Data-Sharing in Financial Contracting: Transparent vs Opaque Algorithms.

Alessio Ozanne*

March 29, 2024

Abstract

Should credit scoring algorithms be public? I study this question in a workhorse model of financial contracting with moral hazard, where the lender uses borrower-shared data to predict the borrower's cost of effort. I show that revealing the allocation rule makes it vulnerable to gaming in the form of strategic withholding of unfavorable information: unraveling forces are absent as both bad and good borrowers withhold data, respectively to escape rationing and to get better pricing (i.e. agency rents). The extent of gaming is more pronounced when the allocation rule relies strongly on data. When data sensitivity is high opacity alleviates gaming, allowing for credit rationing and leading to more price discrimination. When data sensitivity is lower opacity reduces disclosure because of the borrower's hedging behavior against rationing. Ultimately, the lender's transparency choices depend on the predictive power of data, are aimed at maximizing data-sharing, and are in general socially inefficient.

Keywords: Financial Contracting, FinTechs, Data, Algorithms, Opacity.
JEL Classification: D82, G14, G21, G23.

*Toulouse School of Economics. e-mail: alessio.ozanne@tse-fr.eu

1 Introduction

Motivation. Recent advances in digital technology have transformed the way firms store, transmit, process, and analyze information. Notably, a growing number of financial institutions have embraced automated machine learning (ML) algorithms for the task of credit scoring and underwriting[1], leveraging vast datasets that encompass both traditional financial metrics and alternative data sources[2]. The black-box nature of these algorithms has sparked significant public concern, even with accusations of discriminatory practices based on race and gender, and led individuals to adjust their online and offline behaviors in attempts to evade such systems[3]. On the other hand, financial institutions claim that making the algorithms' inner working public would undermine their predictive ability, potentially enabling individuals to game the system through strategic information sharing[4]. Ultimately, algorithm transparency impacts borrowers' willingness to share data - a crucial asset in a market plagued by asymmetric information and agency problems - and has a first-order impact on the allocative efficiency of the credit market. What are the implications of algorithmic opacity and transparency on borrowers' data-sharing behavior? How do these factors affect the allocative efficiency of credit markets, what are their distributional effect, and their impact on social welfare? Despite their significance, these questions have received limited attention in academic research. However, regulators are increasingly addressing this issue, favoring transparency and citing principles of truth, fairness, and equity over market efficiency[5].

[1] A prominent example is FinTech's "3-1-0 model": three minutes to apply, one second to approve, and zero human intervention. Traditional banks, such as J.P. Morgan and Bank of America, have also started to employ automated algorithms (see, e.g., Christine Idzelis, "Bank of America Adopts AI, Finds More Significant Credit Stresses From Covid-19", September 11, 2020), and similarly have done credit scoring systems, such as FICO and Vantage Score.

[2] Alternative sources of data include, but are not limited to, gig economy income, utility bill payments, childcare payments, social media data, digital footprints, web searches, mobile phone usage patterns, e-commerce transactions, payment information, and data from IoT devices, and so on.

[3] Sebastian Siemiatkowski, chief executive of the Swedish online payments company Klarna AB, once said: "Facebook is only as valuable as the information a consumer is willing to share and whether that consumer is willing to connect the financial service to their Facebook data". (See Evelyn M. Rusli, "Bad Credit? Start Tweeting", Wall Street Journal, April 1,2013.))

[4] See, e.g., Suzanne Woolley, "How More Americans Are Getting a Perfect Credit Score", Bloomberg, August 14, 2017.

[5] See, e.g., the Fair Credit Reporting Act and the Equal Credit Opportunity Act put in place by the Federal Trade Commission, and the Digital Services Act, Digital Markets Act and the

Overview of the Model. This paper studies the optimal level of transparency of credit risk algorithms in light of the amount of information that strategic borrowers are willing to disclose. I consider a monopolistic lender and a continuum of borrowers in need of cash to finance a project subject to moral hazard. The lender chooses to provide credit or not and assigns an output-contingent payment to the borrower. The borrowers are heterogeneous in their cost of providing effort and possess hard evidence that can be used to predict it. As mandated by Open Banking and GDPR, data is the borrower's private information unless voluntarily disclosed to the lender. The lender has access to an unbiased estimate of the precision of data and can tailor an algorithm to it, i.e. a mapping from data to allocations. Within this framework, I examine the borrower's voluntary data-sharing decision in two scenarios: when the parameters governing the algorithm are the lender's private information (referred to as opacity) and when they are publicly known (referred to as transparency).

Preview of the Results. The optimal allocation rule, depends on data as follows. The incentive payment to the borrower is increasing in the data realizations and high data realizations are credit rationed. Intuitively, higher data realizations are informative of the borrower having a high cost of effort and thus needing more skin in the game to make the project succeed. However, too high data realizations indicate a low pledgeable income and a negative NPV for the lender, which optimally chooses to deny credit. Moreover, the allocation rule depends on the lender's estimate of the precision of data (which is the lender's private information under opacity). In particular, the higher the precision of data the steeper the payment schedule and the lower the rationing threshold. Intuitively, when data is more precise the incentive scheme and the credit denial decision depend more strongly on it.

I then study the borrower's data-sharing decision when the algorithm is transparent. Contrary to the common wisdom from the literature, I show that transparency makes the allocation rule vulnerable to gaming in the form of strategic withholding of unfavorable information, at the point that credit rationing never happens in equilibrium. This may seem surprising in light of the classic unrav-

Artificial Intelligence Act set up by the European Commission.

eling argument of Grossman (1981) and Milgrom (1981), which loosely states that all available information is transmitted in equilibrium to avoid the lender's averse equilibrium inference following no disclosure. The equilibrium withholding of information in my setting is driven by the very nature of financial data, i.e. data that can be jointly used for rationing and pricing purposes. Rationed borrowers benefit from being pooled with good borrowers as this allows them to access financing. Good borrowers benefit from being pooled with bad borrowers as this allows them to get better prices, that is higher incentive payments. Hence, the lender's equilibrium inference following no disclosure is not so averse, and the unraveling argument does not apply. Only average borrowers below the rationing threshold have incentives to separate from the no-disclosure pool. The extent of gaming - i.e. no disclosure - increases when data is more precise. When data is more precise the rationing threshold is lower and more borrowers are willing to withhold data to escape rationing. This also increases the good borrowers' incentives to pool with bad ones and get better terms.

I then study data-sharing to an opaque algorithm, showing that opacity can lead to both an increase or decrease in the equilibrium amount of information shared compared to a transparent algorithm. In the data-sharing equilibrium under opacity, very good and very bad borrowers still opt to withhold data. Notably, bad borrowers face algorithmic uncertainty when sharing data: the unknown rationing threshold means they may receive either a very high payment or be credit rationed. Conversely, withholding data emerges as a secure strategy: it ensures that a borrower receives credit, albeit with lower expected payments. Consequently, the optimal disclosure strategy for bad borrowers balances the potential benefits of data-sharing against the hedge of withholding. It follows that the amount of information shared in equilibrium can be either higher under opaque or transparent algorithms depending on the precision of data, which determines the rationing threshold and the set of non-disclosed data points under transparency.

Finally, I conduct a welfare comparison highlighting the trade-offs of algorithmic opacity from the lender's point of view. I show that the lender's transparency choices are directed to minimize the information loss resulting from either gaming or hedging. The benefits of one regime over the other stem from an increased ability to personalize agency rents, a better average quality of the no-disclosure pool, and efficient credit rationing. In the end, the model predicts that the most data-sensitive algorithms are kept opaque when transparency choices are delegated to lenders.

I then study the transparency regime that maximizes a number of welfare measures. Transparency maximizes financial inclusion since credit rationing is only possible under opacity. For the same reason, transparency maximizes the project's net present value (NPV), since every borrower possesses a positive NPV project net of the agency rent. Lastly, I study the transparency regime that maximizes (utilitarian) social welfare, intended as the sum of the lender's and the borrower's profits, and show that the lender's transparency choices are socially inefficient since a social planner aims at limiting the amount of data shared in equilibrium.

Structure of the Paper. The remainder of the paper proceeds as follows. In Section 2, I discuss the related literature. Section 3 describes the model. In Section 4, I derive the optimal allocation rule. Sections 5.1 and 5.2 present respectively the data sharing-equilibrium under transparent and opaque rules. Section 6 is dedicated to the welfare comparison of transparency regimes. All the proofs are relegated to the Appendix.

2 Related Literature

Strategic Classification. The paper is related to the literature in economics, computer science, and machine learning studying the manipulation of scoring systems (see Hardt, Megiddo, Papadimitriou and Wootters (2016) for a classic paper in machine learning). In economics, the literature has been ignited by Frankel and Kartik (2019) that studied the manipulation of a linear scoring rule and showed that as incentives to manipulate increase less information is extracted in equilibrium. Frankel and Kartik (2021) and Ball (2022) take a design perspective and study the optimal linear scoring rule. They show that the optimal rule underutilizes data in order to limit the agent's gaming behaviour. Perez-Richet and Skreta (2022) analyze similar frictions when the scoring rule is a Blackwell statistical experiment and an agent can falsify the state of the world. Similarly to the above papers, they show that the informativeness of the experiment should be reduced in order to offset strategic manipulation. A finance application is the paper by Cohn, Rajan and Strobl (2023) in the context of credit ratings.

My paper differs from these papers in at least three respects. First, they consider types of data that can be distorted by agents at some cost. I consider data as hard information that can be shared or held, but not misreported. Second, I specifically focus on financial data used jointly for pricing and rationing purposes. Third, they focus on transparent scoring rules whether I also consider opaque ones and show that opacity can emerge as an alternative strategy to data under-utilization.

Opacity. Some papers have studied the role of opacity in softening strategic agents' gaming behavior in different economic settings. Ederer, Holden and Meyer (2018) study opaque incentive schemes in a principal-agent relationship with multitasking. They show that hiding how the different tasks are rewarded, induces a risk-averse agent to balance effort across tasks. Leitner and Williams (2023) consider opacity in stress testing and show that it can prevent banks from gaming the test but induces them to underinvest in socially desirable assets. Wang, Huang, Jasin and Vir Singh (2023) and Sun (2021) study opacity of algorithmic decision-making in, respectively, hiring and lending, finding contrasting results.

The paper closest to mine is Sun (2021). He studies optimal algorithmic transparency in a competitive lending market where lenders use an opaque predictive algorithm to mitigate adverse selection and borrowers can modify a binary feature at some cost. My paper is different in several aspects. First, his model does not have an interest rate and lenders make a binary choice about credit provision. I allow for data-driven pricing in a model of financial contracting. Second, his lending market is competitive which prevents him from studying the lender's optimal transparency regime and making welfare comparisons. Third, he considers costly manipulation of features whether I consider voluntary data-sharing, consistent with current Open Banking regulation. Fourth, I model algorithmic opacity as uncertainty about the mapping from input data to outcomes, whether he considers borrowers' uncertainty about their project's payoff distributions. My approach seems more in line with reality. Fifth, in his framework, transparency leads to the worst possible outcome for the credit market, whether I highlight possible benefits of transparency, in line with the current debate.

Volontary Disclosure. The paper is related to the large literature on voluntary information disclosure. The classical "unravelling argument" pushed forward by Grossman (1981) and Milgrom (1981) shows that, in equilibrium, a privately informed sender will disclose all the available information to avoid the receiver's averse equilibrium inference. The subsequent literature tried to explain the commonly observed lack of disclosure by introducing disclosure costs (Verrecchia (1983)), uncertainty about the evidence available to the sender (Dye (1985)), information processing costs (Fishman and Hagerty (2003)), receiver's outside information (Harbaugh and To (2020) and Quigley and Walther (2022)) and uncertainty about audience's preferences (Bond and Zeng (2022)).
My paper adds to this literature in two ways. First, I show that when the sender discloses financial information the unraveling argument has no bite. Thus, my paper adds another reason why some information may be withheld in equilibrium. Second, I study information disclosure when the sender does not know how the receiver processes information.

