Specialization and integration in markets for financial information

PRELIMINARY AND INCOMPLETE

Giovanni Cespa,* Junli Zhao,† and Wei Zhao‡

June 3, 2024

Abstract

We argue that when providers of financial information specialize, that is when they either offer certification (paid-for research) to firms or analyst services to investors, their product market decisions are strategic substitutes. This segments the information market, with certified firms displaying more informative security prices than non certified ones. In turn, this compresses the demand for information faced by the analyst, leading it to charge a higher price for information. Conversely, when information sellers integrate, the information monopolist internalizes the negative externality of informative certification and offers investors a more homogeneous cumulative precision and a more precise investment advice at a lower price.

Keywords: Information econonomics, markets for financial information.
JEL Classification Numbers: G10, G12, G14

*Bayes Business School, City, University of London, and CEPR. 106, Bunhill Row, London EC1Y 8TZ, UK. e-mail: giovanni.cespa@gmail.com.

†Bayes Business School, City, University of London. 106, Bunhill Row, London EC1Y 8TZ, UK. e-mail:junli.zhao@city.ac.uk.

‡School of Economics, Renmin University of China. e-mail: wei_zhao@ruc.edu.cn

1 Introduction

Security markets thrive on information. Information is conveyed by firms to perspective investors, to explain the value of the ideas behind their projects and obtain financial backing (White (2010)). Information also helps traders to set up investment strategies, and secure risk-adjusted returns (Admati and Pfleiderer (1986)). Finally, through the concerted actions of investors' trades and firms' disclosure, information is aggregated by asset prices and diffused to the market at large (Diamond and Verrecchia (1991)).

A closer inspection reveals that the way information accrues to markets varies considerably across different securities. At one end of the spectrum, we have securities for which most of the relevant payoff information is facilitated by the security issuer. This is the case, for example, of stocks issued by a large fraction of listed US firms, the corporate bond market, or of some countries' government bonds markets.[1] Somewhat at the other end of the spectrum, we have instead markets in which a considerable amount of information is facilitated by agents that are "closer" to the set of traders that exchange the securities traded therein. A prime example of this latter class are stocks issued by large, liquid, listed firms.

What motivates such a difference? Can economic considerations, such as costs of information production or processing alone determine the structure of a certain sector of the security industry? Alternatively, do the deep parameters of a security market also play a role in explaining these differences?

In this paper we zoom in on the mechanism that allows information to flow from its originators, be them closer to firms like rating agencies or analysts at paid-for research outlets, or more contiguous to secondary markets like sell-side analysts, to prices. More specifically, we analyse the equilibrium that arises in a market where both firms and traders rely on an "information seller" to provide and acquire pay-off relevant information. We consider two cases. In the first case, information providers "specialize," that is one of them provides information to firms (like a paid-for research analyst), while the other one offers

[1] Kirk (2011) reports that about 60% of publicly listed firms in the US "have no analyst coverage[. . .]".

information to traders (like a sell-side analyst). In the second case, they are integrated.

We argue that when providers of financial information specialize, that is when they either offer paid-for research to firms or analyst services to investors, their product market decisions are strategic substitutes. This segments the information market, with "certified" firms displaying more informative security prices than non certified ones. In turn, this compresses the demand for information faced by the sell-side analyst, leading it to charge a higher price for information. Conversely, when information sellers integrate, the information monopolist internalizes the negative externality of informative certification and offers investors a more homogeneous cumulative precision and a more precise investment advice at a lower price.

More in detail. On the surface, when information providers specialize, the contractual arrangements they face leave little room for strategic interaction. This is because the paid-for analyst's clients (firms) differ from the clients of the sell-side analyst (traders). However, as they both ultimately offer insights that improve the forecast of the *same* unknown (the security's payoff) *and* such information is, for all to observe, they de-facto compete on related turfs–their product market decisions are strategic substitutes. We argue that this, in turn, shapes the space of equilibrium entry decisions made by information sellers.

More specifically, we find that paid-for and sell-side analysts compete indirectly (strategically interact) in the market for fundamental information. This affects the fees information sellers charge to their clients. We show that the maximum fee the sell-side analyst charges to traders is negatively influenced by the presence of the paid-for analyst. The information from the sell-side analyst is valuable to traders because of the trading profits it enables them to obtain. As the paid-for analyst publicly releases its certification, less scope is left for speculation based on private information, which in turn diminishes traders' valuation for the signal from the sell-side analyst. Similarly, the maximum fee the paid-for analyst extracts from a firm is *negatively influenced* by the presence of the sell-side analyst. This is because the information sold by the latter facilitates price discovery and improves the price prospects for a non certified firm with good fundamentals. Such an improved outside option for firms, in turn, lowers the value of the paid-for analyst's service, leading sell-side analysts to crowd out their paid-for peers.

We then argue that information sellers' product market strategies in turn condition their entry decisions, impacting the structure of the information market. We show this by studying the set of Nash equilibria of the entry game where information providers incur a fixed cost to produce payoff information. Depending on the uncertainty of the asset payoff and noise trading volume in the asset market, we identify five scenarios. Except for the extreme situations in which uncertainty in the payoff of the asset is very low such that producing information is not cost-efficient, the information market does not collapse and operates with different industry structures.

When noise traders' volume is high, the information market is dominated by the sell-side analyst. This is because all else equal, (i) noise traders' volume increases the profit of informed traders and hence their valuation of the information from the sell-side analyst, and (ii) a sell-side analyst is less concerned about information leakage through prices and provides more information. This, in turn, leads to a lower demand for paid-for research as firms rely more on price discovery by traders to save on certification costs.

When noise traders' volume is low and the uncertainty of payoff is intermediate, only the paid-for analyst enters the information market. This is because, with a low noise traders' volume, asset prices closely reflect the information sold by the sell-side analyst, which reduces investors' valuation of the signal and hence the profit of the sell-side analyst. In the absence of competition, the paid-for analyst can charge a higher disclosure fee on firms and reap a larger revenue.

