this paper connects subjective expectation with asset prices and estimates the belief process using survey data.

This paper is also related to an extensive equilibrium asset pricing literature focusing on (1) rational expectation and aggregate stock market puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006; Gabaix, 2012; Gourio, 2012; Albuquerque et al., 2016), (2) rational expectation and bond markets (Piazzesi and Schneider, 2007; Wachter, 2006; Bansal and Shaliastovich, 2013), (3) the link between stock and bond markets (David and Veronesi, 2013; Campbell et al., 2020; Song, 2017; Zhao, 2017), (4) risk premium and the term structure of equity returns (Hasler and Marfe, 2016; Bansal et al., 2021; Breugem et al., 2022; Gonçalves, 2021a; Li and Xu, 2021), and (5) subjective beliefs in equity and bond markets (Barberis et al., 2015; Adam et al., 2016; Nagel and Xu, 2022a; Zhao , 2020). We extend the literature by providing a unified framework of bond and equity pricing under subjective expectations, and the model matches many stylized facts in these two markets. In particular, while Zhao (2020) shows how subjective expectation resolves some puzzles in bond markets, our paper addresses leading puzzles in equity markets and bond-stock comovements.

The paper continues as follows. Section 2 outlines the framework for expectation formation and asset pricing. Section 3 describes the data and model estimation and calibration. Section 4 shows the empirical results. Section 5 provides some robustness analysis, and Section 6 concludes.

2 Expectation Formation and Asset Prices

In this section, we describe how the agent forms subjective beliefs over real dividend level, real endowment growth, and inflation. Then assuming the CRRA utility, we derive the equilibrium bond and equity prices that are consistent with those subjective beliefs.

2.1 Subjective expectation

We first introduce a framework to illustrate how the agent forms subjective expectations, and we will apply the idea to specific economic forecasts in later subsections. Consider the following state-space model as the data-generating process for the economic outcome y:

$$y_t = Cx_t + \sigma_\epsilon \epsilon_t, \tag{1}$$

$$x_{t+1} = \rho x_t + \sigma_u u_{t+1}, \tag{2}$$

with $i.i.d.$ ϵ_t and u_t following standard normal distribution. The coefficient C reflects how y_t is tied to the structural forces or latent states x_t. Without loss of generality, we assume $C > 0$.

Rational belief updating is defined as Bayesian updating:

$$p(x_t|I_t) \propto p(y_t|x_t, I_{t-1}) \times p(x_t|I_{t-1}), \tag{3}$$

which implies the following dynamics for the posterior belief over x_t (i.e., the Kalman filter):

$$E_t x_t = \rho E_{t-1} x_{t-1} + K(y_t - C\rho E_{t-1} x_{t-1}), \tag{4}$$

with the steady-state Kalman gain $K = \frac{CP}{C^2 P + \sigma_\epsilon^2} > 0$ and P the steady-state variance of the predictive distribution for the latent state.

To model subjective belief, we depart from Equation (3) by assuming that the belief update follows

$$p(x_t|I_t) \propto p(y_t|x_t, I_{t-1})^{1+\theta} \times p(x_t|I_{t-1}), \tag{5}$$

where θ captures the behavioural bias. An agent with subjective expectations evaluates one data observation as if she observed $1 + \theta$ data observations. The belief dynamics under such an update rule have the same form yet different learning gain

$$\tilde{E}_t x_t = \rho \tilde{E}_{t-1} x_{t-1} + \nu(y_t - C\rho \tilde{E}_{t-1} x_{t-1}), \tag{6}$$

where we label $\tilde{E}(\cdot)$ as the subjective expectation and the subjective learning gain as $\nu = \frac{(1+\theta)C\tilde{P}}{(1+\theta)C^2\tilde{P}+\sigma_\epsilon^2}$.[3] When $\nu > K$, that is, $\theta > 0$, the agent overreacts to the news by becoming excessively optimistic (pessimistic) after good (bad) news about the latent state relative to the rational benchmark. Similarly, when $\nu < K$, or $\theta < 0$, the subjective belief underreacts to news, as now the agent puts less weight on the recent data.[4] Hence the belief updating scheme (5) may accommodate rich dynamics as observed in the survey data. For example, survey forecasts display overreaction in long-term earnings growth (Bordalo et al., 2019, 2020b), underreaction in GDP growth and inflation (Coibion and Gorodnichenko, 2015), and underreaction in real dividend level (this paper).

When $\theta > 0$, the belief updating can be micro-founded as the agent has the "belief in the law of small numbers" (Tversky and Kahneman, 1971). Under such cognitive bias, the agent produces forecasts for the future by extrapolating fundamentals; that is, the agent overreacts to the news. The cognitive bias has been widely researched in the literature (e.g., Rabin, 2002) and a formal discussion of (5) is offered by Santosh (2021).[5] However, in this paper, we focus on the forecasts for real dividend level, GDP growth and inflation, which all display underreaction to the news. While underreaction is consistent with the sticky-information model of Mankiw and Reis (2002), noisy-information model of Sims (2003), or learning about the long-run in Farmer et al. (2021), they do not imply the exact form (5) with $\theta < 0$. We provide an alternative approach that can be consistent with underreaction, even when $\theta > 0$.

The idea is that the observed economic outcomes stem from the general equilibrium and may be affected by the actions of unmodeled economic entities. For instance, the central bank has the goal of stabilizing inflation and output gap (Woodford, 2001), and firms are primarily concerned with the stability of dividends (Lintner, 1956; Brav et al., 2005). If the agent has biased beliefs over the

[3] $\tilde{P}$ is the steady-state variance of the *subjective* predictive distribution for the latent state and may differ slightly from P in the Kalman filter.

[4] We formally show this in Equation (IA.7) in Appendix A.1.

[5] Equation (5) has a different psychological foundation than the diagnostic expectation in e.g. Bordalo et al. (2020a), though they both imply agents' overreaction to news. The diagnostic expectation is based on the representativeness heuristic (Kahneman and Tversky, 1972).

central bank's reaction function or the firm management's payout policy, the subjective expectation will be distorted as well.[6] Such bias can be captured by assuming that instead of (1), the agent has the following observation equation in mind:

$$y_t = \tilde{C} x_t + \sigma_\epsilon \epsilon_t, \tag{7}$$

while the observation equation under the rational expectation is still (1). We are interested in the case when $\tilde{C} < C$ (in absolute terms). A lower value for perceived C means that the inflation or output growth is perceived to be less responsive to structural shocks, and the dividend is less responsive to earnings shocks relative to the rational benchmark. In Appendix B, we show that these biases could arise from a standard New Keynesian model where the agent overly estimates the strength of the stabilization motive by the central bank, or from a dividend smoothing model where the agent overly estimates the degree of dividend smoothing.

Now under the perceived model, the subjective learning gain becomes $\nu = \frac{(1+\theta)\tilde{C}\tilde{P}}{(1+\theta)\tilde{C}^2\tilde{P} + \sigma_\epsilon^2}$. Clearly, even when $\theta > 0$, ν can still be smaller than K if $\tilde{C}$ is small enough. An extreme case when $\tilde{C} = 0$, the agent will not react to news ($\nu = 0$). In summary, while the behavioural bias caused by the law of small numbers makes the agent overreact to recent news, the wrong perception of the general equilibrium effect may make the agent underreact.

The above mechanism is only for illustrative purposes. To make the empirical analysis feasible, in subsequent sections when we discuss learning about dividend, inflation, and GDP growth, we directly estimate the subjective learning gain ν from the survey data, and the subjective expectations are still formed according to Equation (6).

[6]The wrong belief may occur due to the difficulty of estimating the central bank's reaction function (e.g., Clarida et al., 2000) or the strength of dividend smoothing (e.g., Leary and Michaely, 2011).

2.2 Subjective beliefs on real dividend level

In this subsection, we describe how the agent forms beliefs over aggregate real dividend. We begin with a two-component dividend model and let the agent learn these two components separately when forming expectations. Specifically, the logarithm of aggregate real dividend can be decomposed as:

$$d_t = d_t^l + d_t^s, \tag{8}$$

where $d_t = \log D_t$ is the log total real dividend and $d_t^l = \log D_t^l$ measures the log real dividend from the sector of long-duration stocks. As a result, $d_t^s = d_t - d_t^l$ quantifies the share of the long-duration dividend in the aggregate dividend. The decomposition (8) extracts useful information from the cross-section of cash-flow duration when modelling the aggregate market.[7] In Section 5.1, we compare the performance of our model with existing models such as the dividend-earning model. We find that separating aggregate dividend according to (8) brings useful information for explaining the subjective dividend expectations, equity forward yields, and bond-stock correlation changes.

We first assume that the total dividend from long-duration stocks d_t^l is more tied to the aggregate economy. It is linked to the total endowment y_t through a leverage parameter λ, with the following state-space model:

$$d_t^l - \lambda y_t = \mu_{d,t} + \sigma_d^l \epsilon_{d,t}^l \tag{9}$$

$$\mu_{d,t+1} = \mu_{d,t} + \sigma_d^\mu \epsilon_{d,t+1}^\mu, \tag{10}$$

where $\epsilon_{d,t}^l$ and $\epsilon_{d,t}^\mu$ are $i.i.d.$ shocks following the standard normal distribution. In Section C of Internet Appendix, we offer empirical evidence that supports the close connection between long-

[7]While traditional literature on dividend learning relies on observations of aggregate dividend or endowment series (see e.g., Johannes et al., 2016), incorporating other informative series into the learning framework can be a promising avenue. For example, Jagannathan and Liu (2019); De La O and Myers (2021); Nagel and Xu (2022a) incorporate earnings information.

duration dividend and aggregate endowment. We show that the long-duration dividend positively and strongly correlates with the real GDP (with or without considering the time trend), and the real GDP level explains 88% of total variations in the long-duration dividend. However, the correlation is weaker for the short-duration dividend, with the real GDP only explaining around 50% of its variations.

Then, following Menzly et al. (2004); Cochrane et al. (2008), we assume that the dividend share is stationary and follows the state-space model:

$$d_t^s = x_{d,t} + \sigma_d^s \epsilon_{d,t}^s \tag{11}$$

$$x_{d,t+1} = \rho_d x_{d,t} + \sigma_d^x \epsilon_{d,t+1}^x, \tag{12}$$

where $\epsilon_{d,t}^s$ and $\epsilon_{d,t}^x$ are $i.i.d.$ shocks following the standard normal distribution. The specification captures the idea that dividends from long-duration stocks cannot deviate permanently from the aggregate dividend and that information from such deviation d_t^s is helpful to infer future dividend. When the share of long-duration dividend is temporarily higher, the aggregate dividend will increase more.

With specified DGPs, the agent forms the subjective expectation following the rule discussed in Section 2.1. Then the agent's posterior beliefs over latent states $\tilde{\mu}_{d,t}$ and $\tilde{x}_{d,t}$ evolve according to:

$$\tilde{\mu}_{d,t+1} = \tilde{\mu}_{d,t} + \nu_d^l (d_{t+1}^l - \lambda y_{t+1} - \tilde{\mu}_{d,t}) \tag{13}$$

$$\tilde{x}_{d,t+1} = \rho_d \tilde{x}_{d,t} + \nu_d^s (d_{t+1}^s - \rho_d \tilde{x}_{d,t}). \tag{14}$$

We note that the subjective learning gains ν_d^l, ν_d^s can be different for two dividend components, and the subjective dividend growth is:

$$\tilde{E}_t \Delta d_{t+1} = \lambda \tilde{E}_t \Delta g_{t+1} + (\rho_d - 1)\tilde{x}_{d,t} + (\nu_d^s - 1)(d_t^s - \rho_d \tilde{x}_{d,t-1}) + (\nu_d^l - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}), \tag{15}$$

where $\Delta g_{t+1} = y_{t+1} - y_t$ is the growth rate of real output. We will specify how its subjective expectation is formed in the next subsection.

2.3 Subjective beliefs on endowment growth and inflation

In an endowment economy with no risk premium, bond yields are driven by investors' expectations of real growth and inflation rate. Given that bond yields contain both a trend component and a cycle component, we use the component models of real GDP growth and inflation following Zhao (2020). Trend in yields are driven by the long-run expectations of growth and inflation, and cyclical movements in yields are driven by investor's beliefs on short-run growth and inflation.

Specifically, the four components of GDP – investment spending, net exports, government spending, and consumption (PCE) – do not move in lockstep with each other; their volatility differs greatly. PCE is very stable and varies less with the business cycle. In contrast, the other three components vary greatly across economic contractions and expansions. Similarly, for inflation, the core inflation is much more stable than other inflation components. With this in mind, we can decompose output growth and inflation via the following accounting identity:

$$\Delta g_t = \Delta g_t^* + Gap_t^g$$
$$\pi_t = \pi_t^* + Gap_t^\pi,$$

where π_t is total inflation and Δg_t is GDP growth. The stable components Δg_t^* and π_t^* are PCE growth and core inflation, and Gap_t^g and Gap_t^π are the volatile components (GDP growth excluding the PCE and GDP deflator excluding the core inflation).

The real consumption growth and core inflation follow:

$$\Delta g_t^* = \mu_{g,t} + \sigma_g \varepsilon_{g,t}^* \tag{16}$$

$$\pi_t^* = \mu_{\pi,t} + \sigma_\pi \varepsilon_{\pi,t}^*, \tag{17}$$

where $\varepsilon_{g,t+1}^*$ and $\varepsilon_{\pi,t+1}^*$ are $i.i.d.$ standard normal shocks. The latent states are assumed to follow the unit-root processes:

$$\mu_{g,t+1} = \mu_{g,t} + \sigma_g^\mu \varepsilon_{g,t+1}^\mu \tag{18}$$

$$\mu_{\pi,t+1} = \mu_{\pi,t} + \sigma_\pi^\mu \varepsilon_{\pi,t+1}^\mu. \tag{19}$$

The two gap components are assumed to contain latent stationary states:

$$Gap_t^i = x_{i,t} + \sigma_i^{gap} \varepsilon_{i,t}^{gap} \tag{20}$$

$$x_{i,t+1} = \rho_i x_{i,t} + \sigma_i^x \varepsilon_{i,t+1}^x, \tag{21}$$

where $i = g, \pi$, and $\varepsilon_{i,t+1}^{gap}$, $\varepsilon_{i,t+1}^x$ are $i.i.d.$ standard normal shocks.

The agent forms beliefs based on the same learning scheme in Section 2.1:

$$\begin{aligned}
\tilde{\mu}_{g,t} &= \tilde{\mu}_{g,t-1} + \upsilon_g^* \left(\Delta g_t^* - \tilde{\mu}_{g,t-1}\right) \\
\tilde{\mu}_{\pi,t} &= \tilde{\mu}_{\pi,t-1} + \upsilon_\pi^* \left(\pi_t^* - \tilde{\mu}_{\pi,t-1}\right) \\
\tilde{x}_{g,t} &= \rho_g \tilde{x}_{g,t-1} + \upsilon_g^{gap} \left(Gap_t^g - \rho_g \tilde{x}_{g,t-1}\right) \\
\tilde{x}_{\pi,t} &= \rho_\pi \tilde{x}_{\pi,t-1} + \upsilon_\pi^{gap} \left(Gap_t^\pi - \rho_\pi \tilde{x}_{\pi,t-1}\right),
\end{aligned}$$

where υ_g^*, υ_π^*, υ_g^{gap} and υ_π^{gap} are subjective learning gains linked to each component. The equilibrium nominal bond yields are linear functions of these four state variables. The impact of $\tilde{\mu}_{g,t}$ and $\tilde{\mu}_{\pi,t}$ on bond yields of all maturities is identical and therefore drives the low-frequency trend movements. The impact of $\tilde{x}_{g,t}$ and $\tilde{x}_{\pi,t}$ is stronger for short-term yields than for the long-term yields. Hence business cycle movements in yields, especially in short-term yields, are mainly attributed to these short-run beliefs.