FinTechs. This paper is linked to the literature on digital disruption in banking (see Vives (2019) for a literature review), and in particular to papers studying the effects of new statistical technologies in the credit market. Alternative forms of data[6] have been shown to outperform traditional ones in predicting loan defaults (see Bazarbash (2019), Berg, Burg, Gombović and Puri (2020) and Iyer, Khwaja, Luttmer and Shue (2016)), and thus can play an important role in expanding access to credit to individuals without a credit history (see Agarwal, Alok, Ghosh and Gupta (2019) and Gambacorta, Huang, Li, Qiu and Chen (2020)). The novel machine learning techniques used by FinTechs to process these data have been shown to have perverse distributional effects due to their flexibility in capturing structural relationships among variables such as race, income, and gender (see Fuster, Goldsmith-Pinkham, Ramadorai and Walther (2022)).
The opacity of these algorithms has raised the concern that individuals may hedge against them by modifying or even hiding their behavior online, with severe con-

[6] Alternative forms of data are any data that is not directly related to a consumer's credit behavior. Examples of alternative data include, but are not limited to, gig economy income, utility bill payments, childcare payments, social media posts, digital footprints, web searches, and payments data.

sequences for their predictive ability and financial inclusion. I contributed to this literature by identifying the welfare effects of borrowers' hedging behavior against algorithmic opacity.

Economics of Data and Privacy. This work belongs to a vast and growing literature on data, privacy, and their implications for markets (see Acquisti, Taylor, and Wagman (2016) and Bergemann and Bonatti (2019) for surveys). Part of this literature analyzes the welfare impact of data regulations such as the GDPR and Open Banking. Ali, Lewis and Vasserman (2023) study price discrimination when consumers have control over their data, finding that GDPR can improve consumer surplus in both monopolistic and competitive markets. He, Huang and Zhou (2023) consider lending market competition when borrowers share data voluntarily and find that Open Banking makes borrowers worse off. My work complements these studies by analyzing the welfare implications of voluntary data sharing in financial contracting, a context where data has welfare effects beyond mere surplus sharing à la Bergemann, Brooks, and Morris (2015)[7] (see also Jansen, Nagel, Yannelis and Zhang (2023) for empirical evidence). A point also made by Liang and Madsen (2024) in the context of the labor market.

3 Model

Economy. The economy is composed of a monopolistic lender and a continuum of penniless borrowers in need of cash to finance a project. Both players are risk-neutral and the borrowers are protected by limited liability. The borrower's project requires a unit of cash and generates a return X with probability $e \in [0, 1]$ and generates nothing otherwise. The probability of success e is chosen by the borrower at a private cost $c(e) = \theta e + \frac{1}{2}\alpha e^2$, and his choice is neither observable nor contractible. Moreover, borrowers are heterogeneous with respect to their cost of effort, which we assume to be uniformly distributed, $\theta \sim U[0, 1]$. In other words, the project is subject to moral hazard in the spirit of Holmstrom and Tirole (1997): the borrowers need monetary incentives to manage the project

[7] Bergemann, Brooks, and Morris (2015) study the welfare consequences of a monopolist having additional data for price discrimination. They show that the granularity of data can achieve every combination of consumer and producer surplus such that the total surplus does not exceed the surplus generated by and efficient trade.

diligently[8]. We make the following assumptions:

Assumption 1 (Cross-Subsidization)**.** $X - \sqrt{4\alpha} \in \left(\frac{1}{2}, 1\right)$.

Assumption 2 (Interior Effort)**.** $\alpha > \frac{1}{2}\left(3 + \sqrt{5}\right)$.

Assumption 1 bounds the NPV of the project and has two implications. First, there is scope for credit rationing: a perfectly informed lender would not provide credit to inefficient borrowers. Second, market conditions allow the lender to operate profitably: an uninformed lender finds it optimal to provide credit.
Assumption 2 assures that, under Assumption 1, borrowers' effort choices are interior solutions.

Data. We assume that the borrower's cost of effort θ is unknown to both players at the time of contracting[9], and that the borrower possesses data $s \in [0, 1]$ that can be used to predict it. The data realization s is the borrower's private information unless disclosed to the lender. Specifically, the borrower chooses a message $m(s) \in \{s, \varnothing\}$, where $m(s) = s$ means sharing data with the lender, while $m(s) = \varnothing$ means withholding data.
The data-generating process (DGP) is truth-or-noise (see Lewis and Sappington (1994)) and such that:

$$s = \begin{cases} \theta & \text{with pr.} \quad \lambda \\ \varepsilon & \text{with pr.} \;\; 1 - \lambda, \end{cases} \tag{1}$$

where $\varepsilon \sim U[0, 1]$ and is independent of θ. The value of λ determines the predictive ability of data: when $\lambda = 1$ the data realization s perfectly predicts θ, while when $\lambda = 0$ data is completely useless. Importantly, we assume that the

[8] Alternatively, one can think that borrowers, in the absence of proper incentives, may deliberately reduce the probability of success in order to enjoy a private benefit.

[9] This assumption excludes the possibility of screening by the lender. An older literature has considered how contractual terms, in particular collateral requirements, can be used to extract the borrower's private information (see, e.g., Bester (1985), Freixas and Laffont (1990) and Besanko and Thakor (1987). A more recent literature has identified the role of data - coupled with new statistical technologies - as a substitute to collateral in solving agency problems and expanding access to finance (see Agarwal, Alok, Ghosh and Gupta (2019) and Gambacorta, Huang, Li, Qiu and Chen (2020)). Because the focus of the paper is on the second topic, we abstract away from the possibility of designing screening contracts.

data-generating process in (1) is a priori unknown and such that λ is uniformly distributed and sufficiently high, $\lambda \sim U[\underline{\lambda}, 1]$. However, the lender, having access to statistical technology and a training dataset, can obtain an unbiased estimate $\widehat{\lambda} = \lambda$ of the parameters governing the DGP. Since, knowing $\widehat{\lambda}$ is equivalent to knowing λ, in what follows we will denote both the precision of data and its estimate as λ. We define a lower bound for the precision of data.

Assumption 3 (Informative Data)**.** $\underline{\lambda} = 2\left(X - \sqrt{4\alpha} - \frac{1}{2}\right)$.

Assumption 3 implies that data will always be used for rationing purposes. This assumption is not necessary for the main results of the paper but excludes full disclosure equilibria that would arise when $\lambda \in [0, \underline{\lambda}]$ and are well understood by the literature.

Algorithm. Given the knowledge of λ and some information set $\mathcal{I}$, the lender decides whether to provide credit to the borrower or not. We denoted the lender's decision to provide or deny credit as $\ell_\lambda(\mathcal{I}) \in \{0, 1\}$, where $\ell_\lambda(\mathcal{I}) = 1$ denotes credit provision. If the lender provides credit, she also offers a payment to the borrower $x_\lambda(\mathcal{I}) \in [0, X]$ in case the project generates a return. The residual return, or pledgeable income, $X - x_\lambda(\mathcal{I})$ is left to the lender. If the project generates nothing, both the borrower and the lender get nothing[10]. We refer to $\big(\ell_\lambda(\mathcal{I}), x_\lambda(\mathcal{I})\big) \triangleq a_\lambda(\mathcal{I})$ interchangeably as an algorithm or an allocation rule, as this map inputs data to allocations. At the optimum, the algorithm will depend on the lender's estimate of λ, so one can think directly at λ as a parameter governing the algorithm. We define transparency and opacity as follows.

Definition 1 (Transparency and Opacity)**.** The lender's algorithm $a_\lambda(s)$ is*:*

- Transparent, if λ is public information,
- Opaque, if λ is the lender's private information.

Timing. The timing of the game is as follows:

[10]Restricting attention to contracts that pay only in case the project success is without loss, since paying nothing when the project fails is optimal for the lender.

$t = 0$. The borrower's type θ, the predictive power of data λ, and data s realize. Data s is observed by the borrower and an unbiased estimate of λ is observed by the lender (resp. everyone) under opacity (resp. transparency).

$t = 1$. The borrower shares or withholds data, choosing a message $m \in \{s, \varnothing\}$.

$t = 2$. The lender allocates credit according to the algorithm $a_\lambda(s) = \big(\ell_\lambda(m), x_\lambda(m)\big)_{m\in\{s,\varnothing\}}$.

$t = 3$. If the borrower gets credit, he observes his type θ, he exerts effort e, and the project's returns are realized.

4 Optimal Algorithm

This section describes the optimal allocation rule in the situation where the lender possesses the borrower's data, i.e. $\mathcal{I} = s$. Let $f_\lambda(\theta|s)$ be the lender's posterior beliefs about the borrower's cost of effort. The lender's optimal algorithm allocates credit to solve:

$$\begin{aligned} &\max_{\substack{x\in[0,X]\\ \ell\in\{0,1\}}} \quad \ell \int_0^1 \Big(e(\theta)(X-x)-1\Big) f_\lambda(\theta|s)\,\mathrm{d}\theta \\ &\text{s.t.} \quad e(\theta) = \operatorname*{argmax}_{e\in[0,1]} ex - \left(\theta e + \frac{1}{2}\alpha e^2\right). \end{aligned} \tag{2}$$

The optimal allocation rule maximizes the NPV of the lender's investment, knowing that the project's probability of success will be determined by the borrowers' heterogeneous effort choices. Thus, the lender takes expectations over the NPV given the data point s and her knowledge of the data-generating process λ. The optimal allocation rule is described in the following Lemma.

Lemma 1 (Optimal Algorithm)**.** *The lender's optimal algorithm is:*

$$\begin{aligned} \ell_\lambda(s) &= \mathbb{1}_{\{s<r(\lambda)\}} && \textit{where } r(\lambda) = 1/2 + \frac{1}{\lambda}\Big(X - \sqrt{4\alpha} - 1/2\Big), \\ x_\lambda(s) &= \frac{X + \mathbb{E}_\lambda(\theta|s)}{2} && \textit{where } \mathbb{E}_\lambda(\theta|s) = \lambda s + (1-\lambda)1/2. \end{aligned} \tag{3}$$

The θ-borrower's utility with data realization s is:

$$V_\lambda^B(s,\theta) = \frac{1}{2\alpha}\ell_\lambda(s)\Big(x_\lambda(s) - \theta\Big)^2. \tag{4}$$

Proof. See the Appendix. ■

The optimal allocation rule is rather intuitive. First, the incentive payment is increasing in the data realization since higher data realizations are informative about the borrower having a high cost of effort and needing greater incentives. The steepness of the repayment schedule $x_\lambda(s)$ is increasing in λ, the more informative is data the more the incentive scheme depends on it. Second, the lender denies credit if the data realization is above a threshold $r(\lambda)$. This happens because for very high data realizations the expected cost of effort, and the resulting incentive payment, are so high that the lender would make a negative NPV investment by providing credit. The rationing threshold is decreasing in data informativeness λ: as λ increases lower and lower data realizations are included in the rationing region. All in all, the higher is the predictive power of data the more the lender's allocation rule relies on it for both rationing and pricing.

5 Voluntary Data-Sharing

In the spirit of Open Banking initiatives, we now consider the case where borrowers can voluntarily share data with the lender, where the latter draws inferences based on both the data being shared and the borrower's disclosure choices. We consider in turn transparent and opaque allocation rules.