When the uncertainty of the payoff and/or noise trading volume is low, the market cannot accommodate the presence of *both* information sellers, and multiple equilibria arise in which either the sell-side analyst enters the market, and the paid-for analyst stays out of it, or the opposite occurs. The intuition is as follows: for low noise trading volume, the sell-side analyst's profits are low, which leads it to not enter; this boosts the paid-for analyst's profit because it can charge a higher certification fee, which leads it to enter. In turn, with the paid-for analyst in the market, the sell-side analyst faces a smaller demand for information, which reinforces its decision to stay out of the market. Conversely, for low uncertainty about the asset payoff, the paid-for analyst's profits are small, which justifies its decision to not enter; this boosts the sell-side analyst's profit, because it can serve a larger market (the mass of uninformed traders is larger in this case), and leads it to enter; in turn, this lowers the profit made by the paid-for analyst, which reinforces its decision to stay out of the market.

With intermediate levels of noise trading volume and substantial uncertainty in asset payoff, we obtain an equilibrium where both information sellers enter the market. The information market is, however, segmented: the certification signals purchased by disclosing firms yields very informative prices, whereas equilibrium prices of non-disclosing firms are noisy signals of firms' fundamentals. This is because when certifying a firm's fundamentals, the paid-for analyst can costlessly increase its profit by boosting the precision of the signal it generates to increase the fee it charges to firms, *without* internalizing the negative effect this has on the sell-side analyst.

Differently from the case with specialized information sellers, an integrated monopolistic information seller internalizes the negative externality of competition between paid-for and sell-side analysts. In particular, the monopolistic information seller scales down the precision of the disclosure signal. This is because, doing so enables it to sell certification to firms first, and a more precise investment advice to investors. When firms purchase the certification, since trader-clients of the monopolistic seller are more informed than clients of the specialized sell-side analyst (the latter only purchases information on assets without disclosure), the monopolistic seller charges a lower price to traders albeit providing more precise information to them. Since revenue from traders is higher when selling information on firms without certification than on firms with certification, the monopolistic seller optimally sells certification to a smaller set of firms. Compared to the specialized information sellers, conditional on entry, we find the total information provided by the monopolistic information seller is less than the total information supplied by the specialized information sellers.

Our model rationalizes the distinct industry structures of information markets for the securities issued by firms with different characteristics. According to Kirk (2011), in the US "[...] About 60% of publicly listed firms have no analyst coverage [...]" and "[...] more than 500 unique companies[...]" purchased paid-for research between 1999 and 2006. Such companies typically display low share turnover and greater uncertainty about their future earnings. Additionally, information on bonds is generated by rating agencies, as bonds are associated with less uncertainty and noise trading. Stocks of liquid, large firms are instead located at the other end of the characteristic spectrum, and naturally sustain information produced by sell-side analysts. Investor-paid information stimulates speculation, leading to a higher trading volume than issuer-paid information does. These implications are consistent with evidence from the transition of payment regimes in the credit rating industry in the 1970s, which we review in Section 5. By revisiting this historical episode, our model also offers a new insight into its cause.

Our paper is related to three strands of the financial economics literature. First, it is related to the literature on certification, specifically credit ratings.[2] Kashyap and Kovrijnykh (2016) analyze how different payment regimes affect the information production activity of credit rating agencies under moral hazard. Bolton et al. (2012) study how issuers shopping for ratings lead to credit rating inflation and potential cures for this problem. Skreta and Veldkamp (2009) show that switching from an "issuer-pays" system to an "investor-pays" system alleviates rating inflation but may lead to a collapse of the rating market. Piccolo and Shapiro (2022) study a stage game between rating agencies and informed traders in the secondary market. They show that information in the secondary market can discipline opportunistic credit rating agencies. However, firms never bypass rating agencies in equilibrium in their setting and hence they do not study the organization of information markets. Focusing on certification in product markets, S. Nageeb et al. (2022) study the optimal

[2]See Sangiorgi and Spatt (2017) for an extensive review of the credit rating literature.

design of the signal that is sold to firms. For a systematic review of how information markets affect product markets, see Bergemann and Ottaviani (2021). We extend this literature by considering the competitive interaction between rating agencies and analysts selling secondary market information.

Second, our paper is related to the literature on the provision of payoff-relevant information in secondary markets. Admati and Pfleiderer (1986) analyze the optimal information and pricing strategies of a monopolistic information seller. When the information seller can both sell information and trade on her own account, Admati and Pfleiderer (1988) show that the optimal policy depends on the degree of risk aversion of other traders and of the information seller. Garcia and Sangiorgi (2011) study the problem of Admati and Pfleiderer (1986) with the presence of strategic risk-averse traders. Cespa (2008) studies how a monopolistic information seller controls the flow of information to traders over time. Relaxing the restriction of selling only to investors, we study how the option of selling to firms affects the analysts' information sales decisions depending on the structure of the information industry. Importantly, taking the entry decisions of information providers as given, none of the above papers studies the driving factors of information markets' industry structures and the influence of the latter on the properties of the underlying secondary market.

Finally, our paper is also related to the literature on the effect of corporate disclosure on asset prices (e.g., Kim and Verrecchia (1991), Diamond and Verrecchia (1991)). For an in-depth review, see Goldstein and Yang (2017). Focusing on the effect of real investment decisions, Gao and Liang (2013), Goldstein and Huang (2020), Goldstein and Yang (2019) study whether disclosure is desirable. Verrecchia (1982) and Diamond (1985) show a crowding-out effect of disclosure on information acquisition in the secondary market. Our paper demonstrates a crowd-out effect of secondary market information on corporate disclosure. Interestingly, this crowding-out effect comes from the fact that information on firms *without* disclosure is produced in the secondary market, which is orthogonal to the crowding-in effect of market information on firms *with* disclosure, studied by Piccolo and Shapiro (2022).

The rest of the paper is organized as follows: in Section 2, we lay out the model's assumptions and in Section 3 we solve for the equilibrium of the model with specialized information sellers. In section 4 we turn our attention to the analysis of the case with integrated information sellers, and the final section contains concluding remarks. Most of the proofs are relegated to the Appendix.

2 Model

There are three periods, a continuum of firms in $[0, 1]$, and two information sellers. Each firm's asset pays off $v \sim \mathcal{N}(\bar{v}, \sigma_v^2)$ at $t = 3$ and is traded by a continuum of investors in the unit interval at $t = 2$. The realization of the random payoff v is unknown to the firms. However, by paying a cost $c > 0$, information sellers, denoted by IS_F and IS_I, discover v and sell signals to firms (IS_F) and traders (IS_I) at $t = 1$. At $t = 0$, information sellers make an entry decision. We often refer to the type of information sold by IS_I as *secondary market* information, and to the type of information sold by IS_F as *certification*. As a consequence, we can think of IS_I as an analyst, and of IS_F as a rating agency.