2.4 Asset prices

We assume that the representative agent has the standard CRRA utility $U(C_t) = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$, where β stands for the agent's subjective discount factor, and γ is the risk-aversion coefficient. The log nominal pricing kernel implied from the CRRA utility is:

$$m_{t+1}^{\$} = \log \beta - \gamma \Delta g_{t+1} - \pi_{t+1}. \tag{22}$$

The equity *spot* yield for the $n-$period dividend strip (claim to the $n-$period ahead aggregate nominal dividend):

$$ey_t^{(n)} = \frac{1}{n}(d_t^{\$} - p_t^{(n)}), \tag{23}$$

where $p_t^{(n)}$ is the log strip price and $d_t^{\$}$ is the log nominal aggregate dividend. For $n = 1$, the time-t equilibrium price of the one-period dividend strip is:

$$P_t^{(1)} = \tilde{E}_t[M_{t+1}^{\$} D_{t+1}^{\$}], \tag{24}$$

where the conditional expectation is taken under subjective belief. Similarly, the price of $n-$period dividend strip is:

$$P_t^{(n)} = \tilde{E}_t[M_{t+1}^{\$} P_{t+1}^{(n-1)}]. \tag{25}$$

Solving the iterations forward, for the $n-$period equity spot yield we obtain:

$$ey_t^{(n)} = \frac{A_e^{(n)}}{n} - (\lambda - \gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + \frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1}), \tag{26}$$

with expression of $A_e^{(n)}$ and solution details given in Appendix A.

Meanwhile, the time-t price of $n-$period nominal discount bond satisfies the recursion:

$$P_{b,t}^{(n)} = \tilde{E}_t[M_{t+1}^{\$} P_{b,t+1}^{(n-1)}]. \tag{27}$$

We solve out the $n-$period nominal bond yield:

$$y_t^{(n)} = \frac{A_b^{(n)}}{n} + \gamma(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + (\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t}). \tag{28}$$

To better connect equity and nominal bond yields, we can write (26) as:

$$ey_t^{(n)} = \frac{A_e^{(n)} - A_b^{(n)}}{n} + y_t^{(n)} - \frac{1}{n}\tilde{E}_t \Delta d_{t+1:t+n}^{\$} = \theta_d^{(n)} + y_t^{(n)} - g_{d,t}^{\$,(n)}, \tag{29}$$

where $g_{d,t}^{\$,(n)} = \frac{1}{n}\tilde{E}_t \Delta d_{t+1:t+n}^{\$}$ is the subjective belief over the life-time nominal dividend growth for the $n-$period dividend strip. The equation follows Equation (4) in Van Binsbergen et al. (2013) by disentangling the equity yield into the (constant) risk premium ($\theta_d^{(n)}$), nominal bond yield ($y_t^{(n)}$), and nominal dividend growth ($g_{d,t}^{\$,(n)}$). Therefore, the time-variations in equity yields are entirely driven by subjective beliefs over real GDP and dividend growth.

3 Data, Estimation and Calibration

3.1 Data

We collect the term structure data of dividend strip yields from Giglio et al. (2021). Using a large cross-section of US stock returns, they estimate an affine model of equity prices and derive the strip yields for the aggregate market. Their method not only accurately replicates the dividend futures data used in recent studies such as Van Binsbergen et al. (2013); Van Binsbergen and Koijen (2017); Bansal et al. (2021), but also extends the length of data substantially.[8] A more extended sample creates an ideal laboratory for us to study the dynamics of the equity term structure over the business cycles. As for the bond term structure, we use the end-of-quarter zero-coupon nominal bond yields from Gürkaynak et al. (2007).

To construct the dividend series used for learning, we obtain firm-level quarterly dividends from the CRSP/Compustat Merged Database for all firms listed on NYSE, NASDAQ, and AMEX. Following De La O and Myers (2021) and Giglio et al. (2021), we focus on ordinary cash dividends. To implement our two-component dividend model (8), we consider firm-level long-term earnings growth median forecasts (LTG) as the benchmark measure of the equity duration, with the data available from the IBES unadjusted summary file.[9] La Porta (1996) and Gormsen and Lazarus (2022) show that such a model-free measure can be interpreted as the equity duration. Since equity duration is defined as the weighted sum of time with the weights given by expected cash-flows, higher long-term expected cash-flows relative to today naturally translate into higher duration.[10] Then, we calcu-

[8]Dividend futures data usually starts from 2003 and hence is not suitable for our study. Van Binsbergen et al. (2012) use option returns to assess the equity term structure, with data going back to 1996. However, option-based data is only available for short maturities up to two-year, and Boguth et al. (2019) find that noises from highly levered option positions may significantly contaminate the inference from option prices.

[9]While IBES data is available at the monthly frequency, we transform it to quarterly frequency by taking the end-of-quarter readings. Results from using the within-quarter average are almost identical.

[10]Existing measures of equity duration (see e.g., Dechow et al., 2004; Weber, 2018; Gonçalves, 2021b) require formal econometric modelling and estimation. We do not take a stand on such modelling issues and prefer to use the model-free duration. In Section 5 we run robustness checks using these measures, and results are quantitatively similar.

late the dividend from the long-duration sector as the following. At the end of each quarter, we assign all dividend-paying firms into either of the two groups based on firms' LTG forecasts in the previous quarter. If a firm's LTG is above or equal to the cross-sectional median of the LTG of all dividend-paying firms, then it is assigned to the long-duration group. Otherwise, it is allocated to the short-duration group. Within each quarter, we sum all dividends from the long-duration and short-duration sectors, respectively. We deflate the obtained two nominal dividend series using the GDP deflator and take a four-quarter trailing summation to remove their seasonality, following the usual practice. By construction, the sum of two dividend series will be the real aggregate dividend. The equity duration data is available from 1981Q3; hence the deseasonalized dividend series ranges from 1982Q4 to 2019Q4. We use the initial 5-year training period for agent's learning, and we start our empirical analysis from 1987Q4.

We use the data for subjective dividend growth to estimate the dividend learning gains, and we extend the 1-year aggregate expected dividend growth data constructed by De La O and Myers (2021) to 2019Q4 using the same empirical steps.[11] Similarly, to estimate learning gains for the real GDP growth and inflation, we rely on the consensus forecasts for 1-year real GDP growth and inflation from the Survey of Professional Forecasters (SPF). The data ranges from 1981Q3 to 2019Q4. Finally, we collect the data on real output growth and GDP deflator from the Bureau of Economic Analysis (BEA). The real personal consumption expenditure (PCE) and core inflation, i.e., the stable components of GDP and total inflation, are also obtained from the BEA. Since the learning gains on growth and inflation may be small (see e.g., Malmendier and Nagel, 2016; Nagel and Xu, 2022a), we need an extended training sample to form reasonable beliefs. Thus we allow the agent to learn these quantities using the data back to 1959Q1.

[11] Their original data spans the period from 2003Q1 to 2015Q3. Their data and our replicated series have a correlation coefficient of 0.92 over the same sample.

3.2 Parameter estimation and calibration

In Panel A of Table 1, we list values used for the standard parameters. The leverage parameter λ is set to 3, the risk aversion coefficient γ is set to 4, and the subjective discount factor β is calibrated to match the average level of the 10-year equity yields in the data. In Panel B, we estimate the persistence of latent states in the state-space models discussed in Section 2, using the maximum likelihood with the Kalman filter.[12] One exception is ρ_g, which we choose to calibrate from the data. The reason is that a persistent $x_{g,t}$ is crucial for our model to match the dynamics of bond yield spread (Zhao , 2020), yet it is challenging to estimate the persistent component directly from the output growth series (e.g., Schorfheide et al., 2018). Therefore, we calibrate ρ_g to maximize the correlation between the model-implied nominal bond yield spread (10-year minus 1-year) and the data counterpart. Finally, to estimate the subjective learning gains for the real GDP growth, inflation, and dividend, we minimize the root mean square errors (RMSE) between the 1-year consensus forecasts from the relevant survey data and the model-implied 1-year expectations, following e.g., Branch and Evans (2006); Cieslak and Povala (2015). Panel C of Table 1 reports the estimated gains. It shall be noted that we do not use any asset price data when determining the parameters governing the subjective belief dynamics.

In this paper, we are particularly interested in whether our model can match the full time-series of dividend expectation data. Figure 1 shows that the model-implied 1-year subjective dividend growth tracks its empirical counterpart well, and the unconditional correlation reaches 0.8. In the right plot, we note that even though we do not use 2-year survey dividend growth in the estimation, the model-implied quantity also closely matches the data. Hence, the estimation results support our model in capturing salient features of subjective dividend growth.

[12] Appendix A provides more details and also reports the volatility parameter estimates that will be used to calculate the constant term in the equity yields.

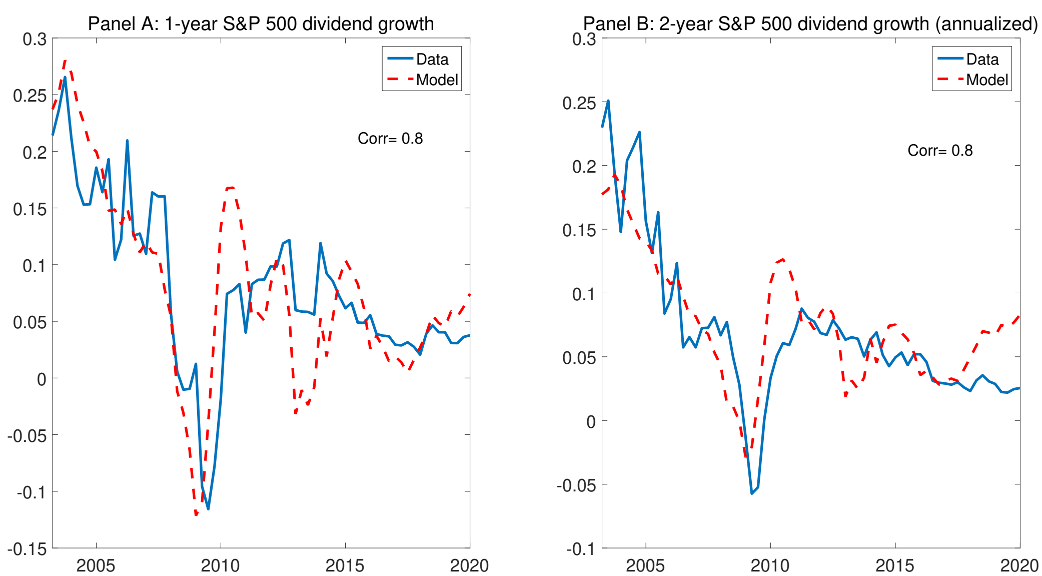
Table 1: Model parameters

The table reports model parameter values. Panel A reports the leverage parameter λ, risk-aversion coefficient γ, and subjective discount factor β. Panel B displays parameter estimates for the latent state persistence in the state-space models discussed in Section 2. Panel C reports the learning gains for real dividend level, real GDP growth, and inflation, all estimated from the corresponding 1-year consensus forecast data. The sample used for estimation ranges from 1981Q3 to 2019Q4 for the real GDP growth and inflation, and from 2003Q1 to 2019Q4 for the real dividend.

Panel A: Leverage and preference						
	λ	γ	β			
	3	4	1.007			
Panel B: Persistence of latent states						
	ρ_g	ρ_π	ρ_d			
	0.941	0.932	0.934			
Panel C: Subjective learning gains						
	ν_d^l	ν_d^s	ν_g^*	ν_g^{gap}	ν_π^*	ν_π^{gap}
	0.164	0.464	0.012	0.065	0.049	0.228

Figure 1: Fit of real dividend growth expectations

The figure plots the model-implied 1-year and 2-year aggregate real dividend growth expectations together with the data. The correlation coefficients are reported in the figure. The sample period is from 2003Q1 to 2019Q4.



Do the model-implied beliefs deviate from the rational expectation in a way consistent with the survey data? We run the test following Coibion and Gorodnichenko (2015) by regressing realized forecast errors on lagged forecast revisions. When the agent underreacts (overreacts) to news, she will insufficiently (overly) adjust the forecast upward after good news. Hence the forecast is likely to be followed by positive (negative) forecast errors, leading to a positive (negative) slope coefficient from the regression. Table 2 reports the regression results from the data and the model. We find that the agent underreacts to news regarding the real dividend level, GDP growth, and inflation. The slope coefficients are positive and (mostly) significant.[13] As argued in Section 2, underreaction is related to the dividend smoothing behaviour and the Fed's dual mandates. The agent chooses to put less weight on recent shocks to real dividend, inflation, and real GDP growth when updating beliefs if she expects that the firm and the Fed will aggressively stabilize future dividend, inflation, and growth.

Table 2: Rationality test of subjective beliefs

The table runs the rationality test of subjective beliefs following Coibion and Gorodnichenko (2015). For each quantity of interest x, the realized forecast errors are regressed on lagged subjective forecast revisions:

$$x_{t+n} - \tilde{E}_t(x_{t+n}) = \alpha + \beta[\tilde{E}_t x_{t+n} - \tilde{E}_{t-n} x_{t+n}] + \epsilon_{t+n}.$$

n is chosen to be 1-year for real dividend, and we run the regression (11) in Coibion and Gorodnichenko (2015) for real GDP growth and inflation. $\tilde{E}_t x_{t+n}$ denotes the subjective forecast from the survey data or implied from the model. The Newey-West t-statistics are reported in parentheses. The sample period for the real GDP growth and inflation is from 1987Q4 to 2019Q4, while that for the real dividend is from 2003Q1 to 2019Q4.

	Real dividend level		real GDP growth		Inflation	
	Data	Model	Data	Model	Data	Model
β	0.24	0.39	0.72	0.38	0.38	0.40
(t)	(2.57)	(2.81)	(1.91)	(1.99)	(1.35)	(2.76)

[13] Coibion and Gorodnichenko (2015) obtain positive and significant estimates for the inflation forecasts on the sample from 1969 to 2014, while our results are based on 1987 to 2019.

4 Empirical Results

This section explores whether our general equilibrium model accounts for leading asset pricing puzzles in equity and bond markets. Since the model nests Zhao (2020), it naturally accounts for key stylized facts in the bond markets. We thus focus more on the equity market by first studying the model implications for the equity term structure. We discuss whether the model generates time-varying bond-stock correlation and return predictability, as usually observed in the data. Finally, we assess model performance in explaining well-known puzzles in the aggregate stock market.