5.1 Transparent Algorithms

In this section, we show that transparent allocation rules are subject to gaming by borrowers in the form of strategic withholding of unfavorable information. When the scoring rule is transparent, the data-sharing equilibrium is described in the following proposition.

Proposition 1 (Equilibrium under Transparency)**.** *With transparent algorithms, there exists an equilibrium where the borrower's data-sharing strategy is:*

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [0, l(\lambda)] \cup [r(\lambda), 1] \triangleq G(\lambda) \\ s & \text{if } s \in \big(l(\lambda), r(\lambda)\big), \end{cases} \tag{5}$$

where $r(\lambda)$ *is defined (3) and* $l(\lambda) = -\big(1 - r(\lambda)\big) + \sqrt{2\big(1 - r(\lambda)\big)}$.

The lender allocates credit according to Lemma 1 to all borrowers that share data, while she allocates credit to all borrowers withholding data according to:

$$\begin{aligned} \ell_\lambda(\varnothing) &= 1 \quad \forall \lambda, \\ x_\lambda(\varnothing) &= \frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2}. \end{aligned} \tag{6}$$

Proof. See the Appendix. ■

Interestingly, some information is withheld in equilibrium. This may seem puzzling in light of the classic unraveling argument that predicts that, in equilibrium, a sender discloses all the available information. Briefly, the unraveling argument states that borrowers with the most advantageous information will surely disclose it, leading the lender to infer from the absence of disclosure that borrowers lack this information. Consequently, borrowers with the next most advantageous information also disclose, creating a cascade effect.

To grasp intuition about the partial disclosure equilibrium in Proposition 1, consider first the lender's inference following no-disclosure:

$$\mathbb{E}_\lambda(\theta|\varnothing) = \lambda\left(\omega(\lambda)\frac{l(\lambda)}{2} + (1-\omega(\lambda))\,\frac{1+r(\lambda)}{2}\right) + (1-\lambda)\frac{1}{2}, \tag{7}$$

where $\omega(\lambda) = \frac{l(\lambda)}{l(\lambda)+1-r(\lambda)}$. The expected cost of effort is proportional to the weighted average of the expected data realizations in the two disjoint sets that compose the no-disclosure pool. Hence, the lender's equilibrium inference following no disclosure is not so averse because borrowers withhold both good and bad evidence. As a consequence, the lender is willing to provide credit following no disclosure. This happens because, by Assumption 1, the overall market conditions are favorable, and good borrowers cross-subsidize bad ones allowing the lender to break even.

Second, consider the θ-borrower's utility from data disclosure, following Lemma 1 this is[11]:

[11] Note that the actual value that the borrower gets from disclosure is $\mathbb{E}(V^B(s,\theta)|s)$, as θ is unknown at the time of the data-sharing decision. Considering the utility in (8) isolates the forces that lead to no disclosure from learning effects.

$$V^B(s,\theta) = \begin{cases} \frac{1}{2\alpha}\Big(x_\lambda(s) - \theta\Big)^2 & \text{if } \ s \in [0, r(\lambda)) \\ 0 & \text{if } \ s \in [r(\lambda), 1]. \end{cases} \tag{8}$$

The utility is non-monotonic in s. It is increasing below the rationing threshold $r(\lambda)$ as higher data realizations get higher incentive payments, it then drops to 0 for data points for which the lender rations credit. This implies that rationed borrowers benefit from being pooled with borrowers with very good evidence as this allows them to get credit. Similarly, borrowers with good evidence benefit from pooling with rationed borrowers as this allows them to get higher incentive payments than what they would get by disclosing data. Jointly, these two forces sustain data withholding in equilibrium. Hence, my model points out that the absence of disclosure can emerge as an equilibrium phenomenon even in markets without disclosure frictions[12] where the information being disclosed is financial data, that is information that can be jointly used for rationing and pricing purposes.

The equilibrium also features some borrowers sharing data. These borrowers are deemed credit-worthy but likely to have a high cost of effort according to their data points. As a consequence, they get incentive payments that are strictly higher than what they would get from no-disclosure. This induces them to separate from the no-disclosure pool by sharing data. The incentives to separate are weaker and weaker for lower and lower data points so that the unraveling stops at some point and prevents full disclosure from happening in equilibrium.
The threshold $l(\lambda)$ that separates the disclosure region from the lower no-disclosure region is determined by an indifference condition. The borrower with data realization $s = l(\lambda)$ gets the same incentive payment after disclosure and no disclosure since the lender's inference is such that $\mathbb{E}_\lambda(\theta|\varnothing) = \mathbb{E}_\lambda(\theta|s = l(\lambda))$. Notably, the threshold $l(\lambda)$ is decreasing in the rationing threshold $r(\lambda)$. This happens for the following reasons. Increasing the rationing threshold reduces the mass of rationed borrowers that withhold data. From the lender's perspective, no disclosure is more likely to come from borrowers with low data points, and thus the

[12]See the "Voluntary Disclosure" sub-section in Section 2 for papers explaining no-disclosure in settings with disclosure costs, limited attention, limited evidence, and so on.

lender reduces the incentive payment for borrowers that withhold data as the pool is more efficient. As a consequence, borrowers with low data points have lower benefits of being pooled with rationed borrowers and thus they disclose more, reducing $l(\lambda)$.

The equilibrium withholding of information can be interpreted as a form of gaming by which borrowers get better terms by hiding unfavorable informatin. The extent of gaming can be measured by the mass of borrowers withholding data, i.e. by the size of the set $G(\lambda)$. Since $r(\lambda)$ and $l(\lambda)$ are respectively decreasing and increasing, the extent of gaming increases when data is more informative and the allocation rule relies more strongly on it. When λ increases, the rule induces more rationing and thus more data-withholding by rationed borrowers. This gives further incentives to borrowers with low data points to withhold them as this allows them to get even higher payments.
Gaming obviously leads to an information loss to the lender, and this loss is increasing in the data-sensitivity of the allocation rule as captured by λ. Interestingly, one can show that if λ were a choice variable, the lender would choose an intermediate data sensitivity balancing the increased predictive ability of the allocation rule with information loss due to the borrower's gaming behavior. This result is related to a number of papers studying the optimal design of allocation rules when agents manipulate data and finding that the optimal rule underutilizes data to attenuate the information loss[13]. In these papers, as in mine, gaming is made possible by the transparency of the allocation rule. Borrowers need to know the rationing threshold $r(\lambda)$ to form the equilibrium data-sharing strategy. In Section 5.2, we explore whether gaming can be attenuated by making the rule opaque instead of less data-sensitive.

Before proceeding with the analysis of opaque scoring rules a comment on equilibrium uniqueness is in order. The equilibrium identified in Proposition 1 is not the unique Perfect Bayesian Equilibrium of the game. Together with that equilibrium, there exists another equilibrium where all the borrowers disclose and the lender allocates them credit according to Lemma 1, and the lender denies credit to borrowers withholding data. I discard this less interesting equilibrium

[13] See the "Strategic Classification" sub-section in Section 2.

as it does not satisfy a number of plausible equilibrium refinements. First, the disclosure strategy is not an equilibrium of a perturbed game where a fraction of borrowers is privacy concerned[14]. Second, the equilibrium is not stable as it is not trembling-hand perfect[15]. Third, the equilibrium preferred by the borrower is the one in Proposition 1. For clarity of exposition, rather than fully considering these model modifications, I prefer to allow for equilibrium multiplicity and focus on the equilibrium that survives these perturbations.

5.2 Opaque Algorithms

In this section, I consider the borrowers' voluntary data-sharing decision when the allocation rule is opaque. I show that opacity makes data-sharing risky, exposing the borrowers to the threat of credit rationing. As a consequence, rational borrowers hedge this risk by withholding information.
When the allocation rule is opaque the data-sharing equilibrium is described in the following proposition.

Proposition 2 (Equilibrium under Opacity)**.** *With opaque algorithms, there exists an equilibrium where the borrower's data-sharing strategy is:*

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [0, a(b)] \cup [b, 1] \triangleq H(b) \\ s & \text{if } s \in \big(a(b), b\big), \end{cases} \tag{9}$$

where $b \in (r(1), 1)$ solves (30) and $a(b) \triangleq -\big(1-b\big) + \sqrt{2\big(1-b\big)} \in (0, r(1))$. The lender allocates credit according to Lemma 1 to all borrowers that share data, while she allocates credit to all borrowers withholding data according to:

$$\begin{aligned} \ell_\lambda(\varnothing) &= 1 \quad \forall \lambda, \\ x_\lambda(\varnothing) &= \frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2}. \end{aligned} \tag{10}$$

Proof. See the Appendix. ■

[14] Alternatively, one can think of privacy-concerned borrowers as not having hard information to disclose as in Dye (1985). See Appendix B for details.

[15] If all the borrowers tremble and do not disclose with some small probability, then the lender finds optimal to provide credit following no disclosure due to Assumption 1. If this is the case then the equilibrium in Proposition 1 emerges.

The nature of the equilibrium data-sharing strategy under opacity is similar to the one under transparency. In particular, borrowers with extremely low and extremely high data realizations withhold data as they expect better terms than if they share them. In other words, the opaque scoring rule is still subject to gaming in the form of strategic no disclosure of information. However, the amount of data transmitted in equilibrium changes with respect to transparency. To see why this happens consider the θ-borrower's expected utility from full disclosure of data[16]:

$$V^B(s,\theta) = \begin{cases} \dfrac{1}{2\alpha}\dfrac{1}{1-\underline{\lambda}}\displaystyle\int_{\underline{\lambda}}^{1}\Big(x_\lambda(s)-\theta\Big)^2 \mathrm{d}\lambda & \text{if } s \in [0, r(1)) \\ \dfrac{1}{2\alpha}\dfrac{1}{1-\underline{\lambda}}\displaystyle\int_{\underline{\lambda}}^{\widehat{\lambda}(s)}\Big(x_\lambda(s)-\theta\Big)^2 \mathrm{d}\lambda & \text{if } s \in [r(1), 1]. \end{cases}$$

where $\widehat{\lambda}(s) \triangleq r^{-1}(s)$ is the highest λ for which the data realization s gets credit. Intuitively, the utility from data-sharing depends on the likelihood of being credit rationed following disclosure. Data realizations in $[0, r(1))$ are safely below the rationing threshold of each scoring rule and thus always get credit after disclosure. For these borrowers, the randomness in λ impacts their utility only through the payment they get. Higher expected payments are granted to higher data realizations and thus the utility is increasing in s. More interestingly, data realizations in $[r(1), 1]$, get credit only if the true rationing threshold is sufficiently high (or, equivalently, if the true data informativeness is sufficiently low). Since higher data realizations are more likely to be credit-rationed, the utility from disclosure is decreasing in s for these borrowers. Hence the utility from disclosure is non-monotonic in s, as under transparency, and this induces borrowers with very low and very high data realization to withhold as the lender's inference following withholding does not lead to rationing (see Section 5.1 for more details). More importantly, the thresholds that separate the disclosure and the no-disclosure regions are now both determined by an indifference condition. The lower threshold $a(b)$ is such that the borrower with data realization $s = a(b)$ gets the same expected payment for under both disclosure and no disclosure, and is such that $\mathbb{E}_\lambda(\theta|\varnothing) = \mathbb{E}_\lambda(\theta|s = a(b))$ for every λ. Consider instead the borrower with data

[16] As we did for Proposition 1, we provide the intuition behind the result abstracting from learning effects: that is we consider $V^B(s,\theta)$ instead of $\mathbb{E}(V^B(s,\theta)|s)$.

realization $s \in [r(1), 1]$. The algorithm opacity exposes them to risk when they share information. On the one hand, disclosing data may lead them to credit rationing whenever the true algorithm applies a sufficiently low rationing threshold. On the other hand, by sharing data they may obtain pretty high incentive payments if they have the chance of getting credit. On the contrary, withholding data is a secure strategy that hedges the borrower against the risk of being rationed. In fact, the lender always provides credit to borrowers withholding data, but she does so at a moderate incentive payment reflecting the average data realization in the no-disclosure pool. For the borrower with data point $s = b$, these two effects perfectly balance out.