More specifically, at $t = 0$, information sellers decide whether or not to enter the market. We denote by $e_j \in \{0, 1\}$ the indicator variable recording IS_j's entry decision: $e_j = 1(= 0)$ when $j \in \{F, I\}$ decides to enter (stay out of the) market. An information seller that enters the market incurs a cost $c > 0$ to discover v.

At $t = 1$, the rating agency IS_F offers each firm m in the unit interval a report that includes a signal of the firm's asset payoff $s_F = v + \epsilon_F$, where $\epsilon \perp v$, $\epsilon_F \sim N(0, \tau_F^{-1})$, at a flat fee $q_F > 0$. Both the precision τ_F *and* the certification fee q_F are posted prior to revealing the signal s_F. Consistent with S. Nageeb et al. (2022), we assume that the firm only pays for IS_F's information if it chooses to have such information disclosed to the market by IS_F. Therefore, if the firm pays the certification fee q_F, the ratings agency publicly releases s_F ($r = s_F$). If the firm does not pay q_F, the report is not made public ($r = \emptyset$).[3] We denote

[3]This assumption captures the intuition that a firm is *per-se* unable to certify the value of its assets, and

by $d_m \in \{0,1\}$ the indicator function recording a firm m's disclosure decision, with $d_m = 1$ ($d_m = 0$) for a firm that decides to have (not to have) s_F disclosed. Firms maximize their asset price net of the certification fee, $p - q_F$.

Additionally, at $t = 1$, investors with CARA utility with risk-aversion ρ can purchase information from the analyst IS_I. We denote by $s_{Ii} = v + \epsilon_i$, the signal that the analyst sells to an investor $i \in [0,1]$ for a fee q_I (signals acquired from IS_I are thus "personalized"). We assume that $\epsilon_i \sim \mathcal{N}(0, \tau_I^{-1})$, $\epsilon_i \perp v$, $\epsilon_i \perp \epsilon_F$, and that ϵ_i is independent across investors. We also denote by λ^* the mass of investors that IS_I optimally chooses to sell information to. Based on Proposition 6.1 in Admati and Pfleiderer (1986) we set $\lambda^* = 1$ without loss of generality.[4] IS_I announces the precision of the signal, τ_I, and the fee q_I before revealing his signal to investors. Liquidity traders post a random market order ("demand liquidity") denoted by $u \sim \mathcal{N}(0, \sigma_u^2)$, where $u \perp v$, $u \perp \epsilon_F$, and $u \perp \epsilon_i$ for all i. The net supply of the risky asset and the risk-free net return are both normalized to 0. We adopt the convention that the average signal $\int_0^1 s_{Ii}\, di$ is equal to v almost surely (SLLN).[5] Finally, we assume that information providers *do not trade in the secondary market.*[6]

At $t = 2$, risk averse investors and liquidity traders post their orders and the firm's (noisy) asset price p obtains. Finally, at $t = 3$ each firm's payoff v is disclosed. In summary, the timeline of the game is as follows:

- At $t = 0$: IS_F and IS_I make their entry decisions $e_F, e_I \in \{0,1\}$. If $e_j = 1$, IS_j pays c and discovers v, $j \in \{F, I\}$.

- At $t = 1$, IS_F

requires a credible third-party to do so.

[4]Admati and Pfleiderer (1986) prove that for an information monopolist selling an *identical* signal to a continuum of traders in a secondary market, it is optimal to restrict the mass information buyers, to control the informational externality occurring through the equilibrium price. However, if the information monopolist can sell *personalized* signals (i.e., signals containing i.i.d. error terms, as we assume) to traders, then he can achieve a higher profit by selling such signals to *all* traders.

[5]In other words, errors cancel out in the aggregate: $\int_0^1 \epsilon_i\, di = 0$. This convention is justified in Section 3.1 of the Technical Appendix of Vives (2008).

[6]Brokers prohibit their employees from trading the securities of firms for which they issued a research report (FINRA: NASD/NYSE Joint Memo on Chinese Wall Policies and Procedures).

- Chooses the firm's report precision τ_F and the disclosure fee q_F.
- A firm m observing s_F decides whether to pay q_F to have the report from IS_F disclosed ($d_m = 1$, in which case $r = s_F$) or not ($d_m = 0$, in which case $r = \emptyset$) to the market.
- If $e_I = 1$, IS_I chooses the private signal precision τ_I, and sells signals s_{Ii} to all investors in a given firm at a fee q_I.

- At $t = 2$, investors trade based on the information provided by IS_F and IS_I, submitting price-contingent orders (generalized limit orders) $X_I(s_F, s_{Ii}, p)$ to the market. Such orders clear the demand for liquidity coming from liquidity traders.

- At $t = 3$, assets pay off.

Figure 1 illustrates the timeline.

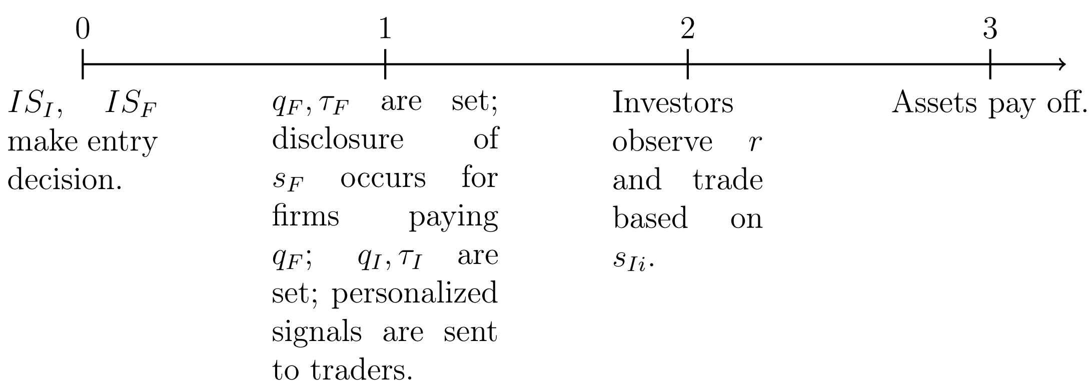

Figure 1: The timeline.

At $t = 2$, all agents observe r and update their beliefs about v. When $r \neq \emptyset$, this is standard Bayesian updating with normal random variables. When firms choose not to pay for the report, $r = \emptyset$, and we assume that agents *do not* update their beliefs.