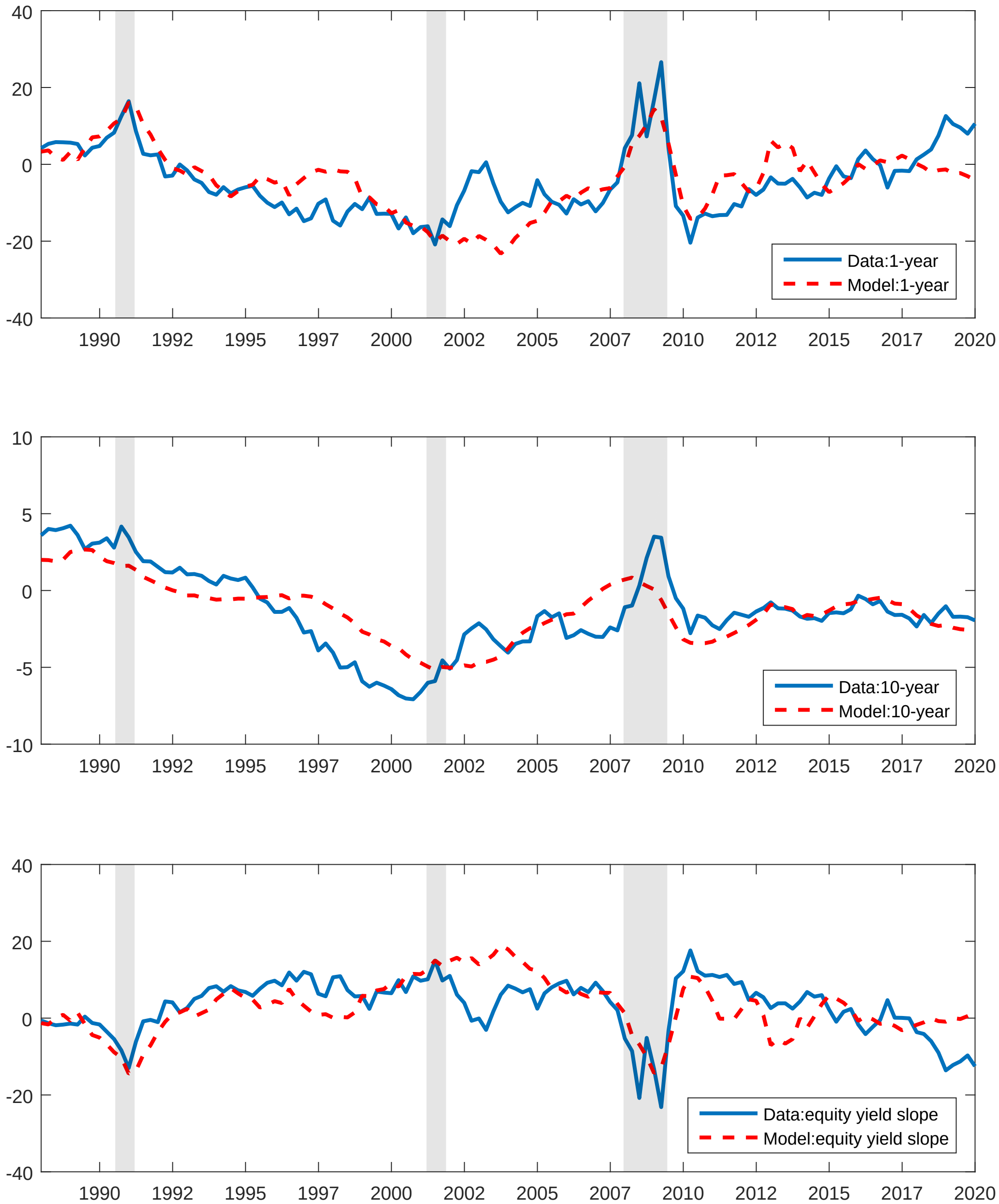
4.1 Term structure of equity yields and returns

4.1.1 Historical dynamics of equity term structure

We start with the business cycle dynamics of the equity term structure. Figure 2 plots the equity yields defined in (26) together with the data from Giglio et al. (2021). The model-implied yields closely track the movements in the data over the entire sample. Comparing the results for 1-year and 10-year, our model can match both the volatile 1-year equity yields and the less volatile 10-year yields. The model also generates a secular decline in equity yields since the late 1980s, followed by an upward trend post-2000, and replicates the equity yield spikes during the recession periods in the 1990s and around 2008. The last row of Figure 2 plots the slopes of the equity term structure, defined as the difference between 10-year and 1-year yields. The time-series plot suggests that model-implied slopes co-move tightly with the data.

Figure 2: Term structure of equity yields

The figures compare the model-implied term structure of equity yields with the data from Giglio et al. (2021). The last row plots the spread between 10-year and 1-year equity yields. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4, and all numbers are in annualized percentage terms.



Panel A of Table 3 reports their unconditional mean, volatility, and correlation. We find that the model-implied moments are close to the empirical counterparts and that the correlation is also high. For example, the 1-year (10-year) model-implied equity yields have a correlation coefficient of 0.68 (0.79) with the data, and the correlation between slopes is 0.59. Overall, the evidence favors the model in terms of fitting the term structure of equity yields.

Table 3: Summary statistics of equity yields

The first three rows of Panel A reports the unconditional mean and standard deviation of equity yields from both our model and data, and their correlation coefficients. Also in Panel A, we report moments of equity yields during expansion and recession periods, identified via the NBER business cycle dating. Panel B reports the decomposition of average slope of equity yields into the components described in Equation (30), i.e., components related to the constant, real GDP growth, and dividend-specific growth. All numbers are in annualized percentage terms, and the sample period is from 1987Q4 to 2019Q4.

Panel A: Moments		Data				Model		
		1Y	10Y	10Y-1Y		1Y	10Y	10Y-1Y
Unconditional	Mean	-4.39	-1.34	3.05		-4.18	-1.34	2.84
	Volatility	8.89	2.70	7.22		8.52	1.92	7.07
	Corr with data					0.68	0.79	0.59
Expansion	Mean	-5.44	-1.51	3.93		-4.94	-1.38	3.56
	Volatility	7.29	2.54	5.95		7.39	1.84	6.04
Recession	Mean	5.82	0.33	-5.50		3.28	-0.92	-4.20
	Volatility	15.25	3.65	12.10		14.22	2.65	11.72
Panel B: Slope decomposition		Const	RGDP	Div-spec	Total			
Expansion		1.05	0.04	2.47	3.56			
Recession		1.05	0.18	-5.43	-4.20			

4.1.2 Procyclical equity slope

The equity yield slope is usually found to be procyclical (see e.g., Van Binsbergen et al., 2013; Bansal et al., 2021); that is, during the recession, the slope is deeply negative, while in normal times, it can be positive. We evaluate whether the conditional moments of model-implied equity yields exhibit similar patterns in Panel A of Table 3. We find that the equity term structure is upward sloping during the expansion period, yet it becomes negative during the recession, with an average equity slope of –5.50%. Our model successfully generates such sign reversal, with an average slope of 3.56% (-4.20%) during the expansion (recession). The model-implied yields also display higher volatilities during the recession, in line with the data.

Our model captures the cyclicality of equity yield slopes in a way different from the previous literature. While most prior studies rely on the procyclical term structure of risk premia to reconcile this evidence (see e.g., Hasler and Marfe, 2016; Gonçalves, 2021a; Li and Xu, 2021; Breugem et al., 2022), their channels may not be coherent with recent survey-based evidence on the importance of cash-flow variations.[14] In contrast, the CRRA utility implies a negligible dividend risk premium in our model and equity yield movements are driven mostly by subjective dividend growth expectations. During recessions, growth expectations are exceptionally lower, with short-term expectation being much lower than its long-run counterpart; therefore, we observe sharp increases in equity yields and procyclical equity yield slopes.

Panel B of Table 3 further explores which factors contribute to the sign reversal of equity slopes by disentangling two economic forces in the equity yield:

$$ey_t^{(n)} = Const^{(n)} \underbrace{-(\lambda-\gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t})}_{RGDP_t^{(n)}} + \underbrace{\frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1})}_{Div-specific_t^{(n)}}. \tag{30}$$

The average term structure of equity yields can be decomposed into (the negative of) the term structure of subjective real GDP growth and real dividend-specific growth. Our results show that during the recession, short-maturity equity yield is higher mainly because the agent perceives lower real dividend-specific growth in the short-run compared to the long-run.

[14] In fact, Table 5 in Van Binsbergen et al. (2013) does find that the dividend growth expectation accounts for a substantial share of equity yield variations. Cassella et al. (2022) document that the term structure of biased beliefs over cash-flows may be empirically consistent with the equity term structure dynamics.

4.1.3 Returns on dividend claims

We then evaluate whether the model also generates reasonable dynamics for returns on dividend claims. Following Van Binsbergen and Koijen (2017), we study the $h-$period realized futures return of the dividend strip with $n-$period maturity, which can be computed as:

$$r^{(n)}_{F,t+1:t+h} = \Delta d^{\$}_{t+1:t+h} + ne y^{(n)}_{t} - (n-h)e y^{(n-h)}_{t+h} - r^{(n)}_{B,t+1:t+h}, \tag{31}$$

with $r^{(n)}_{B,t+1:t+h}$ as the $h-$period realized return of the nominal bond with $n-$period maturity. Under constant dividend risk premium, the time-varying futures returns are driven by time-varying subjective dividend growth.

Figure 3: Strip futures returns: data vs. model

The figure compares the model-implied futures returns of dividend strips with the data calculated from Giglio et al. (2021). Realized dividend growth is calculated from the S&P 500 aggregate dividend. We display results for 2-year and 10-year strip returns, and the sample period is from 1988Q4 to 2019Q4. All numbers are in annualized percentage terms.

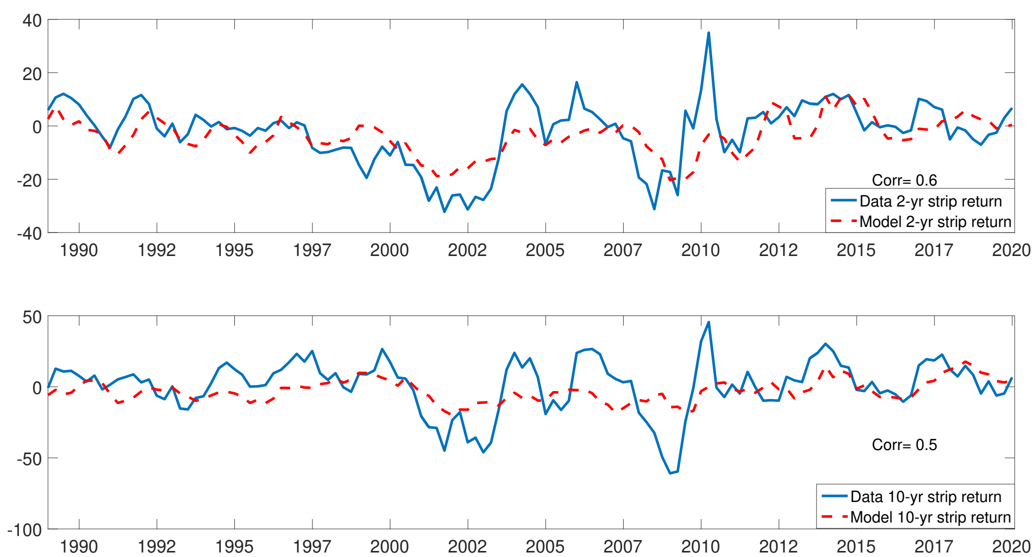


Figure 3 suggests that our model captures ex-post returns in the data reasonably well. The correlation of 2-year realized strip returns reaches 0.6, and that of the 10-year strip is still at a high level of 0.5. An important caveat is that the fit of equity yields in Figure 2 does not necessarily translate into the close fit of returns in Figure 3, as the latter further requires the model to capture the equity yield *changes* well.

4.2 Bond-stock comovement

This subsection addresses the puzzling behavior of bond-stock correlation. A stylized finding from the literature is that the correlation between bond and stock returns changed from positive to negative after 2000. Similar shifts are also observed in the correlation between equity yields and long-term nominal Treasury yields, the "Fed model."[15] We show that changing correlation between real GDP growth expectation and real dividend growth expectation can reconcile the puzzle. Before 2000, the real GDP growth expectation was weakly or negatively correlated with the real dividend growth expectation, suggesting that the real bond was not a hedge since it lost value together with the stock. However, the correlation became positive and sizeable after 2000; thus real bond became a hedge to the stock, leading to a negative bond-stock correlation.

4.2.1 Correlation changes between real GDP and dividend growth expectations

To understand the bond-stock correlation, from the model we have

$$y_t^{(n)} = \theta_b^{(n)} + r_t^{(n)} + \frac{1}{n}\tilde{E}_t \pi_{t+1:t+n}, \tag{32}$$

$$ey_t^{(n)} = \theta_d^{(n)} + r_t^{(n)} - g_{d,t}^{(n)}, \tag{33}$$

$$r_t^{(n)} = cte + \gamma g_t^{(n)}, \tag{34}$$

[15] See discussions on bond-stock return correlations in Baele et al. (2010); David and Veronesi (2013); Campbell et al. (2017, 2020); Kozak (2022); Li et al. (2022), and on the Fed-model in Asness (2003); Campbell and Vuolteenaho (2004); Bekaert and Engstrom (2010); Burkhardt and Hasseltoft (2012).

where $r_t^{(n)}$ is the real bond yield, $g_{d,t}^{(n)}$ is the real dividend growth expectation, $g_t^{(n)}$ is the real GDP growth expectation, $\theta_d^{(n)}$ is the (constant) dividend risk premium, and $\theta_b^{(n)}$ is the (constant) bond risk premium that includes the inflation and real bond risk premium, all at the n-period horizon. Then we can decompose the bond-stock covariance as

$$Cov(y_t^{(n)}, ey_t^{(n)}) = Var(r_t^{(n)}) + Cov(\frac{1}{n}\tilde{E}_t \pi_{t+1:t+n} + \theta_b^{(n)}, ey_t^{(n)}) - \gamma Cov(g_t^{(n)}, g_{d,t}^{(n)}). \tag{35}$$

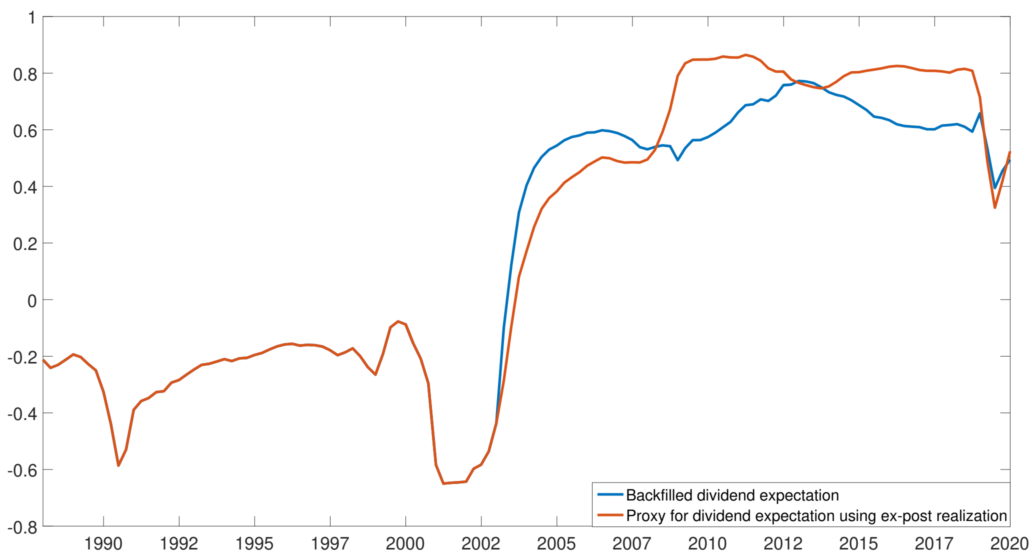
The first term is always positive. Previous literature often attributes sign reversals in bond-stock covariance to the sign changes in the second term, i.e., inflation (or inflation risks in bond yields) has changing correlation with equity yields. We instead study the importance of the final term, which represents a real channel for changes in bond-stock correlation.