6 Welfare Analysis

So far, we quantified the information loss resulting from the voluntary data-sharing decisions of rational borrowers: transparent algorithms allow for gaming, while opaque algorithms induce hedging. This last section analyzes further this trade-off and determines which transparency regime is preferred by the lender and by the credit market as a whole.

6.1 First Best Allocation

We start by studying the first best allocation. We define the first best as the effort and credit provisions that maximize utilitarian social welfare (or social surplus), i.e. the sum of the lender's and the borrower's profits. Moreover, the social planner faces no information frictions knowing the borrower's cost of effort θ (no hidden information problem), and mandating the borrower's effort choices (no hidden action problem).

Lemma 2 (First Best)**.** *The first best level of effort and credit allocation are:*

$$\begin{aligned} e^*(\theta) &= \min\left\{\frac{X-\theta}{\alpha}, 1\right\}, \\ \ell^*(\theta) &= 1. \end{aligned} \tag{11}$$

Proof. See the Appendix. ∎

An implication of Lemma 2 is that the market solution (see Lemma 1) entails inefficiently low levels of effort no matter the lender's information, since

$$e^*(\theta) > \frac{\frac{X+\mathbb{E}_\lambda(\theta|s)}{2} - \theta}{\alpha} \quad \forall\, s \in [0,1],$$

leading to (weakly) excessive credit rationing

$$\ell^*(\theta) \geq \mathbb{1}_{\{s<r(\lambda)\}},$$

since every borrower's type should get credit but borrowers with high data realizations s are credit rationed in the market solution. This happens because every borrower generates a positive surplus, but the lender appropriates only part of it because of the agency rent left to the borrower and finds it optimal to ration credit to borrowers that a planner would fund.

6.2 Lender-Optimal Transparency Regime

First, note that the amount of information being withheld in equilibrium can both be higher or lower under opacity. In particular, opacity increases data-sharing when the predictive power of data is sufficiently high, that is when the allocation rule relies strongly on data. This happens when $1 - \int_{H(b)} \mathrm{d}s > 1 - \int_{G(\lambda)} \mathrm{d}s$, which is the case when the algorithm applies a sufficiently low rationing threshold $r(\lambda) < b$, or, equivalently when the predictive power of data is sufficiently high $\lambda > r^{-1}(b) \triangleq \lambda^*$.

The following proposition shows that the lender's profits are proportional to the amount of information being shared by the borrowers.

Proposition 3. *The lender's profits are higher under opacity iff* $\lambda > r^{-1}(b) \triangleq \lambda^*$.

Proof. See the Appendix. ■

Consider first the case where the predictive power of data is sufficiently low $\lambda < \lambda^*$ so that the lender prefers a transparent algorithm. The optimal algorithm applies a rationing threshold $r(\lambda) > b$ and we can distinguish three sets of borrowers depending on their equilibrium ex-post allocations in the two transparency regimes:

- borrowers with data points $s \in [a(b), b]$ share information in both regimes and thus get credit at the same incentive payments, granting the same profits to the lender;

- borrowers with data points $s \in [0, l(\lambda)] \cup [r(\lambda), 1]$ withhold information in both regimes but are paid lower incentive payments under transparency - as the average borrower in the no disclosure pool is more efficient - which provides higher profits to the lender;

- borrowers with data points $s \in [l(\lambda), a(b)] \cup [b, r(\lambda)]$ withhold data under opacity, but disclose it under transparency, thus, the lender is able to provide personalized incentive payments and increase her profits.

It follows, that when the predictive power of data is low, making the algorithm opaque only reduces the amount of information being disclosed due to the borrowers' hedging behavior. This results in less price personalization, less efficient incentive schemes, and a more risky no-disclosure pool.
Consider next the case where the predictive power of data is higher $\lambda > \lambda^*$ and the lender prefers opacity. Proceeding as above, we can identify groups of borrowers for which opacity allows price discrimination and improves the quality of the no-disclosure pool due to boosted data-sharing resulting in equilibrium. On top of that, opacity allows the lender to deny credit to some borrowers. Borrowers with data points $s \in [r(\lambda), b]$ share information only under opacity, as the rationing threshold is unknown and lower than expected. All in all, when the predictive power of data is sufficiently high, opacity increases price personalization, ameliorates the no-disclosure pool, and allows for credit rationing, ultimately raising the lender's profits.

6.3 Welfare-Optimal Transparency Regime

The following proposition shows that the lender's transparency choices are in general socially inefficient, as a social planner would choose transparency whenever the lender would choose opacity (and vice versa).

Proposition 4. *Social welfare is higher under transparency iff* $\lambda > r^{-1}(b) \triangleq \lambda^*$.

Proof. See the Appendix. ∎

The key to understand this result is to note that social welfare is decreasing in the amount/precision of information available to the lender. This in turn implies that the transparency regime that maximizes social welfare is the one that minimizes the amount of information available to the lender. Consider the situation where the lender owns the borrower's data (Lemma 1) and consider an increase in data precision λ. First, there is a credit rationing effect: an increase in λ reduces credit provision to borrowers with positive surplus projects and so reduces social welfare (see Lemma 2). Second, there is a redistribution effect. As λ increases, incentive schemes become more sensitive to data, causing reallocation of productive effort from better-than-average borrowers (those with $s < \mathbb{E}(\theta)$) to worse-than-average borrowers (with $s > \mathbb{E}(\theta)$) without affecting the overall amount of effort exerted. On the one hand, the NPV of the project - a linear function of effort - remains constant. On the other hand, the overall cost of effort increases because of the convexity of the cost function: a marginal increase in effort surpasses the cost saved by an equivalent reduction. It follows that the re-distributive effects of data are socially inefficient and, as a consequence, the welfare-maximizing transparency regime is the one that minimizes the information available to the lender.

6.4 Additional Welfare Measures

Transparency is optimal when we consider the projects' NPV and financial inclusion (i.e. credit provision) as welfare measures.

Proposition 5. *Credit provision and NPV are (weakly) higher under transparency.*

Proof. See the Appendix. ■

As discussed above, total effort levels and the project's NPV do not depend on the amount of information possessed by the lender. Hence, when we consider NPV as a welfare measure, the optimal transparency regime is the one that maximizes credit provision. Since transparency never induces credit rationing, while opacity does whenever $\lambda > r^{-1}(b)$, a transparent algorithm is weakly preferred to an opaque one when the welfare measure is credit provision or the project's NPV.

7 Conclusion

This paper studied whether lenders' credit risk algorithms should be transparent or opaque. It shows that transparency makes the lender's model vulnerable to gaming by strategic borrowers undermining the model's ability to ration credit efficiently. Not only do bad borrowers hide information to escape rationing, but also very good ones do so to get better contractual terms. Opacity makes credit rationing possible but also induces an information loss due to borrowers' hedging against the uncertainty it generates. Ultimately, both transparency regimes entail a profit loss caused by the information withheld by strategic borrowers, and what is best for the lender or the credit market as a whole depends on economic conditions and data quality.

Some questions remain open for further research. First, the optimal transparency regime may lie in between full transparency and full opacity. Partial disclosure about the model's parameters may grant better outcomes. Second, this paper has analyzed data used in the lender's screening process but may be interesting to extend the analysis to data used in the monitoring process. Third, the paper considers a monopolistic lender, studying a competitive market seems a natural path for future research.

References

Acquisti, Alessandro, Curtis Taylor and Liad Wagman, (2016), *Journal of Economic Literature*, vol 54, No 2, pp. 442-92.

Agarwal, Sumit and Alok, Shashwat and Ghosh, Pulak and Gupta, Sudip, (2019), Financial Inclusion and Alternate Credit Scoring: Role of Big Data and Machine Learning in Fintech, Working Paper.

Ali, S. Nageeb, Greg Lewis and Shoshana Vasserman, (2023), Voluntary Disclosure and Personalized Pricing, *Review of Economic Studies*, Volume 90, Issue 2, March 2023, Pages 538–571.

Ball, Ian, (2022), Scoring Strategic Agents, Working Paper.

Bazarbash, Majid, (2019), FinTech in Financial Inclusion: Machine Learning Applications in Assessing Credit Risk, IMF Working Paper No. 2019/109.

Berg, Tobias, Valentin Burg, Ana Gombović, Manju Puri, (2020), On the Rise of FinTechs: Credit Scoring Using Digital Footprints, *Review of Financial Studies*, vol. 33(7), pages 2845-2897.

Bergemann, Dirk and Alessandro Bonatti, (2019), Markets for Information: An Introduction, *Annual Review of Economics*, Vol. 11:85-107.

Bergemann, Dirk and Morris, Stephen, (2019), Information Design: A Unified Perspective, *Journal of Economic Literature*, 57, issue 1, p. 44-95.

Besanko, David and Anjan Thakor, (1987), Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets, *International Economic Review*, 28, issue 3, p. 671-89.

Bester, Helmut, (1985), Screening vs. Rationing in Credit Markets with Imperfect Information, *American Economic Review*, 75, issue 4, p. 850-55.

Bergemann, Dirk, Benjamin Brooks, and Stephen Morris, (2015), The limits of price discrimination, *American Economic Review*, 105, 921–957.

Bond, Philip and Zeng, Yao, (2022), Silence is safest: Information disclosure when the audience's preferences are uncertain, *Journal of Financial Economics*, 145, issue 1, p. 178-193.

Cowgill, Bo and Tucker, Catherine E., (2020), Algorithmic Fairness and Economics, Columbia Business School Research Paper.

Cohn, Jonathan, Rajan, Uday and Strobl, Gunter (2023), Information Manipulation and Optimal Screening, Working Paper.

Dworczak, Piotr and Martini, Giorgio, (2019), The Simple Economics of Optimal Persuasion, *Journal of Political Economy*, 127, issue 5, p. 1993 - 2048.

Dye, Ra, (1985), Disclosure of Nonproprietary Information, *Journal of Accounting Research*, 23, issue 1, p. 123-145.

Ederer, Florian and Holden, Richard and Meyer, Margaret A., (2018) Gaming and Strategic Opacity in Incentive Provision, *RAND Journal of Economics*, Vol. 49, No. 4, pp. 819-854, Winter 2018.

Fishman, Michael J. and Hagerty, Kathleen M., (2003), Mandatory Versus Voluntary Disclosure in Markets with Informed and Uninformed Customers, *The Journal of Law, Economics, and Organization*, 19, issue 1, p. 45-63.

Frankel, Alex and Kartik, Navin, (2019), Muddled Information, *Journal of Political Economy*, 127, issue 4, p. 1739 - 1776.

Frankel, Alex and Kartik, Navin, (2021), Improving Information from Manipulable Data, *Journal of the European Economic Association*, Volume 20, Issue 1, February 2022, p. 79–115.

Freixas, Xavier and Jean-Jacques Laffont, (1990), Optimal Banking Contracts, in Essays in Honor of Edmond Malinvaud, MIT Press, vol. 2: Macroeconomics, 1990, pp. 33–61.

Fuster, Andreas, Paul Goldsmith-Pinkham, Tarun Ramadorai and Ansgar Walther (2022), Predictably Unequal? The Effects of Machine Learning on Credit Markets, *Journal of Finance*, 77: 5-47.

Grossman, Sanford, (1981), The Informational Role of Warranties and Private Disclosure about Product Quality, *Journal of Law and Economics*, 24, issue 3, p. 461-83.