Assumption 1. The posterior distribution of an asset payoff conditional on $\{r = \emptyset\}$ is the same as its prior distribution: $v|\{r = \emptyset\} \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$.

This assumption is also used by Skreta and Veldkamp (2009). The prior at $t = 1$ can thus be written as $\mathcal{N}(\mu(r), \tau_v(r)^{-1})$, where $\mu(r)$ and $\tau_v(r)$ denote respectively the expectation and precision of the asset payoff depending on the disclosure regime r. With this assumption, the posterior mean $\mu(r)$ and precision $\tau_v(r)$ are given by:

$$\mu(r) = \begin{cases} \frac{\tau_v \bar{v} + \tau_F s_F}{\tau_v + \tau_F} & \text{if } r = s_F \\ \bar{v} & \text{if } r = \emptyset \end{cases}, \quad \tau_v(r) = \begin{cases} \tau_v + \tau_F & \text{if } r = s_F \\ \tau_v & \text{if } r = \emptyset. \end{cases} \tag{1}$$

We look for a sequential Nash equilibrium in information sellers' entry and information sales' decisions: $\{(e_F^*, \tau_F^*, q_F^*), (e_I^*, \tau_I^*, q_I^*)\}$.

3 Solving the model

We solve the model backwards, starting from the determination of the information fees charged by IS_I and IS_F and their associated precisions. We then turn to the determination of the equilibrium where information sellers make their optimal entry decisions. We stipulate and later verify that there exists an equilibrium in which firms adopt a threshold strategy such that they only pay q_F and ask IS_F to disclose the report, if the signal s_F is above a certain *disclosure threshold* $T > 0$: $s_F \geq T$.

3.1 The optimal signal fee and precision

A firm's decision to purchase the report s_F affects the information traders have *before* entering the secondary market. This, in turn, impacts the equilibrium price at $t = 2$, as the following result shows:

Proposition 1. *Given r, $\tau_v(r)$, and the conditional precision of the analyst signal $\tau_I(r)$, there exists a unique linear equilibrium at $t = 2$, in which*

$$p(r) = a_0(r)\mu(r) + a_v(r)v + a_u(r)u, \tag{2}$$

where

$$a_0(r) = \frac{\rho^2 \tau_v(r)}{\rho^2 \tau_I(r) + \rho^2 \tau_v(r) + \tau_u \tau_I(r)^2} \tag{3a}$$

$$a_v(r) = \frac{(\rho^2 + \tau_u \tau_I(r))\tau_I(r)}{\rho^2 \tau_I(r) + \rho^2 \tau_v(r) + \tau_u \tau_I(r)^2} \tag{3b}$$

$$a_u(r) = \frac{\rho\left(\rho^2 + \tau_u \tau_I(r)\right)}{\rho^2 \tau_I(r) + \rho^2 \tau_v(r) + \tau_u \tau_I(r)^2}. \tag{3c}$$

Proof. The proof of this result is standard and relegated to the Appendix. □

The coefficient $a_u(r)$ captures the price impact of a marginal increase in liquidity demand:

$$\frac{\partial p(r)}{\partial u} = a_u(r). \tag{4}$$

As such, it can be interpreted as a measure of market illiquidity.

We denote by $\pi_i(r) = (v - p(r))x_{Ii}$ the profit obtained by a trader in the security market for a given firm's disclosure regime. The corresponding fee charged by IS_I is denoted by $q_I(r)$ and is determined by the following condition:

$$E\left[-\exp\left(-\rho(-q_I(r) + (v - p(r))x_i)\right)\middle| s_{Ii}, p(r), r\right] = E\left[-\exp(-\rho(v - p(r))x_i)\middle| p(r), r\right],$$

which yields the following standard result

$$\begin{aligned} q_I(r) &= \frac{1}{2\rho} \ln \frac{\text{Var}[v|p(r)]}{\text{Var}[v|s_{Ii}, p(r)]} \\ &= \frac{1}{2\rho} \ln \left(1 + \frac{\rho^2 \tau_I(r)}{\rho^2 \tau_v(r) + \tau_u \tau_I(r)^2}\right). \end{aligned} \tag{5}$$

Given that $\lambda^* = 1$, the expression in (5) denotes IS_I's revenue while $\Pi_I(r) \equiv q_I(r) - c$ denotes its profit. Therefore, to maximize profit, IS_I chooses

$$\tau_I(r) \in \arg\max_{\tau_I(r)} \Pi_I(r),$$

obtaining:

$$\tau_I(r) = \rho\sqrt{\frac{\tau_v(r)}{\tau_u}}, \quad q_I(r) = \frac{1}{2\rho}\ln\left(1 + \frac{\rho}{2\sqrt{\tau_v(r)\tau_u}}\right). \tag{6}$$

The following is an immediate implication of (1) and (6):

Lemma 1. *The maximum fee IS_I can charge for the information it sells to traders is inversely related to the information precision of the certification sold by IS_F.*

The intuition for this result is immediate: a higher certification precision improves all traders' information about a firm's payoff v, which lowers each trader's ability to exploit an informational advantage in the secondary market, diminishing the value of IS_I's signal.

3.2 The certification fee

Information sellers target different sets of customers. They thus *do not* directly compete in the information market. However, as shown by Lemma 1, the ability to set the price for the information sold by IS_I is limited by the presence of IS_F. In this section, we show that a similar effect also applies to the price for the certification services sold by IS_F.

Based on the results obtained in Section 3.1, we can compute the certification fee charged by IS_F, q_F. Suppose a firm is *shown* a report s_F. The firm can then choose to pay q_F and have the report disclosed (case $r = s_F$). Alternatively, the firm chooses to avoid paying q_F and prevent the market from knowing more about v (case $r = \emptyset$). As shown by Proposition 1, the equilibrium price changes in these two cases: in the former case $p(s_F)$ obtains, while in the latter case $p(\emptyset)$ obtains. The disclosure fee then cannot be larger than the difference between the firm's expected price with disclosure ($E[p(s_F)|s_F]$) and its expected price without disclosure ($E[p(\emptyset)|s_F]$):[7]

$$q_F(e_I) \equiv E[p(s_F)|s_F] - E[p(\emptyset)|s_F]. \tag{7}$$

[7]Note that both expectations are conditional on s_F since the firm observes the certification signal before deciding whether to pay q_F and have it published.