To show this channel is empirically relevant, Figure 4 plots the rolling correlation between subjective real GDP growth expectation and subjective real dividend growth expectation of S&P 500 in the survey data, both at the one-year horizon.[16] We find that before 2000, the correlation is negative. Higher expected real GDP growth coincides with lower expected real dividend growth, so prices of the real bond and the stock both drop (Equation (34) and (33)). Hence the real bond is not a hedge to the stock before 2000. Strikingly, the correlation jumps to positive after 2000, implying that the real bond becomes a better hedge to the stock. Our model successfully replicates the switch in real correlation. The model-implied growth expectation of real GDP and real dividend has a correlation of -0.1 (0.7) before (after) 2000, compared with the correlation of -0.2 (0.6) in the data. In the following subsections, we quantitatively evaluate the importance of this real channel for generating sign-reversals in bond-stock correlation.

[16]The rolling window is set to 10 years. The subjective real GDP growth is the consensus forecast from the SPF. Since dividend expectation data were unavailable before 2003, we backfill the ex-ante expectation data using ex-post 1-year realized dividend growth. De La O and Myers (2021) show that these two series are close.

Figure 4: Time-varying correlation between expected real GDP growth and real dividend growth

The figure plots the 10-year rolling correlation between subjective real GDP growth and subjective real dividend growth of S&P 500 in the survey data. These forecasts are at the one-year horizon and obtained from the SPF and the IBES. Since dividend expectation data were unavailable before 2003, we backfill the ex-ante expectation data using the ex-post 1-year realized dividend growth when calculating the correlation (blue line). We also display the rolling correlation when we always use the ex-post realization as a proxy for ex-ante expectations (red line). By construction, these two lines coincide before 2003. We use the data before 1987 to display the rolling correlation from 1987Q4 to 2019Q4.



4.2.2 Correlation of equity and bond yields

Panel A of Table 4 reports the correlation between the yield of 10-year nominal bond and the dividend strips at various maturities. In both the data and the model, the bond-stock correlation turns negative after 2000.[17] As argued above, our model replicates this pattern via the real channel. We find that the correlation between real bond and equity yields also exhibits the same sign-reversals. For instance, the 10-year dividend strip has a correlation of 0.26 with the 10-year real bond before 2000, yet such correlation changes to -0.67 afterward.

[17] All our results are robust to other choices of breakpoints, such as 2001Q2 used by Campbell et al. (2020).

To further quantify the importance of the real channel, we use covariance decomposition (35)

$$Cov(y_t^{(N)}, ey_t^{(n)}) = \underbrace{Cov(r_t^{(N)}, r_t^{(n)})}_{real\ rate\ volatility} + \underbrace{Cov(\frac{1}{N}\tilde{E}_t\pi_{t+1:t+N} + \theta_b^{(n)}, ey_t^{(n)})}_{inflation\ real\ effect} - \underbrace{\gamma Cov(g_t^{(N)}, g_{d,t}^{(n)})}_{real\ growth\ cov}, \tag{36}$$

where $N = 10yr$, and we add a label to each component. The three components represent (1) real rate volatility; (2) inflation real effect, which we define as the covariance between expected inflation (and inflation risk premium) and real growth expectations; and (3) covariance between expected real GDP growth and real dividend growth. The sign-reversal we observe for the bond-stock correlation must stem from these three components. For component (2) we note that the prior literature entertains different models of correlation between $\theta_b^{(n)}$ and $\theta_d^{(n)}$ to reconcile the bond-stock return correlation. For instance, David and Veronesi (2013) rely on belief changes and money illusion; Song (2017) embeds procyclical inflation into the long-run risk model; and Campbell et al. (2020) connect inflation with time-varying risk-aversion. We take an orthogonal route because our model has constant risk premium. Thus component (2) reflects the correlation between expected inflation and expected real growth.

In Panel B of Table 4, we report the contribution of each component to the total bond-stock covariance *changes* across two subsamples. The decomposition results suggest that a dominant force driving the bond-stock correlation is indeed the comovements between expected real GDP growth and real dividend growth, which contributes to over 70% of the changes in bond-stock yield covariance after 2000. The explanatory share by the inflation real effect only accounts for around 40%.[18]

[18]The negative contribution from the real rate volatility reflects more volatile beliefs during the latter sample, primarily due to the global financial turmoil around 2008. Excluding the global financial crisis period, we obtain less volatile beliefs and hence a positive explanatory share.

Table 4: Bond-stock yield correlation and covariance decomposition

Panel A reports the correlation between the nominal yield of the 10-year bond and the dividend strip with maturity n. We also report the correlation between the real yield of the 10-year bond and dividend strips from the model. Panel B reports the decomposition results of bond-stock yield covariance (in percentage) based on (36). Results are computed based on the sample before 2000Q1 (1987Q4 to 1999Q4) and after 2000Q1 (2000Q1 to 2019Q4).

n		1Y	5Y	10Y
Panel A: Yield correlation				
Data	Before 2000	0.83	0.85	0.84
	After 2000	-0.47	-0.60	-0.62
Model	Before 2000	0.64	0.85	0.87
	After 2000	-0.62	-0.41	-0.31
Model (real)	Before 2000	0.39	0.29	0.26
	After 2000	-0.41	-0.62	-0.67
Panel B: Decomposition				
Real rate vol		-4.42%	-13.20%	-23.57%
Infl real effect		33.47%	41.19%	43.72%
Real growth cov		70.95%	72.01%	79.86%

4.2.3 Correlation of equity and bond returns

Now we analyze the bond-stock return correlation. The $h-$period return of a nominal bond and a dividend strip with $n-$period maturity are:

$$r^{(n)}_{B,t+1:t+h} = ny^{(n)}_t - (n-h)y^{(n-h)}_{t+h}, \tag{37}$$

$$r^{(n)}_{S,t+1:t+h} = ney^{(n)}_t - (n-h)ey^{(n-h)}_{t+h} + \Delta d^{\$}_{t+1:t+h}. \tag{38}$$

Equation (29) implies that the bond and dividend strip returns are linked:

$$r^{(n)}_{S,t+1:t+h} = r^{(n)}_{B,t+1:t+h} + (\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+1:t+n} + n\theta^{(n)}_d - (n-h)\theta^{(n-h)}_d. \tag{39}$$

Panel A of Table 5 reports the bond-stock return correlation of 10-year nominal bond and the dividend strips at various maturities. Following the literature, we set the holding period to be one year ($h = 4$). The results show that the bond-stock return correlation also turns negative after 2000 in both the data and the model. Furthermore, consistent with the real channel discussed above, the correlation between returns to 10-year real bond and dividend strips inherits the sign changes.

We can run a decomposition similar to (36) to quantify the contribution of the real channel. More explicitly, we extract nominal and real components from the long-term nominal bond return and the dividend strip return:

$$r_{B,t+1:t+h}^{(N)} = INFL_B^{(N)} + RGDP^{(N)}, \tag{40}$$

$$r_{S,t+1:t+h}^{(n)} = INFL_S^{(n)} + RGDP^{(n)} + RDIV^{(n)}, \tag{41}$$

with expressions for each term given in Appendix A (Equation (IA.22)). Absent risk premium variations, the bond and equity returns reflect variations in underlying real cash-flow and inflation expectations. Then, we can decompose the return covariance as:

$$\begin{aligned} Cov(r_{B,t+1:t+h}^{(N)}, r_{S,t+1:t+h}^{(n)}) = & \underbrace{Cov(INFL_B^{(N)}, RGDP^{(n)} + RDIV^{(n)}) + Cov(INFL_S^{(n)}, RGDP^{(N)})}_{Inflation\ real\ effect} \\ & + \underbrace{Cov(INFL_B^{(N)}, INFL_S^{(n)}) + Cov(RGDP^{(N)}, RGDP^{(n)})}_{Inflation\,\&\,real\ rate\ volatility} \\ & + \underbrace{Cov(RGDP^{(N)}, RDIV^{(n)})}_{real\ growth\ cov}. \end{aligned} \tag{42}$$

In Panel B of Table 5, we report the contribution of each component in (42) to the total bond-stock return covariance *changes*. We find that the inflation real effect now explains less of the changes in the bond-stock return covariance than the changes in yield covariance (around 20% versus around 40%, see Table 4). The key reason is that the persistent expected inflation, though it correlates strongly with the level of expected real growth, cannot move enough at high-frequency to explain the return covariance. Thus our results are consistent with recent findings documenting a modest impact of inflation on the bond-stock return correlation (Duffee 2018, 2021; Gomez-Cram and Yaron

2021). More importantly, we find that the covariance of expected *real* growth continues to play a significant role in driving return covariance. This real channel accounts for around 90% of the total covariance changes.

Table 5: Bond-stock return correlation and covariance decomposition

Panel A reports the correlation between returns to 10-year nominal bond and the dividend strip with maturity n. We also report the correlation between returns to 10-year real bond and dividend strips from the model. Panel B reports the decomposition results of bond-stock return covariance (in percentage) based on (42). Results are computed based on the sample before 2000Q1 (1988Q4 to 1999Q4) and after 2000Q1 (2000Q1 to 2019Q4).

n		1Y	5Y	10Y
Panel A: Return correlation				
Data	Before 2000	0.33	0.46	0.43
	After 2000	-0.53	-0.52	-0.51
Model	Before 2000	0.13	0.36	0.41
	After 2000	-0.56	-0.41	-0.17
Model (real)	Before 2000	0.13	0.37	0.42
	After 2000	-0.58	-0.40	-0.15
Panel B: Decomposition				
Infl and real rate vol		-5.40%	-14.15%	-21.52%
Infl real effect		19.13%	21.74%	23.66%
Real growth cov		86.26%	92.41%	97.86%

4.3 Return predictability

We now study the return predictability puzzle. There are two important stylized facts: equity market returns are predictable by lagged dividend-price ratios (e.g., Campbell and Shiller, 1988; Cochrane, 2011), and the strength of predictability decreases from short-term dividend claims to long-term claims (Van Binsbergen et al., 2012). We run two predictive regressions to evaluate whether our model is consistent with the evidence. The first regression is standard, and we predict the market excess returns using the log dividend-price ratio:

$$r_{M,t+1:t+h} - y_t^{(h)} = \alpha + \beta(d_t^{\$} - p_t) + \epsilon_{t+1:t+h}. \tag{43}$$

The second regression is on predicting the strip excess returns using its own lagged equity yields, following Van Binsbergen et al. (2012)

$$r^{(n)}_{S,t+1:t+h} - y^{(h)}_t = \alpha + \beta(d^{\$}_t - p^{(n)}_t) + \epsilon_{t+1:t+h}. \tag{44}$$

The holding period h is set to one year, and the risk-free rate is the h-period nominal bond yield $y^{(h)}_t$. Panel A of Table 6 reports the regression results from the data and the model. During the period from 1987Q4 to 2019Q4, the annual market excess returns are positively predicted by the lagged log dividend-price ratio in the data, with a $t-$statistic of 2.86 and R^2 of 10%. Our model regression generates similar R^2, and the slope coefficient is positive and significant. Moreover, our model reproduces the downward-sloping strength of strip return predictability, as in Van Binsbergen et al. (2012). For instance, in the data, 5-year strip excess returns are strongly predictable with R^2 of 24.7%, yet the R^2 decreases to 17.5% for the 10-year strip. The model-implied term structure of predictability is very similar and downward sloping, with the R^2 decreasing from 26.8% to 18.9%.

What is the source of return predictability and its term structure? To clarify the mechanism, we rewrite the excess return of the $n-$period dividend strip as:

$$r^{(n)}_{S,t+1:t+h} - y^{(h)}_t = Cte + \underbrace{rx^{(n)}_{B,t+1:t+h}}_{bond} + \underbrace{(\tilde{E}_{t+h} - \tilde{E}_t)d^{\$}_{t+n}}_{forecast\ revision}. \tag{45}$$

In our model, the realized strip excess return consists of two components: the maturity-matched realized bond excess return, and the forecast revision for the dividend at maturity. Equity yields that predict strip returns must predict either (or both) of components. Panel B of Table 6 reports the results of using equity yields to predict them, and we document different patterns for short- and long-term strips. For the 5-year strip, its bond component is weakly predictable by the 5-year equity yield, yet the predictability of dividend forecast revision is strong with an R^2 around 10%. However, the pattern reverses for the 10-year strip. The bond component is now significantly predictable with an R^2 of around 9%, yet the predictability of dividend forecast revision is weak. Therefore, the downward-sloping term structure of return predictability is attributed to the downward-sloping term structure of dividend forecast revision predictability. The results are consistent with the agent underreacting to dividend news, as we documented in Table 2. A positive shock to the current dividend (higher equity yield) does not push up too much the agent's dividend expectation today, leaving space for the subsequent dividend forecast revision to increase. Nevertheless, when n becomes larger, since the current news has smaller impact on longer-term dividend expectations, the predictability of dividend forecast revision will become weaker.

Table 6: Return predictability

Panel A reports the results of predictive regressions (43) and (44) in the data and in the model. The dividend strip returns are calculated from the data in Giglio et al. (2021), and the realized dividend growth is calculated from the S&P 500 aggregate dividend. Panel B reports the decomposition results of predictive regression (44) via (45). In brackets, we report the Newey-West t-statistics. The sample period is from 1988Q4 to 2019Q4.

Panel A: Return predictability					
		MKT	5Y	7Y	10Y
	β	0.17	0.34	0.28	0.24
Data	(t)	(2.86)	(5.01)	(4.57)	(4.20)
	$R^2(\%)$	10.29	24.73	20.84	17.50
	β	0.18	0.18	0.15	0.14
Model	(t)	(3.78)	(6.63)	(6.21)	(5.21)
	$R^2(\%)$	12.16	26.79	23.01	18.87
Panel B: Predictability of two return components					
	Bond		0.04	0.05	0.06
	(t)		(1.76)	(1.87)	(2.07)
	$R^2(\%)$		6.48	7.25	8.67
	Forecast revision		0.13	0.10	0.07
	(t)		(2.92)	(2.45)	(1.79)
	$R^2(\%)$		10.15	6.59	3.48

Recent literature exploits the predictable forecast errors (FE) of cash-flow growth to explain the stock return predictability (e.g., Bordalo et al., 2020b; Nagel and Xu, 2022a). To see the link with our proposed channel, we note that the forecast revision in Equation (45) can be rewritten as

$$(\tilde{E}_{t+h} - \tilde{E}_t)d^{\$}_{t+n} = \underbrace{\Delta d^{\$}_{t+1:t+h} - \tilde{E}_t \Delta d^{\$}_{t+1:t+h}}_{FE} + (\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+h+1:t+n}. \tag{46}$$

The prior literature focuses on the predictability of the first term, and it is unclear whether their channel is consistent with the downward-sloping term structure of return predictability. Our analysis extends the literature by showing that (1) predictable forecast error is only one source of return predictability and is embedded in the predictable dividend forecast revision; (2) biases in cash-flow expectations could reconcile downward-sloping term structure of return predictability.