Gambacorta, Leonardo, Yiping Huang, Zhenhua Li, Han Qiu, and Shu Chen, (2020), Data vs Collateral, BIS Working Paper, No 881.

Harbaugh, Richmond and Ted To, (2020), False modesty: When disclosing good news looks bad, *Journal of Mathematical Economics*, 87, issue C, p. 43-55.

Hardt, Moritz, Nimrod Megiddo, Christos Papadimitriou, and Mary Wooters, (2016), Strategic Classification, in Proceedings of the 2016 ACM Conference on Innovations in Theoretical Computer Science, pp. 111–122.

He, Zhiguo, Jing Huang and Jidong Zhou, (2023), Open banking: Credit market competition when borrowers own the data, *Journal of Financial Economics*, Volume 147, Issue 2, Pages 449-474.

Holmstrom, Bengt and Jean Tirole, (1997), Financial Intermediation, Loanable Funds, and the Real Sector, *Quarterly Journal of Economics*, vol. 112, no. 3, 1997, pp. 663–91.

Iyer, Rajkamal, Asim Ijaz Khwaja, Erzo F. P. Luttmer, and Kelly Shue, (2016), Screening Peers Softly: Inferring the Quality of Small Borrowers, *Management Science*, 62.6, 1554–1577.

Jansen, Mark, Fabian Nagel, Constantine Yannelis, and Anthony Lee Zhang, (2023), Data and Welfare in Credit Markets, Working Paper.

Kamenica Emir and Gentzkow, Matthew, (2011), Bayesian Persuasion, *American Economic Review*, 101, issue 6, p. 2590-2615.

Kleinberg, Jon, Jens Ludwig, Sendhil Mullainathan, and Ashesh Rambachan, (2018), Algorithmic Fairness, *AEA Papers and Proceedings*, 108: 22-27.

Lian Annie and Erik Madsen, (2024), Data and Incentives, *Theoretical Economics*, Vol. 19 (1), Pages 407-448.

Leitner, Yaron and Williams, Basil, (2023), Model Secrecy and Stress Tests, *Journal of Finance*, Volume 78, Pages 1055-1095.

Lewis, Tracy and Sappington, David, (1994), Supplying Information to Facilitate Price Discrimination, *International Economic Review*, 35, issue 2, p. 309-27.

Milgrom, Paul, (1981), Good News and Bad News: Representation Theorems and Applications, *Bell Journal of Economics*, 12, issue 2, p. 380-391.

Perez-Richet, Eduardo and Skreta, Vasiliki, (2022), Test Design Under Falsification, *Econometrica*, 90, issue 3, p. 1109-1142.

Quigley, Daniel and Ansgar Walther, (forthcoming), Inside and Outside Information, *Journal of Finance*.

Sun, Jian, (2021), Algorithmic Transparency, Working Paper.

Verrecchia, Robert E., (1983), Discretionary Disclosure, *Journal of Accounting and Economics*, 5, issue 1, p. 179-194.

Vives, Xavier, (2019), Digital Disruption in Banking, *Annual Review of Financial Economics*, Vol. 11:243-272.

Wang, Qiaochu, Yan Huang, Stefanus Jasin and Param Vir Singh, (2023), Algorithmic Transparency With Strategic Users, *Management Science*, Volume 69, Issue 4, Pages 1935-2545.

Appendix

Lemma A.1 (Optimal Allocation for General Information Set)**.** *Given some information set* $\mathcal{I}$*, the lender's optimal contract is:*

$$\begin{aligned} \ell(\mathcal{I}) &= \mathbb{1}_{\left\{\mathbb{E}(\theta|\mathcal{I})<X-\sqrt{4\alpha}\right\}}, \\ x(\mathcal{I}) &= \frac{X+\mathbb{E}(\theta|\mathcal{I})}{2}. \end{aligned} \tag{12}$$

The lender's profits, borrower's utility, and social welfare are given by (16)-(18).

Proof. The borrower's optimal effort level given his repayment in case of success x can be derived by taking the first order condition of his maximization problem in Equation (2):

$$x-(\theta+\alpha e)=0.$$

Solving this condition for e yields:

$$e=\frac{x-\theta}{\alpha}\triangleq e(\theta).$$

Plugging this in the lender's objective function her problem becomes:

$$\max_{\substack{x\in[0,X]\\ \ell\in\{0,1\}}} \quad \ell\left(\frac{x-\mathbb{E}(\theta|\mathcal{I})}{\alpha}(X-x)-1\right)$$

Taking the first order conditions with respect to x we get:

$$\frac{1}{\alpha}\Big((X-x)+\big(x-\mathbb{E}(\theta|\mathcal{I})\big)(-1)\Big)=0,$$

and solving for x we get the optimal repayment to the borrower:

$$x=\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2}\triangleq x(\mathcal{I}). \tag{13}$$

At the optimal incentive payment, the lender's expected profits are:

$$\begin{aligned} V_L(\mathcal{I}) &= \max_{\ell\in\{0,1\}} \ \ell\left(\frac{x(\mathcal{I})-\mathbb{E}(\theta|\mathcal{I})}{\alpha}\big(X-x(\mathcal{I})\big)-1\right), \\ &= \max_{\ell\in\{0,1\}} \ \ell\left(\frac{1}{\alpha}\left[\left(\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2}-\mathbb{E}(\theta|\mathcal{I})\right)\left(X-\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2}\right)\right]-1\right), \\ &= \max_{\ell\in\{0,1\}} \ \ell\left(\frac{1}{\alpha}\left(\frac{X-\mathbb{E}(\theta|\mathcal{I})}{2}\right)^2-1\right). \end{aligned}$$

The lender provides credit only if she makes positive expected profits, which happens when the expected cost of effort is sufficiently low:

$$\ell=\mathbb{1}_{\left\{\mathbb{E}(\theta|\mathcal{I})<X-\sqrt{4\alpha}\right\}}\triangleq\ell(\mathcal{I}). \tag{14}$$

At the optimum, the lender's expected profits are:

$$V_L(\mathcal{I}) = \mathbb{1}_{\left\{\mathbb{E}(\theta|\mathcal{I})<X-\sqrt{4\alpha}\right\}} \left(\left(\frac{X-\mathbb{E}(\theta|\mathcal{I})}{2} \right)^2 - 1 \right). \tag{15}$$

At the optimum, the lender's profits from the θ-borrower are:

$$\begin{aligned} V_L(\mathcal{I},\theta) &= \ell(\mathcal{I}) \Bigg(e(\theta)\big(X - x(\mathcal{I})\big) - 1 \Bigg) \\ &= \mathbb{1}_{\left\{\mathbb{E}(\theta|\mathcal{I})<X-\sqrt{4\alpha}\right\}} \left(\left(\frac{1}{\alpha} \left(\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2} - \theta \right) \frac{X-\mathbb{E}(\theta|\mathcal{I})}{2} \right) - 1 \right), \end{aligned} \tag{16}$$

the θ-borrower's utility is:

$$\begin{aligned} V_B(\mathcal{I},\theta) &= \ell(\mathcal{I}) \left(e(\theta)\big(x(\mathcal{I}) - \theta\big) - \frac{1}{2}\alpha e(\theta)^2 \right) \\ &= \mathbb{1}_{\left\{\mathbb{E}(\theta|\mathcal{I})<X-\sqrt{4\alpha}\right\}} \frac{1}{2\alpha} \left(\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2} - \theta \right)^2, \end{aligned} \tag{17}$$

while the social welfare generated by the θ-borrower is:

$$\begin{aligned} W(\mathcal{I},\theta) &= V_L(\mathcal{I}) + V_B(\mathcal{I}) \\ &= \ell(\mathcal{I}) \left(e(\theta)X - 1 - \left(\theta e(\theta) + \frac{1}{2} e(\theta)^2 \right) \right) \\ &= \ell(\mathcal{I}) \left(\frac{1}{\alpha} \left(\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2} - \theta \right) X - 1 - \frac{1}{2\alpha} \left(\left(\frac{X+\mathbb{E}(\theta|\mathcal{I})}{2} \right)^2 - \theta^2 \right) \right). \end{aligned} \tag{18}$$

■

Lemma A.2 (Inference). *Given a data-sharing strategy of the form:*

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [0,a] \cup [b,1] \triangleq S(a,b) \\ s & \text{if } s \in (a,b) \end{cases}$$

where $0 < a < b < 1$, *the lender's inference is:*

$$\begin{aligned} \mathbb{E}_\lambda(\theta|s) =& \lambda s + (1-\lambda)1/2, \\ \mathbb{E}_\lambda(\theta|\varnothing) =& \lambda \left(\omega(a,b)\frac{a}{2} + \big(1-\omega(a,b)\big)\frac{1+b}{2} \right) + (1-\lambda)\frac{1}{2}, \end{aligned} \tag{19}$$

where $\omega(a,b) = \frac{a}{a+1-b}$.

Proof. Given the data-generating process in (1), after observing s, the lender's posterior beliefs are:

$$f_\lambda(\theta|s) = \lambda\delta(\theta - s) + (1 - \lambda), \tag{20}$$

where

$$\delta(t-a) \triangleq \begin{cases} \infty & \text{if } t - a = 0 \\ 0 & \text{if } t - a \neq 0, \end{cases}$$

is the Dirac delta function satisfying the following property:

$$\int_{a-\varepsilon}^{a+\varepsilon} f(t)\delta(t-a)\mathrm{d}t = f(a) \quad \forall\varepsilon > 0.$$

Hence we have:

$$\begin{aligned} \mathbb{E}_\lambda(\theta|s) &= \int_0^1 \theta f_\lambda(\theta|s)\mathrm{d}\theta, \\ &= \lambda \int_0^1 \theta\delta(\theta - s)\mathrm{d}\theta + (1-\lambda)\int_0^1 \theta \mathrm{d}\theta, \\ &= \lambda s + (1-\lambda)1/2. \end{aligned} \tag{21}$$

The lender's posterior beliefs following $m = \varnothing$ are:

$$\begin{aligned} f_\lambda(\theta|\varnothing) =& \frac{\Pr\big(s \in S(a,b)|\theta\big)f(\theta)}{\Pr\big(s \in S(a,b)\big)} \\ =& \frac{\int_0^a \lambda\delta(\theta - s) + (1-\lambda)\mathrm{d}s + \int_b^1 \lambda\delta(\theta - s) + (1-\lambda)\mathrm{d}s}{\int_0^a \mathrm{d}s + \int_b^1 \mathrm{d}s} \\ =& \lambda\frac{\mathbb{1}\{\theta \le a\} + \mathbb{1}\{\theta \ge b\}}{a + 1 - b} + (1-\lambda). \end{aligned}$$

Thus we have:

$$\begin{aligned} \mathbb{E}_\lambda(\theta|\varnothing) =& \int_0^1 \theta\left(\lambda\frac{\mathbb{1}\{\theta \le a\} + \mathbb{1}\{\theta \ge b\}}{a+1-b} + (1-\lambda)\right)\mathrm{d}\theta \\ =& \lambda\frac{1}{a+1-b}\left(\int_0^a \theta\mathrm{d}\theta + \int_b^1 \theta\mathrm{d}\theta\right) + (1-\lambda)\int_0^1 \theta\mathrm{d}\theta \\ =& \lambda\left(\frac{a}{a+1-b}\frac{a}{2} + \frac{1-b}{a+1-b}\frac{1+b}{2}\right) + (1-\lambda)\frac{1}{2}. \end{aligned}$$

■

Proof of Lemma 1. The lemma follows from Lemma A.1 and Lemma A.2 replacing the expectation in the first line of (19) in equations (13), (14) and(17). ■

Proof of Proposition 1. Consider the candidate borrower's data-sharing strategy:

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [0,a] \cup [b,1] \\ s & \text{if } s \in \big(a,b\big) \end{cases}$$

where a, b are such that $0 < a < b < 1$ and need to be determined.