Using the results of the previous sections, we have

Lemma 2. *The fee charged by IS_F depends on $e_I \in \{0,1\}$, and is given by:*

$$q_F(1) = \frac{\tau_v}{2\tau_v + \tau_I(\emptyset)} \frac{\tau_F(T-\bar{v})}{\tau_v + \tau_F} < q_F(0) = \frac{\tau_F(T-\bar{v})}{\tau_v + \tau_F}. \tag{8}$$

Additionally, the certification fee is strictly increasing in the precision of the certification signal: $\partial q_F(\cdot)/\partial \tau_F > 0$.

Proof. Both $E[p(s_F)|s_F]$ and $E[p(\emptyset)|s_F]$ depend on e_I, that is on whether IS_I enters the market or not. If $e_I = 1$, then by Proposition 1, the value of disclosure to a firm that is shown s_F is given by:

$$\begin{aligned} E[p(s_F)|s_F] - E[p(\emptyset)|s_F] &= a_0(s_F)\mu(s_F) + a_v(s_F)E[v|s_F] - (a_0(\emptyset)\mu(\emptyset) + a_v(\emptyset)E[v|s_F]) \\ &= (1-a_v(s_F))\mu(s_F) + a_v(s_F)\mu(s_F) - ((1-a_v(\emptyset))\mu(\emptyset) + a_v(\emptyset)\mu(s_F)) \\ &= (1-a_v(\emptyset))(\mu(s_F) - \mu(\emptyset)). \end{aligned} \tag{9}$$

Note that the coefficients $a_v(s_F), a_v(\emptyset)$ have different expressions because $\tau_v(r)$, $\tau_I(r)$, and $\mu(r)$ depend on $r \in \{\emptyset, s_F\}$ through equation (1). In the second line, we use $E[u] = 0$. In the third line, using equation (6) and Proposition 1, we have $a_v(r) = (2\tau_v(r) + \tau_I(r))^{-1}(\tau_I(r) + \tau_v(r))$ and $a_0(r) = 1 - a_v(r)$. The result then follows from the fact that $E[v|s_F] = \mu(s_F)$.

The value of disclosure is increasing in s_F. Given the threshold strategy all firms with $s_F \geq T$ decide to disclose. This implies that the maximum fee IS_F can charge makes the marginal firm, i.e. the firm observing $s_F = T$, indifferent between disclosing or not the signal to the market. For this firm the fee charged by IS_F is given by:

$$\begin{aligned} q_F(1) &= (1 - a_v(\emptyset))(\mu(T) - \mu(\emptyset)) \\ &= \frac{\tau_v}{2\tau_v + \tau_I(\emptyset)}(\mu(T) - \mu(\emptyset)) \\ &= \frac{\tau_v}{2\tau_v + \tau_I(\emptyset)} \frac{\tau_F(T - \bar{v})}{\tau_v + \tau_F}. \end{aligned} \tag{10}$$

If $e_I = 0$, then $\tau_I(r) = 0$ since in the absence of IS_I no information accrues to the secondary market, and

$$E[p(s_F)|s_F] = \frac{\tau_v \bar{v} + \tau_F s_F}{\tau_v + \tau_F}, \;\; E[p(\emptyset)|s_F] = \bar{v}. \tag{11}$$

so that the maximum fee IS_F can charge in this case is given by:

$$\begin{aligned} q_F(0) &= \mu(T) - \mu(\emptyset) \\ &= \frac{\tau_F(T - \bar{v})}{\tau_v + \tau_F}. \end{aligned} \tag{12}$$

Therefore, the maximum fee charged by IS_F can be compactly expressed as follows:

$$q_F(e_I) = (1 - a_v(\emptyset)e_I)\frac{\tau_F(T - \bar{v})}{\tau_v + \tau_F}. \tag{13}$$

□

The intuition for the above result is straightforward. For a given firm's fundamentals v, both IS_F and IS_I's signals reduce traders' uncertainty about that firm's payoff. Absent IS_I, thanks to its monopoly power IS_F extracts all the certification value from the marginal firm $(\mu(T) - \mu(\emptyset))$. When IS_I enters the market, the marginal firm anticipates that the information acquired by traders partially reveals the value of its asset. This leads it to value less the certification from IS_F, which reduces the fee that IS_F can charge, the more, the higher is $\tau_I(\emptyset)$.

3.3 The optimal certification fee and precision

The previous sections have shown that even though IS_F and IS_I offer information to different clients (that is, firms and traders), because of the public availability of the certification signal s_F and of the equilibrium price $p(r)$, they strategically interact in the information market. IS_F's decisions depend on IS_I's choices (Lemma 2); the latter, in turn, are affected by IS_F's decisions (Lemma 1). In this section, we pin down the optimal certification fee chosen by IS_F, by determining IS_F's optimal decision about the certification precision, τ_F, and the disclosure threshold T.

We start by defining the objective function of IS_F. Recall that we denote by $d_m \in \{0, 1\}$, the indicator function reflecting whether a firm demands or not the certification signal to be publicly disclosed (respectively $d_m = 1$, and $d_m = 0$). This implies that we can write IS_F's profit upon entering the market, as follows

$$\Pi_F(e_I, q_F, \tau_F) \equiv \int_0^1 d_m q_F(e_I) dm - c. \tag{14}$$

Given the threshold strategy of firms, the share of firms purchasing s_F (the demand function for information faced by IS_F) is given by:

$$\int_0^1 d_m dm = P(s_F \geq T) = P\left(\frac{s_F - \bar{v}}{\sqrt{1/\tau_v + 1/\tau_F}} \geq \frac{T - \bar{v}}{\sqrt{1/\tau_v + 1/\tau_F}}\right),$$

where $E[s_F] = \bar{v}$ and $\text{Var}(s_F) = (\tau_v^{-1} + \tau_F^{-1})^{-1}$. Based on the above, we define the *normalized disclosure threshold* and the demand IS_F faces as follows:

$$X = \frac{T - \bar{v}}{\sqrt{1/\tau_v + 1/\tau_F}} \tag{15a}$$

$$P(s_F \geq T) = 1 - \Phi(X), \tag{15b}$$

where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. Using (15a), the following result is immediate:

Lemma 3. *The demand for certification is decreasing in* τ_v^{-1} *and in* τ_F^{-1}.

Firms demand certification services to reduce traders' uncertainty about the value of their payoff. As $1/\tau_v$ or $1/\tau_F$ grow, so does the uncertainty of the certification signal ($\text{Var}[s_F] = 1/\tau_v + 1/\tau_F$), which lowers the usefulness of certification.