The return predictability caused by underreaction to fundamental news is also consistent with the momentum literature, which shows that the momentum may be due to market's underreaction to cash-flow news (see e.g., Chan et al., 1996; Chordia and Shivakumar, 2006). In our equilibrium model, underreaction to dividend news leads to positively autocorrelated dividend forecast revision. Equation (45) implies that such underreaction also causes price momentum. Therefore, our channel fits well the evidence showing the tight relation between price momentum and fundamental momentum (Chordia and Shivakumar, 2006).

4.4 Puzzles about the aggregate stock market

In this subsection, we revisit several aggregate stock market puzzles via our equilibrium model. We model the aggregate market portfolio as the portfolio of dividend strips as:

$$P_t = \sum_{n=1}^{H_t} P_t^{(n)}, \tag{47}$$

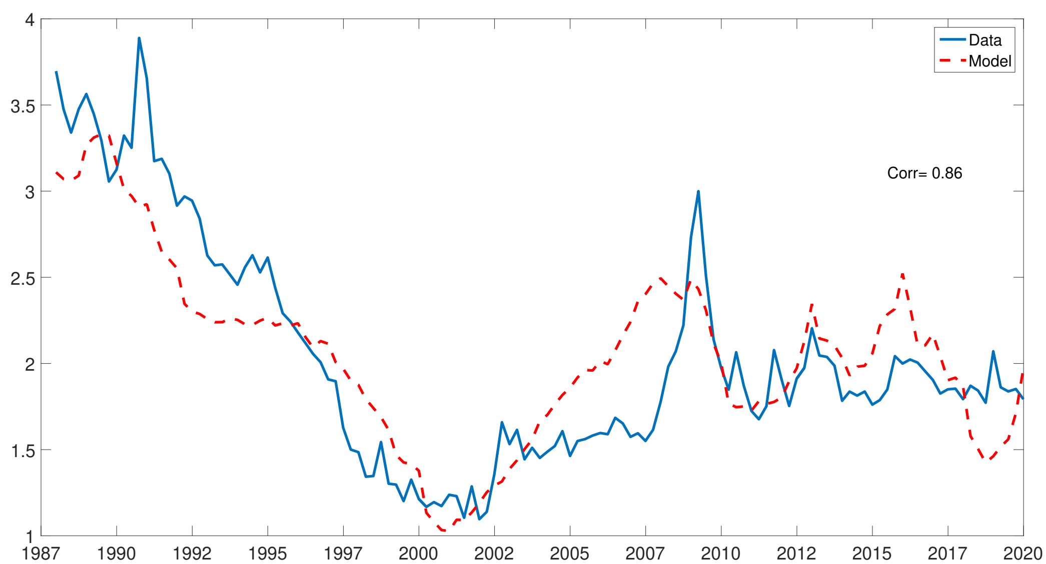
where $P_t^{(n)}$ is the price of the $n-$period dividend strip, and the time-varying H_t may be interpreted as the life expectancy of the aggregate portfolio. We do not manually set values for H_t, nor clarify the mechanisms for its time-variations (e.g., Fama and French, 2004; Chen, 2011). Instead, we adopt a simple approach by modeling it as a function of our equity duration measures for the aggregate market:

$$H_t = a + bLTG_t, \tag{48}$$

where LTG_t is calculated as the value-weighted average of long-term growth forecasts over all firms. The specification captures the time-varying cash-flow duration of the aggregate stock market in a parsimonious way. We can pin down parameters a and b by minimizing the RMSE between the aggregate price-dividend ratio in the data and the model. Although we use the data when estimating a and b, the time-variations of the model-implied aggregate price-dividend ratio are driven entirely by the strip yield variations and exogenous movements in aggregate equity duration.[19]

Figure 5: Aggregate dividend-price ratio: data vs. model

The figure compares the model-implied aggregate dividend-price ratio with the data. The model-implied quantity is obtained following the method in Section 4.4. The correlation coefficient between the model and data is reported in the plot. The sample period is from 1987Q4 to 2019Q4, and the numbers are in annualized percentage terms.



[19] Model performance is similar if we use a fixed cut-off level when calculating (47), following e.g., Van Binsbergen (2021).

Figure 5 shows that the model-implied aggregate dividend-price ratio is close to the data, with a correlation coefficient of 0.86. In Table 7, we find that they are equally persistent, with the AR(1) coefficients around 0.95. Meanwhile, the market log dividend-price ratio implied by the model has an annualized volatility of 26% and is close to 30% in the data. With the time-series of aggregate dividend-price ratio, we can also calculate the implied market returns. Our model generates an average market return of around 12%, replicating the high equity return (8%) in the data. Market returns are also volatile in our model, with an annualized volatility of 14%. Finally, we replicate sign switches of the correlation between long-term nominal bond returns and aggregate stock returns after 2000, with the correlation coefficient changing from 0.29 to -0.42.

Table 7: Moments for aggregate market

The table reports the moments of the aggregate stock market, including the annualized mean and volatility of market returns, volatility and AR(1) coefficient of market log dividend-price ratio, and the correlation between 10-year nominal bond returns and aggregate stock returns. We also report the correlation between the model and the data regarding the log dividend-price ratio and market returns. The sample period is from 1987Q4 to 2019Q4.

	$E(r_M)$	$\sigma(r_M)$	$\sigma(d-p)$	$\rho(d-p)$	$Corr(r_M, r_B \mid t < 2000)$	$Corr(r_M, r_B \mid t \geq 2000)$
Data	0.08	0.16	0.30	0.95	0.39	-0.64
Model	0.12	0.14	0.26	0.96	0.29	-0.42
$Corr(dp^{data}, dp^{model})$	0.86					

5 Robustness analysis

5.1 Comparison with existing models of dividend expectations

We compare the term structure of dividend expectations implied from the two-component model (8) with those implied from Nagel and Xu (2022a); De La O and Myers (2021). We also consider a variation of our model that omits two components in (8) and choose to learn directly from the aggregate dividend levels. After obtaining the dividend expectations from those alternative models, we calculate the model-implied equity forward yields by assuming the CRRA utility and compare them with the data.[20] Such an exercise would shed more light on the value added by our dividend model.

Table IA.3 displays their implications for the term structure of equity forward yields and the bond-stock yield correlation.[21] The left part reports the correlation between the implied term structure of equity *forward* yields from different models and the data. Our two-component model outperforms existing models in terms of explaining the equity term structure. Meanwhile, the right part of Table IA.3 suggests that existing models cannot explain sign-reversals in bond-stock correlation. They imply the positive bond-stock correlation throughout the whole sample.

5.2 Other measures of duration

The baseline measure of equity duration is the analyst forecast for long-term earnings growth. As a robustness check, we experiment with alternative duration measures proposed in the literature, including those discussed in Dechow et al. (2004); Weber (2018); Gonçalves (2021b). In addition, we consider the book-to-market ratio as a duration measure following Lettau and Wachter (2007). After constructing the dividend series sorted over these duration measures, Table IA.4 reports the moments of model-implied equity yields and their correlation with the data. Even if we use different measures of equity duration, the results show that the model successfully replicates key moments of the data, and the time-series correlation coefficients are also high. In a related exercise, while still using LTG as the duration measure, we change the construction of long-duration dividends by using the 40th or 60th cross-sectional percentile of LTG as the breakpoint. Results remain similar, as found in the last two rows of Table IA.4.

[20] It should be noted that the focus of Nagel and Xu (2022a); De La O and Myers (2021) is not to explain the equity term structure, but to understand other asset pricing facts. Appendix D.1 gives more details on the comparison.

[21] Since they do not model the bond market, we use bond yield data for calculating the model-implied equity spot yields and the bond-stock correlation.

6 Conclusion

Motivated by the finding that future returns but not future cash flows are predictable by current price-dividend ratios, research in macro-finance has, over the past three decades and within the rational expectations framework, been trying to come up with a force that moves prices but not expected future cash flows. This principle has guided equilibrium asset pricing literature and has given rise to the model of time-varying risk attitude (habit formation) or time-varying risks (long-run risk or disaster risk).

However, new empirical findings on subjective expectations, the term structure of bond and equity yields, and the correlation of stocks and bonds pose severe challenges to existing rational models. Subjective expectations of cash flows are found to be the most important drivers of equity and bond prices. In contrast, subjective return expectations are not as important as predicted by the rational models. Meanwhile, dividend and bond risk premia in the rational model cannot explain equity and bond yield spread movements observed in data. Furthermore, using the inflation risk premium to explain the change in stock-bond correlation implies too much inflation risk in equity returns.

We offer a unified bond and equity pricing framework consistent with these empirical findings. The movements of equity/bond yields are driven by subjective dividend/GDP growth expectations, and subjective risk premium is negligible. The model-implied long- and short-yields of dividend strips and bonds, and their spreads are close to the data (both time-series dynamics and moments). Long-term Treasury bonds switched from risky assets to safe assets after the late 1990s, mainly due to a shift in correlation between real GDP growth and real dividend growth expectations from negative to positive, and only partially driven by the procyclical inflation. Equity/strip returns are predictable, but the strength of predictability decreases from short-term to long-term claims due to predictable subjective forecast revisions. The channel is also consistent with the literature on equity momentum. Finally, our framework quantitatively matches several major aggregate stock market puzzles, such as the persistent and volatile price-dividend ratios and excess volatility of stock returns.

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For Online Publication

Internet Appendix for "Expectation-Driven Term Structure of Equity and Bond Yields"

A Model Solution Details

A.1 Derivation of rational and subjective beliefs

For the steady-state Kalman filter, the posterior (or the filtered) distribution for x_t after observing y_t is:

$$p(x_t|I_t) \propto \exp(-\frac{(y_t - Cx_t)^2}{2\sigma_\epsilon^2}) \times \exp(-\frac{(x_t - \rho E_{t-1}x_{t-1})^2}{2P}), \tag{IA.1}$$

with P the steady-state conditional variance of the predictive distribution under the Kalman filter. We then calculate the posterior mean for x_t:

$$E_t x_t = \rho E_{t-1}x_{t-1} + \frac{CP}{C^2P + \sigma_\epsilon^2}(y_t - \rho E_{t-1}x_{t-1}). \tag{IA.2}$$

Note that by definition, P solves:

$$P = \frac{\rho^2\sigma_\epsilon^2 P}{C^2P + \sigma_\epsilon^2} + \sigma_u^2. \tag{IA.3}$$

Then we solve for the *subjective* posterior distribution:

$$p(x_t|I_t) \propto \exp(-\frac{(1+\theta)(y_t - \tilde{C}x_t)^2}{2\sigma_\epsilon^2}) \times \exp(-\frac{(x_t - \rho \tilde{E}_{t-1}x_{t-1})^2}{2\tilde{P}}), \tag{IA.4}$$

with $\tilde{P}$ the steady-state conditional variance of the predictive distribution under the subjective learning. We then calculate the subjective posterior mean for x_t:

$$\tilde{E}_t x_t = \rho \tilde{E}_{t-1}x_{t-1} + \underbrace{\frac{(1+\theta)\tilde{C}\tilde{P}}{(1+\theta)\tilde{C}^2\tilde{P} + \sigma_\epsilon^2}}_{\nu}(y_t - \rho\tilde{E}_{t-1}x_{t-1}). \tag{IA.5}$$

Note that by definition, $\tilde{P}$ solves:

$$\tilde{P} = \frac{\rho^2 \sigma_\epsilon^2 \tilde{P}}{(1+\theta)\tilde{C}^2 \tilde{P} + \sigma_\epsilon^2} + \sigma_u^2. \tag{IA.6}$$

To formally show under-(over-) reaction when $\nu < K$ ($\nu > K$), we can derive the following relation between the expectation wedge and news:

$$\tilde{E}_t x_{t+1} - E_t x_{t+1} = \frac{\nu - K}{K} \underbrace{(E_t x_{t+1} - E_{t-1} x_{t+1})}_{news} + \rho(1-\nu)(\tilde{E}_{t-1} x_t - E_{t-1} x_t). \tag{IA.7}$$

It shall be noted that to clarify interpretation, our definition of news refers to the innovations to rational beliefs obtained from the Kalman filter (so that two terms on the right-hand side of are uncorrelated). In contrast, Coibion and Gorodnichenko (2015) define the news as innovations to subjective beliefs. We show that the two notions of news are highly correlated (see simulation results in Table IA.5). To derive the equation, from the Kalman filter we can write y_t as:

$$y_t = \frac{E_t x_{t+1} - \rho E_{t-1} x_t}{K} + \rho E_{t-1} x_t = \frac{E_t x_{t+1} - E_{t-1} x_{t+1}}{K} + E_{t-1} x_{t+1},$$

which is replaced into (6). After subtracting both sides by $E_t x_{t+1}$, we can obtain (IA.7).