We first determine the borrower's value from data-sharing and data-withholding. The lender's optimal allocation and the resulting utility for a borrower of type θ are given in Lemma A.1 replacing the inference in Lemma A.2 equation (19). Taking expectations over θ we get the borrower's value from data-withholding and data-sharing:

$$\begin{aligned} V_\lambda^B(\varnothing) &= \mathbb{1}_{\{\mathbb{E}_\lambda(\theta|\varnothing) < X - \sqrt{4\alpha}\}} \frac{1}{2\alpha} \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2} - \theta \right)^2 f_\lambda(\theta|s)\, \mathrm{d}\theta, \\ V_\lambda^B(s) &= \mathbb{1}_{\{s < r(\lambda)\}} \frac{1}{2\alpha} \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|s)}{2} - \theta \right)^2 f_\lambda(\theta|s)\, \mathrm{d}\theta, \end{aligned} \tag{22}$$

where

$$r(\lambda) = 1/2 + \frac{1}{\lambda} \left(X - \sqrt{4\alpha} - 1/2 \right). \tag{23}$$

We now determine the equilibrium thresholds (a, b). Suppose the lender provides credit after no data-sharing, that is $\mathbb{E}_\lambda(\theta|\varnothing) < X - \sqrt{4\alpha}$ (this will be verified later). Since all the data realizations $s \in [r(\lambda), 1]$ are credit rationed if shared, the borrower prefers to withhold them to get $V_\lambda^B(\varnothing) > 0$. Hence, in equilibrium, we have $b = r(\lambda)$. The threshold $a \in \big(0, r(\lambda)\big)$ can be computed as the data realization that solves $V_\lambda^B(a) = V_\lambda^B(\varnothing)$. The equation reduces to $\mathbb{E}_\lambda(\theta|a) = \mathbb{E}_\lambda(\theta|\varnothing)$ and further simplifies to:

$$a = \frac{a}{a+1-b}\frac{a}{2} + \frac{1-b}{a+1-b}\frac{1+b}{2}.$$

The above equation has two solutions: one is negative and thus cannot be an equilibrium threshold, while the other one is:

$$a = -\big(1-b\big) + \sqrt{2\big(1-b\big)} \triangleq a(b),$$

It follows that the equilibrium thresholds of the data-sharing strategy are:

$$\begin{aligned} a &= a(r(\lambda)) \triangleq l(\lambda), \\ b &= r(\lambda). \end{aligned} \tag{24}$$

where $r(\lambda) \in (1/2, 1)$ by Assumptions 1 and 2 and $a(r(\lambda)) \in (0, r(\lambda)$. ■

Proof of Proposition 2. Consider the candidate borrower's data-sharing strategy:

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [0,a] \cup [b,1] \\ s & \text{if } s \in \big(a,b\big) \end{cases}$$

where a,b are such that $0 < a < b < 1$ and need to be determined.

We first determine the borrower's value from data-sharing and data-withholding. For a given λ, the lender's optimal allocation can be computed as in Proposition 1, so that the resulting values for the borrower are given by equations (22). Taking expectation over λ we get the borrower's value from data-withholding and data-sharing. In particular, suppose the lender provides credit after no data-sharing for every λ, that is $\mathbb{E}_\lambda(\theta|\varnothing) < X - \sqrt{4\alpha}$ for any $\lambda \in [\underline{\lambda}, 1]$ (this will be verified later), the borrower's value from data-withholding is:

$$\begin{aligned} V^B(\varnothing) &= \int_{\underline{\lambda}}^1 V_\lambda^B(\varnothing)\, \frac{1}{1-\underline{\lambda}} \mathrm{d}\lambda \\ &= \frac{1}{1-\underline{\lambda}} \frac{1}{2\alpha} \int_{\underline{\lambda}}^1 \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2} - \theta \right)^2 f_\lambda(\theta|s)\, \mathrm{d}\theta \mathrm{d}\lambda \end{aligned} \tag{25}$$

while the value from data-sharing is:

$$\begin{aligned} V^B(s) &= \int_{\underline{\lambda}}^1 V_\lambda^B(s)\, \frac{1}{1-\underline{\lambda}} \mathrm{d}\lambda \\ &= \frac{1}{1-\underline{\lambda}} \frac{1}{2\alpha} \int_{\underline{\lambda}}^1 \mathbb{1}_{\{s<r(\lambda)\}} \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|s)}{2} - \theta \right)^2 f_\lambda(\theta|s)\, \mathrm{d}\theta \mathrm{d}\lambda \\ &= \frac{1}{1-\underline{\lambda}} \frac{1}{2\alpha} \int_{\underline{\lambda}}^{\min\{1,\widehat{\lambda}(s)\}} \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|s)}{2} - \theta \right)^2 f_\lambda(\theta|s)\, \mathrm{d}\theta \mathrm{d}\lambda, \end{aligned} \tag{26}$$

where

$$\widehat{\lambda}(s) \triangleq r^{-1}(s) = \frac{1}{s - \frac{1}{2}} \left(X - \sqrt{4\alpha} - \frac{1}{2} \right) \tag{27}$$

is the inverse function of the rationing threshold $r(\lambda)$ defined in (23).

We now determine the conditions that defines the equilibrium thresholds (a,b). Suppose that $a \in \big(0, r(1)\big)$ and $b \in \big(r(1), 1\big)$ so that $\widehat{\lambda}(a) > 1$ and $\widehat{\lambda}(b) < 1$. The thresholds a,b can be computed as the data realizations that solve the following system of equations:

$$\begin{cases} \int_{\underline{\lambda}}^1 \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|a)}{2} - \theta \right)^2 f_\lambda(\theta|a)\, \mathrm{d}\theta \mathrm{d}\lambda = \int_{\underline{\lambda}}^1 \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2} - \theta \right)^2 f_\lambda(\theta|a)\, \mathrm{d}\theta \mathrm{d}\lambda \\ \int_{\underline{\lambda}}^{\widehat{\lambda}(b)} \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|b)}{2} - \theta \right)^2 f_\lambda(\theta|b)\, \mathrm{d}\theta \mathrm{d}\lambda = \int_{\underline{\lambda}}^1 \int_0^1 \left(\frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2} - \theta \right)^2 f_\lambda(\theta|b)\, \mathrm{d}\theta \mathrm{d}\lambda. \end{cases} \tag{28}$$

The first equation has a unique solution for $a \in (0, r(1))$, this is:

$$a = -(1-b) + \sqrt{2(1-b)} \triangleq a(b).$$

for which we have:

$$\begin{aligned} \mathbb{E}_\lambda\big(\theta|\varnothing\big) &= \mathbb{E}_\lambda\big(\theta|a(b)\big) \\ &= \lambda\big(a(b)\big) + (1-\lambda)\frac{1}{2} \end{aligned} \tag{29}$$

for every λ. Substituting (29) in the right-hand side of the second equation in (28) we get the equilibrium condition for b:

$$L(b) = R(b), \tag{30}$$

where

$$\begin{aligned} L(b) &\triangleq \int_{\underline{\lambda}}^{\widehat{\lambda}(b)} \int_0^1 \left(\frac{X + \lambda b + (1-\lambda)\frac{1}{2}}{2} - \theta \right)^2 f_\lambda(\theta|b)\, \mathrm{d}\theta \mathrm{d}\lambda \\ R(b) &\triangleq \int_{\underline{\lambda}}^{1} \int_0^1 \left(\frac{X + \lambda\big(a(b)\big) + (1-\lambda)\frac{1}{2}}{2} - \theta \right)^2 f_\lambda(\theta|b)\, \mathrm{d}\theta \mathrm{d}\lambda. \end{aligned}$$

We now show that $L(b)$ and $R(b)$ cross in the interval $(r(1), 1)$. To see this note that:

$$\begin{aligned} &\lim_{b \to r(1)} L(b) = \int_{\underline{\lambda}}^{1} \int_0^1 \left(\frac{X + \lambda\, r(1) + (1-\lambda)\frac{1}{2}}{2} - \theta \right)^2 f_\lambda(\theta|r(1))\, \mathrm{d}\theta \mathrm{d}\lambda, \\ &\lim_{b \to r(1)} R(b) = \int_{\underline{\lambda}}^{1} \int_0^1 \left(\frac{X + \lambda\big(a(r(1))\big) + (1-\lambda)\frac{1}{2}}{2} - \theta \right)^2 f_\lambda(\theta|r(1))\, \mathrm{d}\theta \mathrm{d}\lambda, \\ &\lim_{b \to 1} L(b) = 0, \\ &\lim_{b \to 1} R(b) = \int_{\underline{\lambda}}^{1} \int_0^1 \left(\frac{X + (1-\lambda)\frac{1}{2}}{2} - \theta \right)^2 f_\lambda(\theta|1)\, \mathrm{d}\theta \mathrm{d}\lambda. \end{aligned} \tag{31}$$

The first line in (31) follows from the fact that $\widehat{\lambda}(r(1)) = 1$ (see (27)), while the third line follows from the fact that $\widehat{\lambda}(1) = \underline{\lambda}$ (see (27) and Assumption 3). The second and last lines are trivial. It follows that:

$$\begin{aligned} \lim_{b \to r(1)} L(b) &> \lim_{b \to r(1)} R(b), \\ \lim_{b \to 1} L(b) &< \lim_{b \to 1} R(b). \end{aligned} \tag{32}$$

The first line in (32) follows from the fact that the integrand on the right-hand side is greater than the integrand on the left-hand side for every λ since Assumption 2 implies that $r(1) \in (1/2, 1)$ so that $r(1) > a(r(1)) = -(1-r(1)) + \sqrt{2(1-r(1))}$. The second line is trivial. ■

Proof of Lemma 2. The social planner solves:

$$\max_{\substack{e\in[0,1]\\ \ell\in\{0,1\}}} \quad \ell\left(eX-1-\left(\theta e+\frac{1}{2}\alpha e^2\right)\right). \tag{33}$$

The first order conditions with respect to e is:

$$X-\theta-\alpha e=0,$$

and is solved for $e=\frac{X-\theta}{\alpha}$, which is the optimal effort level whenever it is lower than 1. We denote the optimal effort level as

$$e^*(\theta)\triangleq\min\left\{1,\frac{X-\theta}{\alpha}\right\}.$$

Social welfare at the optimal effort level is

$$\begin{aligned}W(e^*(\theta)) &= \min\left\{(X-\theta)-\frac{1}{2}\alpha,\frac{1}{2\alpha}(X-\theta)^2\right\}-1\\ &= \begin{cases}(X-\theta)-\frac{1}{2}\alpha-1 & \text{if } \theta\le X-\alpha\\ \frac{1}{2\alpha}(X-\theta)^2-1 & \text{if } \theta> X-\alpha.\end{cases}\end{aligned}$$

Note that

$$W(e^*(1))\ge\frac{1}{2\alpha}(X-1)^2-1>0,$$

where the second inequality is equivalent to $X>1+\sqrt{2\alpha}$ which is satisfied under Assumptions 1 and 2. Since $W(e^*(\theta))$ is a continuous and decreasing function of θ and $W(e^*(1))>0$, social welfare is positive for every θ and it is socially efficient to provide credit to every borrower, i.e.

$$\ell^*(\theta)=1\quad\forall\theta\in[0,1].$$

■

Proof of Proposition 3. Fix some $s\in[0,1]$ and some $z\in(1/2,1)$, and define the following functions

$$\begin{aligned}a(z) &\triangleq-(1-z)+\sqrt{2(1-z)},\\ m_z(s) &\triangleq\begin{cases}\varnothing & \text{if } s\in[0,a(z)]\cup[z,1]\\ s & \text{if } s\in\big(a(z),z\big),\end{cases}\\ K(s) &\triangleq\frac{1}{\alpha}\left(\frac{X-\big(\lambda s+(1-\lambda)\frac{1}{2}\big)}{2}\right)^2,\\ H(z) &\triangleq\big(a(z)+1-z\big)K(a(z))+\int_{a(z)}^{z}K(s)\mathrm{d}s.\end{aligned} \tag{34}$$

The proof of the proposition follows from the following three claims.