Additionally, we can express the information fee IS_F charges as a function of the normalized threshold X:

$$\begin{aligned}
q_F(e_I) &= (1 - a_v(\emptyset)e_I)\frac{\tau_F(T-\bar{v})}{\tau_v+\tau_F} \\
&= (1 - a_v(\emptyset)e_I)\frac{\tau_F(T-\bar{v})}{\sqrt{\tau_v+\tau_F}}\frac{1}{\sqrt{\tau_v+\tau_F}}\frac{\sqrt{\tau_v\tau_F}}{\sqrt{\tau_v\tau_F}} \\
&= (1 - a_v(\emptyset)e_I)\sqrt{\frac{\tau_F}{\tau_v(\tau_v+\tau_F)}}X. \qquad (16)
\end{aligned}$$

According to (16), for a given level of precision τ_F, there is a one-to-one mapping between the certification fee q_F and the normalized threshold X (and hence T) used by firms. That is, given τ_F, IS_F essentially controls the threshold used by firms, T, by choosing q_F. It turns out to be easier to solve IS_F's problem by letting it optimally choose τ_F and X, which (due to (16)) amounts to having the rating agency choose the price for the certification service s_F. Using the expressions for the demand function for information faced by IS_F ((15b)) and the information fee q_F ((16)), we thus write IS_F's profit maximization problem as follows:

$$\begin{aligned}
&\max_{e_F, X, \tau_F} \quad \Pi_F(e_I^*, e_F, X, \tau_F) = \qquad (17) \\
&\max_{e_F, X, \tau_F}\left(\left(1 - \frac{\tau_v+\tau_I^*(\emptyset)}{2\tau_v+\tau_I^*(\emptyset)}e_I^*\right)\sqrt{\frac{\tau_F}{\tau_v(\tau_v+\tau_F)}}(1-\Phi(X))X - c\right)e_F,
\end{aligned}$$

where the starred variables $((e_I^*, \tau_I^*(\emptyset))$ in the above expression reflect the optimal entry and precision decisions of IS_I. The following result simplifies IS_F's objective function in (18):

Lemma 4. $\tau_F^* = +\infty$.

Proof. This result follows from the fact that for any normalized threshold X and IS_I's entry decision e_I, Π_F is increasing in τ_F because, as observed in Lemma 2, q_F is increasing in τ_F. □

For given information acquisition cost c, increasing τ_F increases the demand for certification (Lemma 3), and the certification fee (Lemma 2). Hence, IS_F optimally chooses $\tau_F^* = +\infty$. A direct implication of Lemma 4, is that IS_F's profit maximization problem reduces to:

$$\max_{e_F, X} \quad \Pi_F(e_I^*, e_F, X) = \max_{e_F, X} \quad \left(\left(1 - \frac{\tau_v + \tau_I^*(\emptyset)}{2\tau_v + \tau_I^*(\emptyset)} e_I^*\right) (1 - \Phi(X))X - c \right) e_F, \tag{18}$$

In (18), the fraction of firms that disclose the report is controlled by IS_F via the normalized threshold X. All else equal, a higher threshold improves the certification value, thereby increasing q_F, but decreases IS_F's demand for information $(1 - \Phi(X))$. Facing this trade-off, IS_F optimally sets X^*:

Lemma 5. *There exists a unique optimal normalized threshold X^* above (below) which firms decide to have the signal s_F published by IS_F (keep it private). Such threshold obtains as the unique solution to:*

$$X = \frac{1 - \Phi(X)}{\phi(X)}, \tag{19}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution.

Equation (19) has a unique solution $X^* \approx 0.75$, since the RHS is monotonically decreasing over the real line.[8] Given X^*, the optimal threshold T^* is given by

$$T^* = \bar{v} + \sqrt{\tau_v} X^*. \tag{20}$$

We now turn to define the profit function for IS_I. Since IS_F only sells information to (certifies) a fraction $1 - \Phi(X^*) \approx 0.23$ of the firms in the market, IS_I faces a market in

[8] The function $(1 - \Phi(X))X$ is concave on $(-\infty, 2)$ and convex on $(2, +\infty)$. The first-order condition gives a local maximum (X^*) for $X \in (-\infty, 2)$, which is also the global maximum because the derivative of $(1 - \Phi(X))X$ is always negative for $X \in [2, +\infty)$.

which traders have different priors about firms' asset payoffs. The mass of investors trading the stock of those firms that do *not* purchase s_F, i.e., $1-(1-\Phi(X^*))$ is "uninformed" in that it only knows that a firm's payoff $v \sim \mathcal{N}(\bar{v}, \sigma_v^2)$. The complementary mass of investors, instead, trades the stock of a firm that belongs to the mass of certified firms, i.e. $(1-\Phi(X^*))$ and thus holds the additional information conveyed by s_F. As shown in Proposition 1 and Lemma 1, this implies that the fee IS_I can charge differs for these two groups of investors, which in turn is reflected into its profit function:

$$\Pi_I(e_I, e_F^*, X^*) = \left((1-(1-\Phi(X^*))e_F^*)\frac{1}{2\rho}\ln\left(1+\frac{\rho}{2\sqrt{\tau_u \tau_v}}\right) + (1-\Phi(X^*))e_F^*\frac{1}{2\rho}\ln\left(1+\frac{\rho}{2\sqrt{\tau_u\tau_v(s_F)}}\right) - c\right)e_I, \tag{21}$$

where, once again, the starred variables (e_F^*, X^*) in the above expression reflect the optimal entry and threshold decisions of IS_F. More in detail: when IS_F is in the market, a mass $(1-\Phi(X^*))e_F^*$ of firms disclose their rating $(r = s_F)$. To these firms' investors, IS_I charges $(2\rho)^{-1}\ln(1+\rho/(2\sqrt{\tau_u\tau_v(s_F)}))$. Conversely, a mass $1-(1-\Phi(X^*))e_F^*$ chooses not to disclose their rating. The optimal fee charged by IS_I to these firms' investors is instead given by: $(2\rho)^{-1}\ln(1+\rho/(2\sqrt{\tau_u\tau_v}))$.