A.2 Equilibrium prices and returns

A.2.1 Bond prices

We first derive the equilibrium nominal bond prices. Conjecture that the log price of $n-$period nominal bond follows:

$$p_{b,t}^{(n)} = -A_b^{(n)} - B_b^{(n)} \widetilde{\mu}_{g,t} - C_b^{(n)} \widetilde{x}_{g,t} - D_b^{(n)} \widetilde{\mu}_{\pi,t} - E_b^{(n)} \widetilde{x}_{\pi,t}. \tag{IA.8}$$

The pricing of one-period bond implies:

$$p_{b,t}^{(1)} = \tilde{E}_t m_{t+1}^{\$} + \frac{1}{2} \widetilde{Var}_t m_{t+1}^{\$}, \tag{IA.9}$$

from which we can solve out the coefficients:

$$
\begin{aligned}
A_b^{(1)} &= -\log\beta - \frac{1}{2}\gamma^2\tilde{\sigma}_g^2 - \frac{1}{2}\gamma^2(\tilde{\sigma}_g^{gap})^2 - \frac{1}{2}\tilde{\sigma}_\pi^2 - \frac{1}{2}(\tilde{\sigma}_\pi^{gap})^2 \\
B_b^{(1)} &= \gamma \\
C_b^{(1)} &= \gamma\rho_g \\
D_b^{(1)} &= 1 \\
E_b^{(1)} &= \rho_\pi, \qquad \text{(IA.10)}
\end{aligned}
$$

with

$$
\tilde{\sigma}_g^2 = \tilde{P}_{\mu g} + \sigma_g^2, (\tilde{\sigma}_g^{gap})^2 = \tilde{P}_{xg} + (\sigma_g^{gap})^2, \tilde{\sigma}_\pi^2 = \tilde{P}_{\mu\pi} + \sigma_\pi^2, (\tilde{\sigma}_\pi^{gap})^2 = \tilde{P}_{x\pi} + (\sigma_\pi^{gap})^2. \qquad \text{(IA.11)}
$$

Then, from the pricing of $n-$period nominal bond:

$$
p_{b,t}^{(n)} = \tilde{E}_t(m_{t+1}^{\$} + p_{b,t+1}^{(n-1)}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1}^{\$} + p_{b,t+1}^{(n-1)}), \qquad \text{(IA.12)}
$$

we can solve out the iteration for coefficients

$$
\begin{aligned}
A_b^{(n)} &= A_b^{(n-1)} - \log\beta - \frac{1}{2}(B_b^{(n-1)}v_g^* + \gamma)^2\tilde{\sigma}_g^2 - \frac{1}{2}(C_b^{(n-1)}v_g^{gap} + \gamma)^2(\tilde{\sigma}_g^{gap})^2 - \frac{1}{2}(D_b^{(n-1)}v_\pi^* + 1)^2\tilde{\sigma}_\pi^2 \\
&\quad - \frac{1}{2}(E_b^{(n-1)}v_\pi^{gap} + 1)^2(\tilde{\sigma}_\pi^{gap})^2 \\
B_b^{(n)} &= B_b^{(n-1)} + \gamma \\
C_b^{(n)} &= C_b^{(n-1)}\rho_g + \gamma\rho_g \\
D_b^{(n)} &= D_b^{(n-1)} + 1 \\
E_b^{(n)} &= E_b^{(n-1)}\rho_\pi + \rho_\pi. \qquad \text{(IA.13)}
\end{aligned}
$$

We have the following explicit formula for coefficients:

$$
B_b^{(n)} = n\gamma, C_b^{(n)} = \frac{\rho_g(1-\rho_g^n)}{1-\rho_g}\gamma, D_b^{(n)} = n, E_b^{(n)} = \frac{\rho_\pi(1-\rho_\pi^n)}{1-\rho_\pi} \qquad \text{(IA.14)}
$$

A.2.2 Dividend strip prices

Then we solve for the equilibrium dividend strip yield (log price-dividend ratio). Since it is in real terms, we conjecture that $p^{(n)}_{e,t} - d^{\$}_t$ has the following functional form:

$$p^{(n)}_{e,t} - d^{\$}_t = -A^{(n)}_e - B^{(n)}_e \widetilde{\mu}_{g,t} - C^{(n)}_e \widetilde{x}_{g,t} - D^{(n)}_e \widetilde{x}_{d,t} - E^{(n)}_e (d^l_t - \lambda y_t - \tilde{\mu}_{d,t-1}) - F^{(n)}_e (d^s_t - \rho_d \tilde{x}_{d,t-1}). \quad \text{(IA.15)}$$

For the one-period strip, its log price-dividend ratio follows:

$$p^{(1)}_{e,t} - d^{\$}_t = \tilde{E}_t(m_{t+1} + \Delta d_{t+1}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1} + \Delta d_{t+1}). \quad \text{(IA.16)}$$

We solve out the coefficients:

$$\begin{aligned}
A^{(1)}_e &= -\log\beta - \frac{1}{2}(\lambda-\gamma)^2\tilde{\sigma}^2_g - \frac{1}{2}(\lambda-\gamma)^2(\tilde{\sigma}^{gap}_g)^2 - \frac{1}{2}(\tilde{\sigma}^l_d)^2 - \frac{1}{2}(\tilde{\sigma}^s_d)^2 \\
B^{(1)}_e &= \gamma - \lambda \\
C^{(1)}_e &= (\gamma-\lambda)\rho_g \\
D^{(1)}_e &= 1 - \rho_d \\
E^{(1)}_e &= 1 - \nu^l_d \\
F^{(1)}_e &= 1 - \nu^s_d, \quad \text{(IA.17)}
\end{aligned}$$

with

$$(\tilde{\sigma}^l_d)^2 = \tilde{P}_{\mu d} + (\sigma^l_d)^2, (\tilde{\sigma}^s_d)^2 = \tilde{P}_{xd} + (\sigma^s_d)^2. \quad \text{(IA.18)}$$

Similarly, the $n-$period strip price solves:

$$p^{(n)}_{e,t} - d^{\$}_t = \tilde{E}_t(m_{t+1} + \Delta d_{t+1} + p^{(n-1)}_{e,t+1} - d^{\$}_{t+1}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1} + \Delta d_{t+1} + p^{(n-1)}_{e,t+1} - d^{\$}_{t+1}), \quad \text{(IA.19)}$$

from which we obtain

$$\begin{aligned}
A_e^{(n)} &= A_e^{(n-1)} - \log\beta - \frac{1}{2}(\lambda-\gamma-B_e^{(n-1)}v_g^*)^2\tilde{\sigma}_g^2 - \frac{1}{2}(\lambda-\gamma-C_e^{(n-1)}v_g^{gap})^2(\tilde{\sigma}_g^{gap})^2 - \frac{1}{2}(v_d^l)^2(\tilde{\sigma}_d^l)^2 \\
&\quad - \frac{1}{2}(v_d^s)^2(1-D_e^{(n-1)})^2(\tilde{\sigma}_d^s)^2 \qquad \text{(IA.20)} \\
B_e^{(n)} &= B_e^{(n-1)} + \gamma - \lambda \\
C_e^{(n)} &= C_e^{(n-1)}\rho_g + (\gamma-\lambda)\rho_g \\
D_e^{(n)} &= D_e^{(n-1)}\rho_d + 1 - \rho_d \\
E_e^{(n)} &= 1 - v_d^l \\
F_e^{(n)} &= 1 - v_d^s.
\end{aligned}$$

We thus have the following explicit formula for coefficients:

$$B_e^{(n)} = n(\gamma-\lambda), C_e^{(n)} = \frac{\rho_g(1-\rho_g^n)}{1-\rho_g}(\gamma-\lambda), D_e^{(n)} = 1-\rho_d^n, E_e^{(n)} = 1-v_d^l, F_e^{(n)} = 1-v_d^s. \qquad \text{(IA.21)}$$

According to (IA.13) and (IA.20), we need the following parameters to calculate the constant term for the equity and bond yields. Their estimates are obtained from the state-space model estimation.

$\tilde{\sigma}_g$	$\tilde{\sigma}_g^{gap}$	$\tilde{\sigma}_\pi$	$\tilde{\sigma}_\pi^{gap}$	$\tilde{\sigma}_d^l$	$\tilde{\sigma}_d^s$
0.011	0.006	0.004	0.002	0.142	0.065

Finally, to decompose bond and equity strip return as in (40) and (41), from the return definitions we have (ignoring the constant):

$$\begin{aligned}
r_{B,t+1:t+h}^{(N)} &= \underbrace{\tilde{E}_t\pi_{t+1:t+N} - \tilde{E}_{t+h}\pi_{t+h+1:t+N}}_{INFL_B^{(N)}} + \underbrace{\gamma(\tilde{E}_t\Delta g_{t+1:t+N} - \tilde{E}_{t+h}\Delta g_{t+h+1:t+N})}_{RGDP^{(N)}}, \\
r_{S,t+1:t+h}^{(n)} &= \underbrace{\pi_{t+1:t+h}}_{INFL_S^{(n)}} + \underbrace{\gamma(\tilde{E}_t\Delta g_{t+1:t+n} - \tilde{E}_{t+h}\Delta g_{t+h+1:t+n})}_{RGDP^{(n)}} + \underbrace{(\tilde{E}_{t+h} - \tilde{E}_t)\Delta d_{t+1:t+n}}_{RDIV^{(n)}}. \qquad \text{(IA.22)}
\end{aligned}$$

B Micro-founded models for (7)

We present a standard New Keynesian model where the agent perceives excessive controls on the inflation and the output gap by the central bank, and a dividend-smoothing model where the agent perceives excess smoothing behavior by the firm management. Both will make the magnitude of $\tilde{C}$ to be smaller than its rational benchmark C.

First, a standard three-equation New Keynesian model is given as the following:

$$\hat{y}_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + E_t\hat{y}_{t+1} + z_{u,t}, \tag{IA.23}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\hat{y}_t, \tag{IA.24}$$

$$i_t = \phi_\pi\pi_t + \phi_y\hat{y}_t + z_{i,t}, \tag{IA.25}$$

where $\hat{y}_t$ is the output gap, π_t is the inflation, and i_t is the nominal short rate. The Taylor rule coefficients ϕ_π and ϕ_y describe how the central bank reacts to inflation and output gap. We denote the demand shock as $z_{u,t}$ and the monetary shock as $z_{i,t}$. They follow the AR(1) process

$$z_{u,t+1} = \rho_u z_{u,t} + \epsilon_{u,t+1}, \tag{IA.26}$$

$$z_{i,t+1} = \rho_i z_{i,t} + \epsilon_{i,t+1}. \tag{IA.27}$$

Conjecture the following solutions of endogenous variables as functions of exogenous variables

$$\pi_t = C_{\pi u} z_{u,t} + C_{\pi i} z_{i,t}, \tag{IA.28}$$

$$\hat{y}_t = C_{yu} z_{u,t} + C_{yi} z_{i,t}. \tag{IA.29}$$

Replacing them into the New Keynesian model yields the solution for coefficients

$$\begin{aligned}
C_{\pi u} &= \frac{\kappa}{\frac{\kappa(\phi_\pi - \rho_u)}{\sigma} + (1-\beta\rho_u)(1 + \frac{\phi_y}{\sigma} - \rho_u)} \\
C_{yu} &= \frac{1}{1 + \frac{\phi_y}{\sigma} - \rho_u + \frac{\kappa(\phi_\pi - \rho_u)}{\sigma(1-\beta\rho_u)}} \\
C_{\pi i} &= \frac{-\kappa/\sigma}{\frac{\kappa(\phi_\pi - \rho_i)}{\sigma} + (1-\beta\rho_i)(1 + \frac{\phi_y}{\sigma} - \rho_i)} \\
C_{yi} &= -\frac{1/\sigma}{1 + \frac{\phi_y}{\sigma} - \rho_i + \frac{\kappa(\phi_\pi - \rho_i)}{\sigma(1-\beta\rho_i)}}.
\end{aligned} \tag{IA.30}$$

By attaching a measurement error to inflation, we arrive at the state-space representation in the main text

$$\pi_t = Cx_t + \sigma_\epsilon\epsilon_t, \tag{IA.31}$$

$$x_{t+1} = \rho x_t + \sigma_u u_{t+1}, \tag{IA.32}$$

with $x_t = [z_{u,t}, z_{i,t}]'$, $C = [C_{\pi u}, C_{\pi i}]$. Similar representation can also be obtained for the output gap.

In this system, the agent learns demand and monetary shocks from the observation, but we assume that the agent perceives $\tilde{\phi}_\pi > \phi_\pi$, with ϕ_π the coefficient under the rational measure. That is, the agent overly estimates the strength of the inflation stabilization by the central bank. Since all parameters in C will shrink towards zero as ϕ_π increases, we thus obtain lower C (in absolute terms) relative to the rational benchmark. Similarly, if the agent perceives $\tilde{\phi}_y > \phi_y$, we obtain the same prediction.

Second, we consider the dividend smoothing model proposed by Lintner (1956)

$$D_{t+1} = \alpha + C \times D^*_{t+1} + (1-C) \times D_t + \epsilon_{t+1}, \tag{IA.33}$$

$$D^*_{t+1} = TP \times E_{t+1}, \tag{IA.34}$$

$$E_{t+1} = \bar{E} + \rho(E_t - \bar{E}) + \eta_{t+1}, \tag{IA.35}$$

where D^*_{t+1} is the target dividend level and is equal to target payout ratio (TP) times the earnings E_{t+1}. To form the state-space model, we have added the state equation (IA.35) that describes the earnings process. For this system, parameter C governs how the firm management adjusts the dividend facing earnings shocks. When the agent perceives smoother dividend relative to the rational benchmark, C will be lower and the dividend is less responsive to earnings shock under the subjective measure.

C Properties of dividends from long-duration portfolios

We provide empirical justification for the novel two-component model (8). To show that long-duration dividends are more related to the real GDP level, in the data we estimate the following regression for log dividends from short or long-duration portfolios:

$$\log D^i_t = \alpha_{0i} + \alpha_{1i} t + \beta_i \log(RGDP), \qquad i = \{S, L\}. \tag{IA.36}$$

As a comparison, we also estimate the above regression without the time trend; that is, we estimate the cointegrating relation between the level series.

Table IA.1 reports the results. After removing the deterministic trend in the level data, we find that short-duration dividends are negatively related to the real GDP, with a t-statistic of -4.87. In contrast, the long-duration dividend significantly and positively correlates to the real GDP. The high R^2 indicates that the real GDP can explain 90% of the fluctuations in long-duration dividends. The outcome does not change when we ignore the time trend. Although the short-duration dividend now positively relates to the real GDP, the R^2 is only 42%, suggesting that more than half of the fluctuations in the short-duration dividend are not related to the real GDP. Notably, the estimates for long-duration dividends are still positive, and the R^2 is close to 90%.

Table IA.1: Relation between the real GDP and the dividends from short- and long-duration portfolios

	with time trend		without time trend	
	Short-duration	Long-duration	Short-duration	Long-duration
β	-2.08	0.96	1.05	2.24
(t)	(-4.87)	(2.86)	(10.21)	(31.06)
$R^2(\%)$	57.8	88.0	41.5	86.8

Table IA.2 lists the average sector composition for short- and long-duration portfolios, where the sector classification follows the Global Industry Classification Standard (GICS) of S&P and MSCI. On average, the long-duration portfolio is mainly populated by stocks from the consumption and technology sectors—they account for around 60% of the stocks in the long-duration portfolio.

Table IA.2: Sector composition for short- and long-duration portfolios

Short-duration		Long-duration	
Financials	28.7%	Consumer discretionary	21.6%
Utilities	17.3%	Consumer staples	19.5%
Industrials	11.9%	Technology	18.4%
Materials	9.4%	Energy	10.8%
Consumer discretionary	9.0%	Financials	7.1%
Consumer staples	6.7%	Health care	7.0%
Technology	5.4%	Materials	6.6%
Energy	4.9%	Industrials	4.4%
Others	4.4%	Utilities	3.6%
Health care	2.3%	Others	1.0%

D Additional Results

D.1 Comparison with existing models

We compare the term structure of dividend expectations implied from our model with those implied from the following learning framework.