Claim 1. *For a given λ, the lender's equilibrium profits under transparency and opacity are, respectively:*

$$V_T^L(\lambda) = H(r(\lambda)) - 1, \tag{35}$$

and

$$V_O^L(\lambda) = \begin{cases} H(b) - 1 & \text{if } b < r(\lambda) \\ H(b) - 1 - \displaystyle\int_{(r(\lambda),b)} K(s) - 1 \,\mathrm{d}s & \text{if } b \geq r(\lambda). \end{cases} \tag{36}$$

where $H(z)$ is defined in (34), $r(\lambda)$ is defined in (3), and b solves (30).

Proof. From Lemma A.1, we know the lender's realized profits at the optimum $V_L(\mathcal{I}, \theta)$ when she faces a borrower of type θ and holds some information $\mathcal{I}$ (see Equation (16)). From Proposition 1 and Proposition 2, we know the equilibrium disclosure strategies under transparency and opacity are respectively $m_{r(\lambda)}(s)$ and $m_b(s)$ where $m_z(s)$ is defined in (34), $r(\lambda)$ is defined in (3), and b solves (30). Hence, we can compute the equilibrium profits of the lender for a given λ by integrating $V_L(m_z(s), \theta)$ at the equilibrium disclosure strategies in both transparency regimes.
First, consider the case of transparency. For a given λ, the lender's equilibrium expected profits are:

$$\begin{aligned}
V_T^L(\lambda) &= \int_{[0,1]} \int_{[0,1]} V_L\big(m_{r(\lambda)}(s), \theta\big)\, f_\lambda(\theta, s)\,\mathrm{d}\theta\,\mathrm{d}s, \\
&= \int_{[0,1]} \ell_\lambda(m_{r(\lambda)}(s)) \left[\frac{1}{\alpha} \left(\frac{X + \mathbb{E}_\lambda(\theta|m_{r(\lambda)}(s))}{2} - \mathbb{E}_\lambda(\theta|s) \right) \frac{X - \mathbb{E}_\lambda(\theta|m_{r(\lambda)}(s))}{2} - 1 \right] \mathrm{d}s, \\
&= \int_{[0,a(r(\lambda))]\cup[r(\lambda),1]} \frac{1}{\alpha} \left(\frac{X + \mathbb{E}_\lambda(\theta|\varnothing)}{2} - \mathbb{E}_\lambda(\theta|s) \right) \frac{X - \mathbb{E}_\lambda(\theta|\varnothing)}{2} \mathrm{d}s \,+ \\
&\quad + \int_{\left(a(r(\lambda)),r(\lambda)\right)} \frac{1}{\alpha} \left(\frac{X + \mathbb{E}_\lambda(\theta|s)}{2} - \mathbb{E}_\lambda(\theta|s) \right) \frac{X - \mathbb{E}_\lambda(\theta|s)}{2} \mathrm{d}s - 1, \\
&= \big(a(r(\lambda)) + 1 - r(\lambda)\big) \frac{1}{\alpha} \left(\frac{X - \mathbb{E}_\lambda(\theta|\varnothing)}{2} \right)^2 + \int_{\left(a(r(\lambda)),r(\lambda)\right)} \frac{1}{\alpha} \left(\frac{X - \mathbb{E}_\lambda(\theta|s)}{2} \right)^2 \mathrm{d}s - 1, \\
&= H(r(\lambda)) - 1,
\end{aligned} \tag{37}$$

where $H(r(\lambda))$ is defined in (34). The second line uses the lender's realized profits as computed in Lemma A.1 (Equation (16)). The third and fourth lines use the definition of $m_{r(\lambda)}(s)$ in (34) and the optimal credit allocation in Proposition 1. The fifth line follows from

$$\int_{[0,a(r(\lambda))]\cup[r(\lambda),1]} \mathbb{E}_\lambda(\theta|s)\,\mathrm{d}s = \big(a(r(\lambda)) + 1 - r(\lambda)\big)\mathbb{E}_\lambda(\theta|\varnothing). \tag{38}$$

The last line uses the fact that $\mathbb{E}_\lambda(\theta|\varnothing) = \mathbb{E}_\lambda(\theta|s = a(r(\lambda)))$ by Proposition 1.

Next, consider the lender's profits under opacity. We distinguish two sub-cases depending on whether there is ex-post credit rationing for the λ considered: i) $b < r(\lambda)$, and ii) $r(\lambda) \leq b$. In case i) the lender's expected profits are:

$$\begin{aligned} V_O^L(\lambda) &= \int_{[0,1]} \int_{[0,1]} V_L\big(m_b(s), \theta\big)\, f_\lambda(\theta, s)\, \mathrm{d}\theta\, \mathrm{d}s, \\ &= H(b) - 1. \end{aligned} \tag{39}$$

This is due to the fact that there is no credit rationing, i.e. $\ell_\lambda(m_b(s)) = 1$ for all $s \in [0,1]$ and the lender's profits can be computed as in (37) replacing b to $r(\lambda)$.
In case ii) the lender's expected profits are:

$$\begin{aligned} V_O^L(\lambda) &= \int_{[0,1]} \int_{[0,1]} V_L\big(m_b(s), \theta\big)\, f_\lambda(\theta, s)\, \mathrm{d}\theta\, \mathrm{d}s, \\ &= H(b) - 1 - \int_{(r(\lambda),b)} \int_{[0,1]} \left[\frac{1}{\alpha} \left(\frac{X + \mathbb{E}_\lambda(\theta|s)}{2} - \theta \right) \frac{X - \mathbb{E}_\lambda(\theta|s)}{2} - 1 \right] f_\lambda(\theta, s) \mathrm{d}\theta \mathrm{d}s, \\ &= H(b) - 1 - \int_{(r(\lambda),b)} K(s) - 1\ \mathrm{d}s, \end{aligned} \tag{40}$$

where $H(b)$ and $K(s)$ are defined in (34). Note that data realizations $s \in [r(\lambda), b]$ are disclosed and do not get credit yielding nil profits to the lender, i.e. $\ell_\lambda(m_b(s)) = 0$ for $s \in \big(r(\lambda), b\big)$ and $\ell_\lambda(m_b(s)) = 1$ for $s \in [0, r(\lambda)] \cup [b, 1]$. The second line follows from adding and subtracting to the first line the profits that the lender would get by providing credit to borrowers with $s \in (r(\lambda), b)$ and performing computations similar to (37). ■

Claim 2. *$H(z)$ is strictly increasing.*

Proof. We have

$$\begin{aligned} H'(z) &= \big(a'(z) - 1\big) K(a(z)) + \big(a(z) + 1 - z\big) K'(a(z)) a'(z) + K(z) - K(a(z)) a'(z), \\ &= \big(a(z) + 1 - z\big) a'(z) K'(a(z)) + K(z) - K(a(z)), \end{aligned} \tag{41}$$

where

$$\begin{aligned} a(z) + 1 - z &= \sqrt{2(1-z)}, \\ a'(z) &= 1 - \frac{1}{\sqrt{2(1-z)}} < 0, \\ K'(a(z)) &= -\lambda \frac{1}{\alpha} \left(\frac{X - (\lambda a(z) + (1-\lambda)\frac{1}{2})}{2} \right) < 0, \\ K(z) - K(a(z)) &= -\lambda \frac{1}{\alpha} \big(z - a(z)\big) \left(\frac{X - \left(\lambda \frac{a(z)+z}{2} + (1-\lambda)\frac{1}{2}\right)}{2} \right) < 0. \end{aligned} \tag{42}$$

Noting that:

$$
\begin{aligned}
\big(a(z)+1-z\big)a'(z) &= \sqrt{2(1-z)}\left(1-\frac{1}{\sqrt{2(1-z)}}\right), \\
&= -\big(1-\sqrt{2(1-z)}\big), \\
&= -\big(z-a(z)\big),
\end{aligned}
\tag{43}
$$

we have:

$$
\begin{aligned}
H'(z) &= \frac{\lambda^2}{2\alpha}\big(z-a(z)\big)\left(\frac{a(z)+z}{2}-a(z)\right), \\
&= \frac{\lambda^2}{4\alpha}\big(z-a(z)\big)^2 > 0.
\end{aligned}
$$

■

Claim 3. $V_T^L(\lambda) > V_O^L(\lambda) \iff r(\lambda) > b$.

Proof. For $r(\lambda) > b$ we have $V_T^L(\lambda) > V_O^L(\lambda)$ since $H(z)$ is increasing in z. For $b > r(\lambda)$ we have $V_O^L(\lambda) > V_T^L(\lambda)$ since $H(z)$ is increasing in z and the integrand in the definition of $V_O^L(\lambda)$ in (36) is negative for $s > r(\lambda)$ by Lemma 1. ■

The proof of Proposition 3 follows from Claims 1-3. ■

Proof of Proposition 4. Fix some $s \in [0,1]$ and some $z \in (1/2, 1)$, and define the following functions:

$$
\begin{aligned}
M(s) &\triangleq \frac{1}{\alpha}\left(\frac{X-\big(\lambda s+(1-\lambda)\frac{1}{2}\big)}{2}\right)X - \frac{1}{2\alpha}\left(\frac{X+\lambda s+(1-\lambda)\frac{1}{2}}{2}\right)^2, \\
J(z) &\triangleq \big(a(z)+1-z\big)M\big((a(z)\big) + \int_{a(z)}^{z} M(s)\ \mathrm{d}s + \frac{1}{6\alpha}.
\end{aligned}
\tag{44}
$$

The proof of the proposition follows from the following three claims.