Based on our results, for a given optimal entry decision by IS_I, IS_F's profit $\Pi_F(e_I^*, e_F)$ reads as follows:

$$\Pi_F(e_I^*, e_F) = \left(\left(1-\frac{\tau_v+\tau_I^*(\emptyset)}{2\tau_v+\tau_I^*(\emptyset)}e_I^*\right)\sqrt{\frac{1}{\tau_v}}(1-\Phi(X^*))X^* - c\right)e_F. \tag{22}$$

As IS_F reveals all information for firms with $s_F \geq T$, it is immediate that $\tau_v(s_F) = \tau_v + \tau_F = +\infty$. Thus, IS_I won't sell information on firms that disclosed the report from IS_F and its profit becomes

$$\Pi_I(e_I, e_F^*) = \left(1-(1-\Phi(X^*))e_F^*)\frac{1}{2\rho}\ln\left(1+\frac{\rho}{2\sqrt{\tau_u\tau_v}}\right) - c\right)e_I. \tag{23}$$

The following result relates the profit expressions (22) and (23) to a change in the deep parameters of the asset market:

Corollary 1. At optimum:

1. $\Pi_F(e_I^*, 1)$ is decreasing in τ_v and (weakly) increasing in τ_u.

2. $\Pi_I(1, e_F^*)$ is decreasing in τ_v and τ_u.

Proof. From (22), it is immediate to see that $\Pi_F(e_I^*, e_F)$ is decreasing in τ_v. Noticing that $\tau_I^*(\emptyset)$ decreases with τ_u (equation (6)), it follows that $\Pi_F(e_I^*, e_F)$ is increasing in τ_u when $e_I^* = 1$ and does not depend on τ_u when $e_I^* = 0$. The result on $\Pi_I(e_I, e_F^*)$ is obtained similarly. □

The rationale for this result is as follows. Large payoff uncertainty (small τ_v) implies a large value of information and hence higher profit for both information sellers. Indeed, both the certification and the signal fee, $q_F(e_I)$, $q_I(r)$ are decreasing in τ_v (see, respectively, (16) and (6)). Conversely, a higher liquidity demand volume ($E[|u|] \propto \sigma_u = \sqrt{1/\tau_u}$) has different effects of on the profits of IS_F and IS_I. This is because, all else equal, larger noise traders' volume (1) increases the profit of informed traders and hence their valuation of the information from IS_I and, (2) reduces the leakage of the information sold by IS_I through asset prices, leading to more information provision by IS_I. This, however, ultimately lowers the demand for ratings as firms rely more on price discovery in the secondary market to save on certification costs (because the certification fee is increasing in τ_u according to (16)).

3.4 The equilibrium of the entry game

We are now ready to characterize the set of Nash equilibria of the entry game. We solve for the equilibrium by iterative elimination of dominated strategies. Given that $\Pi_F(e_I^*, e_F)$ is weakly decreasing in e_I^*, entry ($e_F = 1$) is the dominant strategy for IS_F if $\Pi_F(1, 1) > 0$. Conversely, $e_F = 0$ is the dominant strategy for IS_F if $\Pi_F(0, 1) < 0$. Similarly, entry

($e_I = 1$) is the dominant strategy for IS_I if $\Pi_I(1,1) > 0$, and $e_I = 0$ is the dominant strategy if $\Pi_I(1,0) < 0$. Depending on parameter values, five types of equilibrium outcomes arise in our model:

Proposition 2. *(i) When $\Pi_F(0,1) < 0$ and $\Pi_I(1,0) < 0$, there is a unique Nash equilibrium in which both information sellers stay out of the market.*

(ii) When $\Pi_F(1,1) > 0$ and $\Pi_I(1,1) < 0$, there is a unique Nash equilibrium in which only IS_F enters the market.

(iii) When $\Pi_F(1,1) < 0$ and $\Pi_I(1,1) > 0$, there is a unique Nash equilibrium in which only IS_I enters the market.

(iv) When $\Pi_F(1,1) > 0$ and $\Pi_I(1,1) > 0$, there is a unique Nash equilibrium in which both information sellers enter the market.

(v) When $\Pi_F(1,1) < 0$, $\Pi_I(1,1) < 0$, $\Pi_F(0,1) > 0$, and $\Pi_I(1,0) > 0$, there exist two pure strategy Nash equilibria (PSNE)[9] where either only IS_F or only IS_I enters the market.

Proof. The proof is in the Appendix. □

Multiple equilibria obtain if and only if the market can accommodate only one information seller, i.e., when $\Pi_I(1,1) < 0$ and $\Pi_F(1,1) < 0$, and $\Pi_I(1,0) > 0$ or $\Pi_F(0,1) > 0$, and arise because the entry decisions are made simultaneously at $t = 0$, and are not conditional on information provision and pricing strategies. The intuition is as follows: when, conditional on IS_F' entry, IS_I's entry decision is dominated, IS_I stays out of the market; this has a positive effect on IS_F, because it can charge a higher certification fee (Lemma 2), which vindicates its decision to enter. In turn, with IS_F in the market, IS_I faces a smaller demand for information, which reinforces its decision to stay out of the market. Conversely, when,

[9] In case (v), there also exists a a unique mixed strategy Nash equilibrium (MSNE) in which IS_F and IS_I enter the market with probability $\alpha_F = (\Pi_I(0,1) - \Pi_I(1,1))^{-1}\Pi_I(0,1)$ and $\alpha_I = (\Pi_F(1,0) - \Pi_F(1,1))^{-1}\Pi_F(1,0)$, respectively. If both information sellers enter, they make negative profits. If only one information seller enters, the entrant makes a positive profit. The entry probabilities are chosen such that information sellers are indifferent between entering and staying out of the market.

conditional on IS_I's entry decision, IS_F's entry decision is dominated (Lemma 2), IS_F stays out of the market; this boosts IS_I's profit, because it can serve a larger market (effectively, *no* firm is offered a rating in this case), and leads it to enter; in turn, this lowers the profit made by IS_F, which reinforces its decision to stay out of the market (Lemma 2). A mixed-strategy equilibrium also exists in which the two information sellers enter with probabilities α_F and α_I such that both are indifferent between entry and staying out of the market.