First, we consider the constant-gain learning as in Nagel and Xu (2022) (their Equation (1)):

$$\Delta d_t = \mu_d + \epsilon_t. \tag{IA.37}$$

The agent learns about the mean growth rate μ_d according to

$$\tilde{\mu}_{d,t+1} = \tilde{\mu}_{d,t} + 0.018 \times (\Delta d_{t+1} - \tilde{\mu}_{d,t}). \tag{IA.38}$$

The term structure of dividend expectation is flat with

$$\tilde{E}_t \Delta d_{t+j} = \tilde{\mu}_{d,t}, \forall j. \tag{IA.39}$$

The second framework is the dividend-earning model as in De La O and Myers (2021)

$$e_{t+1} = x_t + \epsilon_{t+1} \tag{IA.40}$$

$$x_{t+1} = \mu + x_t + (1-\theta)\epsilon_{t+1} \tag{IA.41}$$

$$d_{t+1} = (1-\omega)e_{t+1} + \omega d_t + \epsilon^d_{t+1}, \tag{IA.42}$$

where e_{t+1} (d_{t+1}) is the log aggregate earning (dividend). The model-implied term structure of dividend growth is

$$\tilde{E}_t \Delta d_{t+j} = \omega^{j-1}(1-\omega)(-\theta\epsilon_t - (d_t - e_t)). \tag{IA.43}$$

De La O and Myers (2021) use $\omega = 0.66$ and $\theta = 0.6$ when inferring the expected dividend growth.

Finally, we consider a variation of our two-component model (8). We drop two components but similar to (9) and (10), we use the following framework

$$d_t - \lambda y_t = \mu_{d,t} + \sigma_d \epsilon_{d,t}, \tag{IA.44}$$

$$\mu_{d,t+1} = \mu_{d,t} + \sigma^\mu_d \epsilon^\mu_{d,t+1}, \tag{IA.45}$$

where d_t now is the log aggregate dividend. We apply the same learning rule as in Section 2.2 but omit the information from the cross-section.

We still assume the CRRA utility when calculating the asset prices. The left part of Table IA.3 reports the correlation between the implied term structure of equity *forward* yields from different models and the data. We find that our two-component model outperforms existing models in terms of explaining the equity term structure. In particular, we find that omitting the cross-sectional information will substantially worsen the model performance even in our framework, suggesting the usefulness of separating total dividend into two components. The right part of Table IA.3 reports the correlation between the 10-year nominal bond yield in the data and the model-implied spot yield of dividend strips. Note that since those papers do not model the bond market, we directly input the bond yield data when calculating the spot yield and the correlation. Consistent with their performance of explaining equity term structure, existing models cannot generate realistic sign-reversals in bond-stock correlation. They actually imply a positive bond-stock correlation throughout the whole sample.

Table IA.3: Comparison with existing learning models

The table compares the performance of our model with existing learning models. We consider learning about the mean in Nagel and Xu (2022), the dividend-earning model in De La O and Myers (2021), and one modification of our model that ignores the two components in the total dividend. The table reports the correlation between the implied term structure of equity *forward* yields from different models and the data. The table also reports the correlation between the 10-year nominal bond yield in the data and the model-implied spot yield of dividend strips. The sample period is from 1987Q4 to 2019Q4.

	Corr with equity fwd yields			Bond-stock corr (before 2000)			Bond-stock corr (after 2000)		
Maturity	1Y	10Y	10Y-1Y	1Y	5Y	10Y	1Y	5Y	10Y
Data				0.83	0.85	0.84	-0.47	-0.60	-0.62
Learning about the mean (NX2022)	-0.53	-0.41	N/A	0.59	0.89	0.95	0.91	0.95	0.90
Dividend-earning model (DM2021)	0.47	0.46	0.42	0.64	0.74	0.81	-0.09	0.14	0.41
Ignoring two components	0.17	0.53	-0.01	0.52	0.91	0.92	0.53	0.79	0.87
Two-component model	0.66	0.77	0.59	0.68	0.88	0.91	-0.54	-0.31	-0.12

D.2 Robustness analysis using alternative duration measures

Table IA.4: Robustness: alternative decomposition of aggregate dividends

The table reports the unconditional mean and standard deviation of equity yields from both our model and data, and their correlation coefficients. For the model-implied quantities, we change our way of decomposing aggregate dividend in (8) based on different measures of equity duration. These include the measures proposed by Weber (2018), Gonçalves (2021a), and the book-to-market ratio in Lettau and Wachter (2007). Alternatively, when decomposing dividend using our baseline measure of duration (LTG), we change the breakpoint to the 40th or 60th cross-sectional percentile. The sample period is from 1987Q4 to 2019Q4.

		1Y	10Y	10Y-1Y
Data	Mean	-4.39	-1.34	3.05
	Volatility	8.89	2.70	7.22
	Mean	-3.13	-1.33	1.79
Weber (2018)	Volatility	10.35	2.68	8.09
	Corr.	0.59	0.82	0.43
	Mean	-6.67	-1.34	5.34
Gonçalves (2020a)	Volatility	10.73	1.84	9.00
	Corr.	0.62	0.71	0.50
	Mean	-4.02	-1.34	2.68
Book-to-market	Volatility	8.70	1.86	7.10
	Corr.	0.62	0.85	0.47
	Mean	-5.33	-1.34	3.98
40th	Volatility	7.79	1.49	6.54
	Corr.	0.64	0.71	0.55
	Mean	-4.95	-1.34	3.61
60th	Volatility	10.25	2.05	8.73
	Corr.	0.63	0.72	0.56

Table IA.5: Correlation between subjective and rational news

The table reports the average correlation between rational news defined in (IA.7) and the subjective news when learning gain is ν instead of the Kalman gain K. For each path, we simulate both the rational news and subjective news under the specified learning gains in the table, with the sample length identical to our sample period from 1987Q4 to 2019Q4 (129 observations). The simulation is repeated 100,000 times, and reported numbers are the average correlation over all simulated paths.

		Panel A: $\rho = 0.9$								
		Kalman gain K								
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
	0.01	1	0.99	0.99	0.96	0.91	0.82	0.72	0.63	0.56
	0.05	0.99	1	0.99	0.98	0.94	0.87	0.78	0.70	0.64
	0.1	0.99	0.99	1	0.99	0.97	0.92	0.85	0.77	0.71
	0.2	0.96	0.98	0.99	1	0.99	0.97	0.93	0.87	0.82
Subjective	0.3	0.91	0.94	0.97	0.99	1	0.99	0.97	0.94	0.89
gain ν	0.4	0.82	0.87	0.92	0.97	0.99	1	0.99	0.97	0.94
	0.5	0.72	0.78	0.85	0.93	0.97	0.99	1	0.99	0.98
	0.6	0.63	0.70	0.77	0.87	0.94	0.97	0.99	1	0.99
	0.7	0.56	0.64	0.71	0.82	0.89	0.94	0.98	0.99	1
		Panel B: $\rho = 1$								
		Kalman gain K								
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
	0.01	1.00	0.99	0.98	0.95	0.86	0.70	0.53	0.38	0.30
	0.05	0.99	1.00	0.99	0.97	0.93	0.83	0.69	0.56	0.47
	0.1	0.98	0.99	1.00	0.99	0.96	0.90	0.80	0.69	0.60
	0.2	0.95	0.97	0.99	1.00	0.99	0.97	0.92	0.84	0.77
Subjective	0.3	0.86	0.93	0.96	0.99	1.00	0.99	0.97	0.93	0.87
gain ν	0.4	0.70	0.83	0.90	0.97	0.99	1.00	0.99	0.97	0.94
	0.5	0.53	0.69	0.80	0.92	0.97	0.99	1.00	0.99	0.97
	0.6	0.38	0.56	0.69	0.84	0.93	0.97	0.99	1.00	0.99
	0.7	0.30	0.47	0.60	0.77	0.87	0.94	0.97	0.99	1.00

E Model with Ambiguity

E.1 Equilibrium bond and equity yields

We extend the analysis to consider the agent's fear over model misspecification of the real GDP growth and dividend growth. We show that the extended model better explains the dynamics of bond and equity yields relative to the benchmark model in Section 2. To begin with, we assume that the representative agent has a recursive multiple-priors preference (see e.g., Epstein and Schneider , 2003).

$$V_t(C_t) = \min_{p_t \in \mathscr{P}_t} \mathbb{E}^{p_t}[U(C_t) + \beta V_{t+1}(C_{t+1})], \quad \text{(IA.46)}$$

where $\mathscr{P}_t$ denotes the set of alternative models (probability measures) and the CRRA utility function $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$. The agent is ambiguous about real endowment growth and dividend growth. The set of alternative measures is generated by different mean growth rates around respective reference mean values. We assume that the reference model is the posterior distribution obtained from agent's learning over the real GDP growth and real dividend, as discussed in Subsection 2.2 and 2.3, but the agent evaluates future prospects under the *worst-case* measure. More explicitly, the agent will select the lowest real GDP growth and dividend growth forecasts when pricing assets.[1] Hence, the worst-case beliefs over the real GDP and dividend growth are:

$$\begin{aligned}
\tilde{E}_t \Delta g_{t+1} &= \tilde{\mu}_{g,t} + \rho_g \tilde{x}_{g,t} - a_{g,t} \\
\tilde{E}_t \Delta d_{t+1} &= \lambda(\tilde{\mu}_{g,t} + \rho_g \tilde{x}_{g,t} - a_{g,t}) + (\rho_d - 1)\tilde{x}_{d,t} + (\nu_d^s - 1)(d_t^s - \rho_d \tilde{x}_{d,t-1}) + (\nu_d^l - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - a_{d,t},
\end{aligned} \quad \text{(IA.47)}$$

where $a_{g,t}$ denotes the ambiguity over real endowment growth. The ambiguity over total real dividend growth consists of two parts arising from distorting real endowment growth ($\lambda a_{g,t}$) and dividend-specific growth ($a_{d,t}$). We assume that they follow the standard AR(1) processes:

$$\begin{aligned}
a_{g,t+1} &= \mu_{ag} + \rho_{ag} a_{g,t} + \sigma_{ag} \epsilon_{ag,t+1} \\
a_{d,t+1} &= \mu_{ad} + \rho_{ad} a_{d,t} + \sigma_{ad} \epsilon_{ad,t+1},
\end{aligned} \quad \text{(IA.48)}$$

with $i.i.d.$ standard normal shocks $\varepsilon_{ag,t+1}$, $\varepsilon_{ad,t+1}$. The equilibrium $n-$period bond and equity yields

[1] The worst-case distortion for dividend growth rests on the assumption that dividend shocks are positively correlated with endowment shocks. However, when learning from past data, we do not ask the agent to consider such correlation. This assumption greatly simplifies our analysis because it avoids additional parameters that are hard to pin down under correlated learning. Croce et al. (2015) consider a similar setting where the agent ignores some shock correlations when pricing assets.

are then solved out as:

$$y_t^{(n)} = \frac{A_b^{(n)}}{n} + \gamma(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + (\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t}) - \frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}\gamma a_{g,t}, \tag{IA.49}$$

$$\begin{aligned} ey_t^{(n)} = {} & \frac{A_e^{(n)}}{n} - (\lambda-\gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + \frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) \\ & - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1}) + \frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}(\lambda-\gamma)a_{g,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t}, \end{aligned} \tag{IA.50}$$

with constant $A_b^{(n)}$ and $A_e^{(n)}$ given by iterations similar to those in Subsection A.2.

E.2 Ambiguity parameters

We use standard empirical measures for the ambiguity to calculate related parameters and equilibrium yields. First, the ambiguity over real GDP growth is constructed from the Survey of Professional Forecasters (SPF). For each quarter, we calculate the ambiguity by dividing the interquartile range of forecasts for the next year real GDP growth by two (see also Drechsler , 2013; Ilut and Schneider , 2014; Zhao , 2017). Second, a new variable that we need to obtain from the survey data is the dividend-specific ambiguity $a_{d,t}$. From Equation (IA.47), as long as the ambiguity over aggregate real dividend growth is empirically available, we can back out $a_{d,t}$ after removing the real GDP ambiguity part. To achieve this, we resort to firm-level earnings survey data from the IBES. Given that the IBES summary file does not provide the upper and lower quartiles of analyst forecasts for each firm, we retrieve them from the IBES unadjusted detail file.[2] For each firm and quarter, we collect individual analyst forecasts of future earnings per share (EPS) for multiple forecasting horizons. For each forecasting horizon, we obtain the upper and lower quartiles of analyst forecasts, and then we apply linear interpolations to obtain the forecasts at the 1-year horizon. After multiplying those interpolated forecasts with the shares outstanding in each quarter and aggregate over all stocks, we obtain the 25th and 75th percentiles of predicted 1-year-ahead earnings levels for the aggregate market. Ambiguity over aggregate cash-flows is then calculated as one-half of the log difference between these quartiles.

In spite of using earnings survey data when estimating dividend ambiguity, we show empirically that the obtained measure is sensible. Figure IA.1 displays reasonable time-variations in our

[2] We do not use the dividend forecast in the IBES detail file when constructing the ambiguity measure, primarily because the dividend forecast is only available after 2003 and this will shorten the period for our analysis substantially. Also, the average number of analysts providing dividend estimates in the IBES detail file is much smaller than that for the earnings, which may yield inaccurate measure.

ambiguity measure within the sample, with a correlation of 0.62 with the ambiguity over real GDP growth. Meanwhile, Ilut and Schneider (2014) suggest that valid empirical measure of ambiguity should not exceed twice the volatility of the forecasted time-series itself (see their Section III.B). In compliance with their ambiguity bound, we find that the sample average of ambiguity over annual real dividend growth is around 4% while the volatility of realized annual real dividend growth is around 7%. Finally, with the empirical measures in hand, we obtain ambiguity parameters by matching the simulated moments with the mean, volatility, and AR(1) coefficients from the ambiguity data. Table IA.6 displays the parameters.

Figure IA.1: Ambiguity over aggregate dividend growth

The figure plots the annualized ambiguity over 1-year-ahead aggregate dividend growth. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4.