Claim 1. *For a given λ, social welfare under transparency and opacity are, respectively:*

$$
W_T(\lambda) = J(r(\lambda)) - 1, \tag{45}
$$

and

$$
W_O(\lambda) = \begin{cases} J(b)-1 & \text{if } b < r(\lambda) \\ J(b)-1-\displaystyle\int_{(r(\lambda),b)}\int_{[0,1]} \widehat{W}(s,\theta) f_\lambda(\theta,s)\ \mathrm{d}\theta \mathrm{d}s & \text{if } b \geq r(\lambda). \end{cases} \tag{46}
$$

where $J(z)$ is defined in (44) and

$$
\widehat{W}(s,\theta) \triangleq \frac{1}{\alpha}\Big(x_\lambda(s)-\theta\Big)X - \frac{1}{2\alpha}\Big(x_\lambda(s)^2-\theta^2\Big) - 1. \tag{47}
$$

Proof. From Lemma A.1, we know the realized social welfare at the optimum $W(\mathcal{I},\theta)$ when the lender faces a borrower of type θ and holds some information $\mathcal{I}$ (see Equation (18)). From Proposition 1 and Proposition 2, we know the equilibrium disclosure strategies under transparency and opacity are respectively $m_{r(\lambda)}(s)$ and $m_b(s)$ where $m_z(s)$ is defined in (34), $r(\lambda)$ is defined in (3), and b solves (30). Hence, we can compute the equilibrium social welfare for a given λ by integrating $W(m_z(s),\theta)$ at the equilibrium disclosure strategies in both transparency regimes.
First, consider the case of transparency. Expected social welfare is:

$$
\begin{aligned}
W_T(\lambda) &= \int_{[0,1]}\int_{[0,1]} W\big(m_{r(\lambda)}(s),\theta\big)\, f_\lambda(\theta,s)\,\mathrm{d}\theta\,\mathrm{d}s,\\
&= \int_{[0,1]} \ell_\lambda(m_{r(\lambda)}(s))\left[\frac{1}{\alpha}\Big(x_\lambda\big(m_{r(\lambda)}(s)\big)-\mathbb{E}_\lambda(\theta|s)\Big)X-1-\frac{1}{2\alpha}\Big(x_\lambda\big(m_{r(\lambda)}(s)\big)^2-\mathbb{E}_\lambda(\theta^2|s)\Big)\right]\mathrm{d}s,\\
&= \int_{[0,a(r(\lambda))]\cup[r(\lambda),1]} \frac{1}{\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|\varnothing)}{2}-\mathbb{E}_\lambda(\theta|s)\right)X-\frac{1}{2\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|\varnothing)}{2}\right)^2\mathrm{d}s\,+\\
&\quad+\int_{\left(a(r(\lambda)),r(\lambda)\right)} \frac{1}{\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|s)}{2}-\mathbb{E}_\lambda(\theta|s)\right)X-\frac{1}{2\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|s)}{2}\right)^2\mathrm{d}s\,+\\
&\quad+\frac{1}{2\alpha}\int_{[0,1]}\mathbb{E}_\lambda(\theta^2|s)\mathrm{d}s-1,\\
&= \big(a(r(\lambda))+1-r(\lambda)\big)\left[\frac{1}{\alpha}\left(\frac{X-\mathbb{E}_\lambda(\theta|\varnothing)}{2}\right)X-\frac{1}{2\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|\varnothing)}{2}\right)^2\right]\,+\\
&\quad+\int_{\left(a(r(\lambda)),r(\lambda)\right)} \frac{1}{\alpha}\left(\frac{X-\mathbb{E}_\lambda(\theta|s)}{2}\right)X-\frac{1}{2\alpha}\left(\frac{X+\mathbb{E}_\lambda(\theta|s)}{2}\right)^2\mathrm{d}s\;+\frac{1}{6\alpha}-1,\\
&= J(r(\lambda))-1,
\end{aligned}
\tag{48}
$$

where $J(r(\lambda))$ is defined in (44). The second line uses the social welfare as computed in Lemma A.1 (Equation (18)). The third and fourth lines use the definition of $m_{r(\lambda)}(s)$ in (34) and the optimal credit allocation in Proposition 1. The fifth line follows from (38) and the fact that $\int_{[0,1]}\mathbb{E}_\lambda(\theta^2|s)\mathrm{d}s=\int_{[0,1]}\lambda s^2+(1-\lambda)\frac{1}{3}\mathrm{d}s=\frac{1}{3}$. The last line uses the fact that $\mathbb{E}_\lambda(\theta|\varnothing)=\mathbb{E}_\lambda(\theta|s=a(r(\lambda)))$ by Proposition 1.
Next, consider social welfare profits under opacity. We distinguish two sub-cases depending on whether there is ex-post credit rationing for the λ considered: i) $b<r(\lambda)$, and ii) $r(\lambda)\le b$. In case i) expected social welfare is:

$$
\begin{aligned}
W_O(\lambda) &= \int_{[0,1]}\int_{[0,1]} W\big(m_b(s),\theta\big)\, f_\lambda(\theta,s)\,\mathrm{d}\theta\,\mathrm{d}s,\\
&= J(b)-1.
\end{aligned}
\tag{49}
$$

Since there is no credit rationing, i.e. $\ell_\lambda(m_b(s)) = 1$ for all $s \in [0,1]$, social welfare can be computed as in (48) replacing b to $r(\lambda)$.
In case ii) expected social welfare is:

$$
\begin{aligned}
W_O(\lambda) &= \int_{[0,1]} \int_{[0,1]} W\big(m_b(s), \theta\big)\, f_\lambda(\theta, s)\, \mathrm{d}\theta\, \mathrm{d}s, \\
&= J(b) - 1 - \int_{(r(\lambda),b)} \int_{[0,1]} \left[\frac{1}{\alpha}\Big(x_\lambda(s) - \theta\Big)X - 1 - \frac{1}{2\alpha}\Big(x_\lambda(s)^2 - \theta^2\Big)\right] f_\lambda(\theta, s)\, \mathrm{d}\theta \mathrm{d}s, \\
&= J(b) - 1 - \int_{(r(\lambda),b)} \int_{[0,1]} \widehat{W}(s,\theta) f_\lambda(\theta, s)\ \mathrm{d}\theta \mathrm{d}s,
\end{aligned}
\tag{50}
$$

where $J(b)$ is defined in (44). Note that data realizations $s \in [r(\lambda), b]$ are disclosed and do not get credit yielding nil social welfare, i.e. $\ell_\lambda(m_b(s)) = 0$ for $s \in \big(r(\lambda), b\big)$ and $\ell_\lambda(m_b(s)) = 1$ for $s \in [0, r(\lambda)] \cup [b, 1]$. The second line follows from adding and subtracting to the first line the social welfare generated by providing credit to borrowers with $s \in (r(\lambda), b)$ and performing computations similar to (48). ■

Claim 2. *J(z) is strictly decreasing.*

Proof. We have

$$
\begin{aligned}
J'(z) &= \big(a'(z) - 1\big)M(a(z)) + \big(a(z) + 1 - z\big)M'(a(z))a'(z) + M(z) - M(a(z))a'(z), \\
&= \big(a(z) + 1 - z\big)a'(z)M'(a(z)) + M(z) - M(a(z)),
\end{aligned}
\tag{51}
$$

where

$$
\begin{aligned}
a(z) + 1 - z &= \sqrt{2(1-z)}, \\
a'(z) &= 1 - \frac{1}{\sqrt{2(1-z)}} < 0, \\
M'(a(z)) &= -\lambda \frac{1}{2\alpha}\left(X + \frac{X + (\lambda a(z) + (1-\lambda)\frac{1}{2})}{2}\right) < 0, \\
M(z) - M(a(z)) &= -\lambda \frac{1}{2\alpha}\big(z - a(z)\big)\left(X + \frac{X + \Big(\lambda \frac{a(z)+z}{2} + (1-\lambda)\frac{1}{2}\Big)}{2}\right).
\end{aligned}
\tag{52}
$$

Noting that:

$$
\begin{aligned}
\big(a(z) + 1 - z\big)a'(z) &= \sqrt{2(1-z)}\left(1 - \frac{1}{\sqrt{2(1-z)}}\right) \\
&= -\big(1 - \sqrt{2(1-z)}\big) \\
&= -\big(z - a(z)\big)
\end{aligned}
\tag{53}
$$

we have:

$$J'(z) = -\frac{\lambda^2}{4\alpha}\big(z - a(z)\big)\left(\frac{a(z)+z}{2} - a(z)\right)$$
$$= -\frac{\lambda^2}{8\alpha}\big(z - a(z)\big)^2 < 0.$$

■

Claim 3. $W_T(\lambda) < W_O(\lambda) \iff r(\lambda) > b$.

Proof. For $r(\lambda) > b$ we have $W_O(\lambda) > W_T(\lambda)$ since $J(z)$ is decreasing in z. Consider now $b > r(\lambda)$. Note that $\widehat{W}(s,\theta)$ can be rewritten as

$$\begin{aligned}\widehat{W}(s,\theta) &= \frac{1}{\alpha}\big(x_\lambda(s) - \theta\big)X - \frac{1}{2\alpha}\Big(x_\lambda(s)^2 - \theta^2\Big) - 1,\\ &= \frac{1}{\alpha}\big(x_\lambda(s) - \theta\big)X - \frac{1}{\alpha}\Big(\theta + \frac{1}{2}\left(x_\lambda(s) - \theta\right)\Big) - 1,\\ &= \frac{1}{\alpha}\big(x_\lambda(s) - \theta\big)\big(X - \theta\big) - \frac{1}{2\alpha}\big(x_\lambda(s) - \theta\big)^2 - 1.\end{aligned} \tag{54}$$

Since $\widehat{W}(s,\theta)$ is convex in θ, by Jensen's inequality, we have:

$$\begin{aligned}\int_{[0,1]} \widehat{W}(s,\theta) f_\lambda(\theta,s)\mathrm{d}\theta &\geq \widehat{W}\left(s, \int_{[0,1]} \theta f_\lambda(\theta,s)\mathrm{d}\theta\right),\\ &= \frac{1}{\alpha}\big(x_\lambda(s) - \mathbb{E}_\lambda(\theta|s)\big)\big(X - \mathbb{E}_\lambda(\theta|s)\big) - \frac{1}{2\alpha}\big(x_\lambda(s) - \mathbb{E}_\lambda(\theta|s)\big)^2 - 1,\\ &= \frac{3}{8\alpha}\left(X - \mathbb{E}_\lambda(\theta|s)\right)^2 - 1 > 0,\end{aligned} \tag{55}$$

where the last inequality is satisfied even when $\mathbb{E}_\lambda(\theta|s) = 1$ under Assumption 1 and 2. It follows that, for $b > r(\lambda)$, we have $W_T(\lambda) > W_O(\lambda)$ since $J(z)$ is decreasing in z and the integral in the definition of $W_O(\lambda)$ in (46) is positive. ■

The proof of Proposition 4 follows from Claims 1-3. ■

Appendix B

Consider the following modification of the model described in Section 3. There exists a mass $\pi \in (0,1)$ of privacy-concerned borrowers that never share data. The remaining mass $1-\pi$ share data strategically. In other words, borrowers have a random disclosure cost $c \in \{0,\infty\}$ where $\Pr(c = \infty) = \pi$ and the randomness in disclosure cost is independent of θ or s. Alternatively, one can think of privacy-concerned borrowers as not having hard information to disclose to the lender as in Dye (1985).

Consider the following data-sharing strategy:

$$m(s) = \begin{cases} \varnothing & \text{if } s \in [\widehat{s}(\lambda), 1] \\ s & \text{if } s \in \big[0, \widehat{s}(\lambda)\big). \end{cases} \tag{56}$$

The lender's inference after no data-sharing can be shown to be:

$$\mathbb{E}(\theta|\varnothing) = \omega(\pi)\frac{1}{2} + \big(1 - \omega(\pi)\big)\left(\lambda\frac{\widehat{s}(\lambda)+1}{2} + (1-\lambda)\frac{1}{2}\right), \tag{57}$$

where $\omega(\pi) = \pi/\big(\pi + (1-\pi)(1-\widehat{s}(\lambda))\big)$. The optimal contract is as in Lemma 1 with $\mathbb{E}(\theta|\varnothing)$ given by (57).

The data-sharing strategy is not an equilibrium if $\pi \in (\underline{\pi}, 1]$. To see this consider a deviation of the borrower with data realization $s = 0$. By sharing data, the lender's inference after disclosure is:

$$\mathbb{E}(\theta|s=0) = (1-\lambda)\frac{1}{2}. \tag{58}$$

Since $\widehat{s}(\lambda) > 1/2$ by Assumption 1, we have that

$$\mathbb{E}(\theta|\varnothing) > 1/2 > (1-\lambda)\frac{1}{2} = \mathbb{E}(\theta|s=0),$$

so that the borrower with data realization $s = 0$ gets a strictly higher payment after no data-sharing. This is a profitable deviation if the lender is willing to provide credit after no data-sharing, i.e. if $\mathbb{E}(\theta|\varnothing) < X - \sqrt{4\alpha}$. This happens if π is sufficiently high.