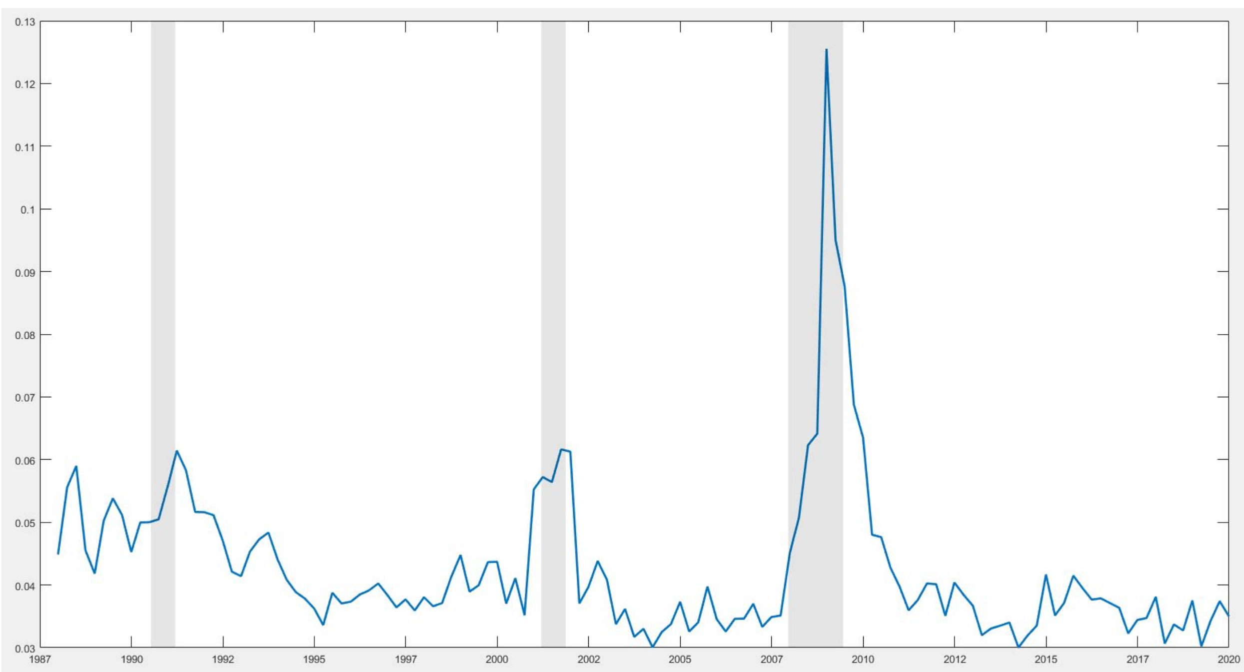


Table IA.6: Parameters for ambiguity processes

μ_{ag}	ρ_{ag}	σ_{ag}	μ_{ad}	ρ_{ad}	σ_{ad}
0.0004	0.65	0.05	0.0002	0.978	0.09

E.3 Asset pricing implications

We then discuss the asset pricing performance from the extended model. First, Table IA.7 reports the model-implied bond yield statistics, and they closely match the data. Second, the left panel of Figure IA.2 displays the time-series fit for the term structure of equity yields. Comparing this with Figure 2, we see that introducing the ambiguity helps improve the model's explanatory power for equity yields, especially during the 2008 global financial crisis. Table IA.8 summarizes the moments for data and model-implied quantities. The correlation coefficients between the data and the model indeed increase relative to the benchmark model. Furthermore, the right panel of Figure IA.2 plots the time-series fit for the term structure of equity *forward* yields, defined as the difference between equity and nominal bond yields. The model explains well both the level and variability of equity forward yields, as can be confirmed in the right panel of Table IA.8.

Table IA.7: Term structure of nominal bond yields: data vs. model

The table reports the mean and standard deviation of nominal bond yields. These numbers are in annualized percentage terms. We report statistics from both our model and the data, and also their correlation coefficients. The sample period is from 1987Q4 to 2019Q4.

		1Y	2Y	5Y	7Y	10Y
Data	Mean	3.40	3.63	4.25	4.57	4.92
	Volatility	2.59	2.56	2.35	2.24	2.14
Model	Mean	4.84	4.89	4.95	4.94	4.91
	Volatility	1.77	1.71	1.62	1.58	1.56
	Corr.	0.89	0.92	0.94	0.95	0.95

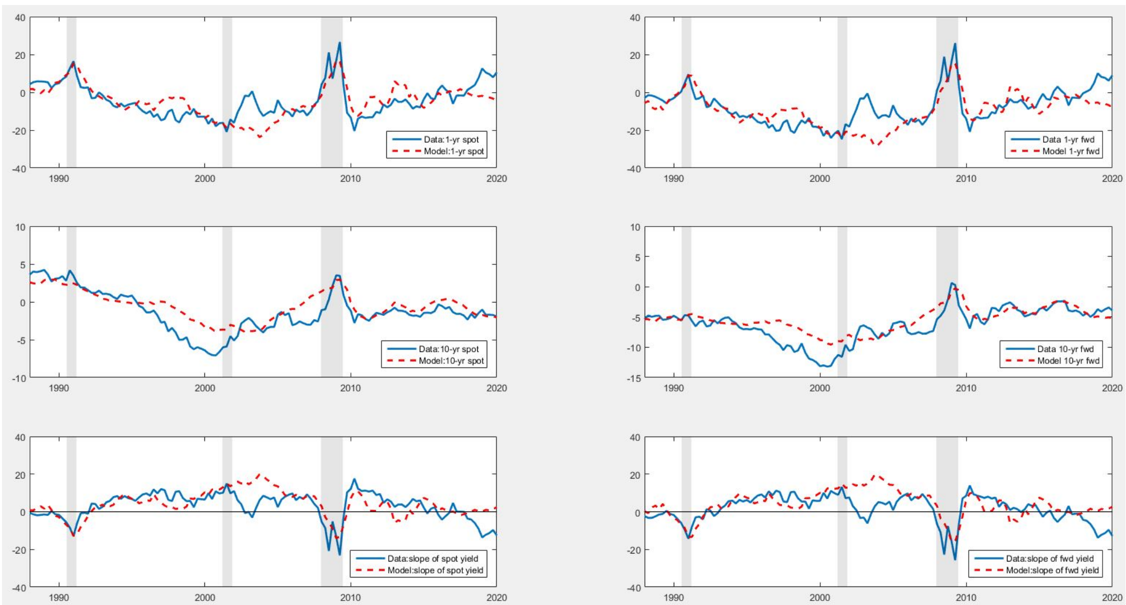
Table IA.8: Summary statistics of equity yields

The table reports the mean and standard deviation of spot and forward yields of dividend strips. These numbers are in annualized percentage terms. We report statistics from both our model and the data, and also their correlation coefficients. Sample period is from 1987Q4 to 2019Q4.

		Spot			Forward		
		1Y	10Y	10Y-1Y	1Y	10Y	10Y-1Y
Data	Mean	-4.39	-1.34	3.05	-7.79	-6.26	1.53
	Volatility	8.89	2.70	7.22	9.14	2.80	7.18
Model	Mean	-4.60	-0.55	4.05	-9.44	-5.46	3.97
	Volatility	8.39	1.89	6.92	8.65	1.96	7.09
	Corr	0.68	0.85	0.59	0.67	0.81	0.59

Figure IA.2: Term structure of equity spot and forward yields

The figure compares the model-implied spot (left panel) and forward yields (right panel) of dividend strips with the data from Giglio et al. (2021). The forward yields in the data are computed by subtracting the spot yields with the maturity-matched zero-coupon nominal Treasury bond yields. The last row plots the spread between 10-year and 1-year spot or forward yields. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4, and all numbers are in annualized percentage terms.



Third, following Van Binsbergen et al. (2013), we run variance decomposition on forward equity yields to understand determinants of their time-variations. We can write the forward yields as:

$$\begin{aligned} ef_t^{(n)} = Const^{(n)} & \underbrace{-\lambda(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t})}_{RGDP_t^{(n)}} + \underbrace{\frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1})}_{Div-specific_t^{(n)}} \\ & \underbrace{-(\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t})}_{Infl_t^{(n)}} + \underbrace{\frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}\lambda a_{g,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t}}_{Ambiguity_t^{(n)}}, \end{aligned} \tag{IA.51}$$

from which we obtain the following decomposition:

$$\begin{aligned} var(ef_t^{(n)}) = & cov(ef_t^{(n)}, RGDP_t^{(n)}) + cov(ef_t^{(n)}, Div-specific_t^{(n)}) \\ & + cov(ef_t^{(n)}, Infl_t^{(n)}) + cov(ef_t^{(n)}, Ambiguity_t^{(n)}). \end{aligned} \tag{IA.52}$$

Table IA.9 illustrates the proportion of total forward yield variability explained by each component. Consistent with our mean decomposition results in Table 3, the subjective real dividend-specific growth contributes over 90% to the yield volatility at the 1-year horizon. Interestingly, the importance of subjective real GDP growth increases steadily with the horizon. For the 10-year forward yield, it explains 23% of total yield variance while the proportion of dividend-specific growth decreases to 59%. A similar pattern is observed for the ambiguity part, which explains around 10% of total variance at the 10-year horizon. Zooming in on different economic regimes, we find that the explanatory power of subjective real GDP growth is stronger during the expansion period, yet the ambiguity channel is more important during the recession. For instance, it explains 18% of the 10-year forward yield variance.

Table IA.9: Variance decomposition of forward equity yields

The table reports the model-based variance decomposition (IA.52), where forward yields are decomposed to the components related to the real GDP growth, dividend-specific growth, inflation, and ambiguity. The decomposition is run over the full sample from 1987Q4 to 2019Q4, or over expansion and recession periods identified via the NBER business cycle dating. The decomposition is done for the dividend strip with the maturity of 1-year, 5-year, 7-year, and 10-year.

		1Y	5Y	7Y	10Y
Unconditional	RGDP	0.04	0.11	0.15	0.23
	Div.-spec.	0.93	0.80	0.73	0.59
	Infl.	-0.01	-0.01	0.01	0.05
	Ambiguity	0.05	0.10	0.12	0.13
Expansion	RGDP	0.04	0.12	0.17	0.26
	Div.-spec.	0.95	0.84	0.76	0.60
	Infl.	-0.01	-0.01	0.01	0.06
	Ambiguity	0.01	0.05	0.06	0.08
Recession	RGDP	0.03	0.09	0.11	0.15
	Div.-spec.	0.94	0.78	0.71	0.61
	Infl.	-0.01	0.01	0.03	0.06
	Ambiguity	0.04	0.11	0.14	0.18

Incorporating ambiguity into the model also improves the fit for equity returns. Figure IA.3 shows that the correlation coefficients of 2-year (10-year) realized strip returns slightly increase, and the model matches better the significant return crash during the global financial crisis. The model further matches two stylized facts regarding return variations: (1) long-term dividend strips co-move more strongly with the market returns (Van Binsbergen and Koijen , 2017; Gonçalves , 2021b); (2) return volatilities of long-term dividend strips are higher than those of short-term strips (Lettau and Wachter , 2007; Van Binsbergen and Koijen , 2017). Previous studies reconcile the facts via the idea that long duration assets have higher exposures to discount rate variations (see e.g., Campbell and Vuolteenaho , 2004; Brennan and Xia , 2006; Lettau and Wachter , 2007; Gonçalves , 2021b). Such an explanation may not be consistent with recent literature that casts doubt on the relevance of discount rate variations at both short and long horizons. Indeed, if discount rate variability per se does not contribute much to price volatility, we might expect it will explain little about the above patterns for return variations. Because our model imposes minimal discount rate variations, we have to use variations in beliefs over cash-flows to match these two stylized facts. Table IA.10 reports the results, where Panel A estimates CAPM betas of strip futures returns to gauge the magnitude of comovements and Panel B calculates volatilities of futures returns. Results imply that our model replicates well the upward-sloping term structure of both CAPM betas and volatilities, although the model-implied return volatilities are slightly smaller than the data.

Figure IA.3: Strip futures returns: data vs. model

The figure compares the model-implied futures returns of dividend strips with the data calculated from Giglio et al. (2021). We display results for 2-year and 10-year strip returns, and the sample period is from 1988Q4 to 2019Q4. All numbers are in annualized percentage terms.

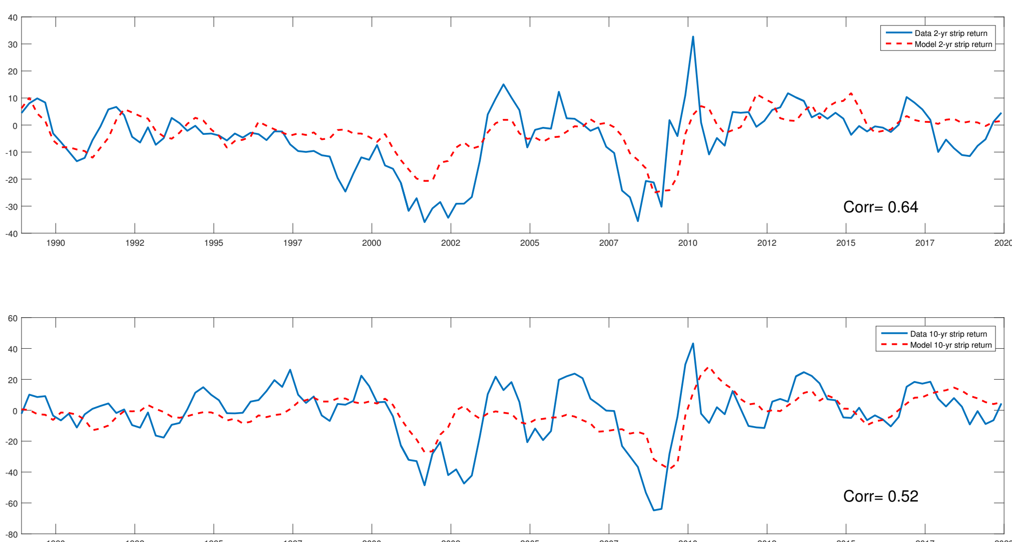


Table IA.10: Strip return comovements and volatilities

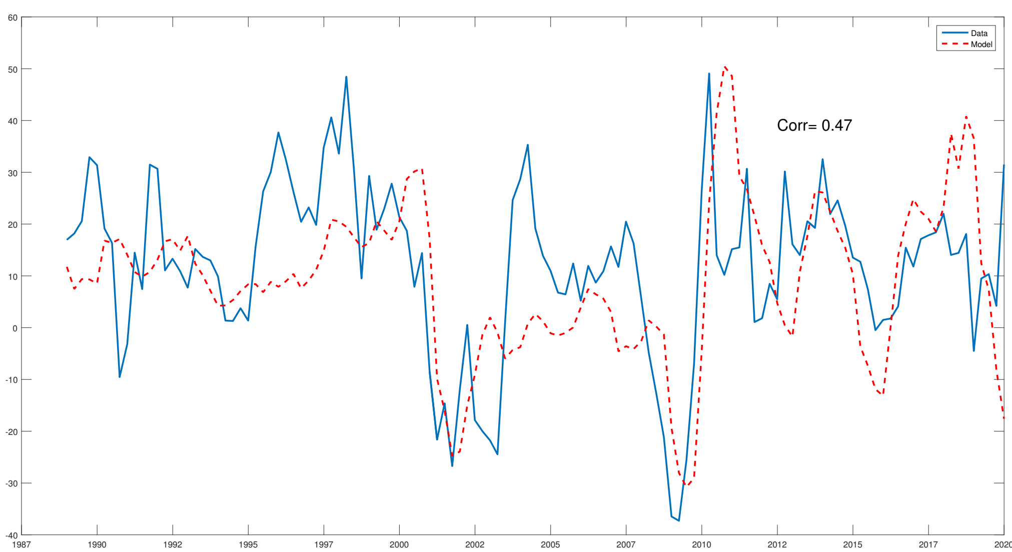
Panel A reports the model-implied CAPM betas of strip futures returns, and the Newey-West standard errors are in parentheses. Panel B reports the volatilities of strip futures returns in annualized percentage terms. The sample period is from 1988Q4 to 2019Q4.

	2Y	5Y	7Y	10Y
	Panel A: CAPM betas			
Data	0.47	0.80	0.88	0.93
	(0.10)	(0.10)	(0.10)	(0.11)
Model	0.25	0.44	0.53	0.61
	(0.06)	(0.06)	(0.06)	(0.08)
	Panel B: return volatilities			
Data	11.94	16.37	17.56	18.84
Model	7.33	8.30	9.40	10.81

Finally, Figure IA.4 plots the model-implied market returns together with the data. They are close with each other and the correlation coefficient is 0.47.

Figure IA.4: Aggregate stock returns: data vs. model

The figure compares the model-implied aggregate market returns with the data. The sample period is from 1987Q4 to 2019Q4. Plotted numbers are in annualized percentage terms.



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