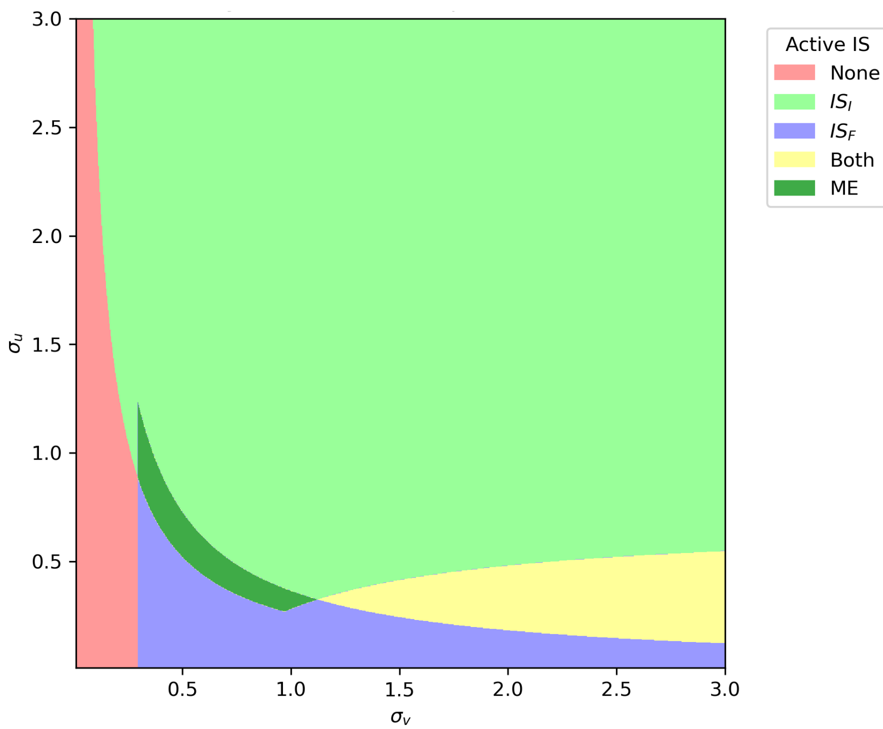

Figure 2: Information sellers' equilibrium entry decisions as a function of σ_v and σ_u. In the red region: $e_I = 0, e_F = 0$; in the green region: $e_I = 1, e_F = 0$; in the purple region: $e_I = 0, e_F = 1$; in the yellow region: $e_I = 1, e_F = 1$; in the dark green region multiple, pure strategy Nash equilibria obtain where $e_I = 0$, and $e_F = 1$, $e_I = 1$, and $e_F = 0$, or IS_I, and IS_F randomize over their entry decisions. Parameter values: $\rho = 5$, $c = 0.05$. $\sigma_v^2 = 1/\tau_v$ $\sigma_u^2 = 1/\tau_u$.

Figure 2 provides a graphical illustration of Proposition 2. The figure draws on the results of a numerical simulation in which we solve for the entry decisions maximizing (22) and (23) for different values of $\{\sigma_v, \sigma_u\}$:

$$\{\sigma_v, \sigma_u\} \in \{\{0.01, 0.013, \ldots, 3\} \times \{0.01, 0.013, \ldots, 3\}\}.$$

This figure demonstrates how asset characteristics shape the information market. The red, light green, purple, and yellow regions in the figure respectively reflect equilibria in which none of the information sellers is in the market ($e_I = 0, e_F = 0$), only the analyst enters ($e_I = 1, e_F = 0$), only the rating agency enters ($e_I = 0, e_F = 1$), both IS_I and IS_F enter ($e_I = 1, e_F = 1$). Finally, the dark green region reflects the asset characteristics that are conducive to multiple Nash equilibria in which either IS_I or IS_F only enters the market.

More in detail, in the red region, where uncertainty in asset payoff (σ_v) is low, producing information is not cost-effective for either information seller: the value of certification to firms and the value of private signals both shrinks to 0 when σ_v vanishes. In the purple region, with substantial asset payoff uncertainty but low liquidity demand volume, only IS_F is in the market. This is because, with a low liquidity demand volume, asset prices closely reflect the information sold by IS_I, which reduces investors' valuation of s_{Ii} and IS_I's profit. As shown in Corollary 1, a higher σ_u increases the profit of IS_I but decreases the profit of IS_F. This explains why the investor-pay regime, represented by the light green region, occurs when liquidity demand volume is sufficiently high.

When the uncertainty in the asset payoff is high and liquidity traders' demand volume falls in the intermediate range, both information sellers are active in the market, which is captured by the yellow region. As argued in Proposition 3, in this case the information market is segmented: IS_I only sells information to firms without disclosure. For the parameter values in the dark green region, there are two pure-strategy Nash equilibria arise. In one equilibrium, only IS_F enters. In the other equilibrium, only IS_I enters the market. This situation occurs when the information market can only accommodate either IS_I or IS_F but not both. Our numerical analysis shows that this region's magnitude increases with ρ and c.

Since the two information sellers release different amounts of information, aggregate trading volume is also different under different market regimes. Following He and Wang (1995), we define the expected trading volume from informed investors as

$$E\left[\int_0^1 |X_i(p(r), r, s_{Ii})| di\right], \text{ where } X_i(p(r), r, s_{Ii}) = \frac{E[v|p(r), r, s_{Ii}] - p(r)}{\rho \text{Var}[v|p(r), r, s_{Ii}]}, \tag{24}$$

where $X_i(p(r), r, s_{Ii})$ denotes secondary market investors' demand for the asset conditional on observing the asset price $p(r)$, the report r, and the signal s_{Ii}. Using the results of Proposition 1, we have

$$X_i(p(r), r, s_{Ii}) = \begin{cases} -u & \text{if } r = s_F \\ \frac{\tau_I}{\rho}(s_{Ii} - v) - u & \text{if } r = \emptyset. \end{cases} \tag{25}$$

When $r = s_F$, since $\tau_F = +\infty$, the asset price is fully revealing, which eliminates speculative trading motives of the secondary market investors. They simply act as market makers and absorb liquidity demand (u). Without disclosure ($r = \emptyset$), if IS_I enters the market, secondary market investors, besides accommodating liquidity orders, also speculate on their personalized signal. If $r = \emptyset$ and IS_I stays out of the market ($\tau_I = 0$), there is no speculative trading as all investors are equally uninformed. We thus have the following result that ranks equilibria on trading volume.

Corollary 2. Aggregate trading volume descends in the following order: investor-pays regime, investor-and-issuer-pays regime, and issuer-pays regime.

We summarize the previous discussion with the following:

Proposition 3. *At a Nash equilibrium of the market:*

1. *When only IS_F is in the market, a mass $1 - \phi(X^*)$ of firms has a fully revealing price. The complementary mass $\phi(X^*)$ has a completely uninformative equilibrium price. The market for such firms' securities is infinitely liquid (Issuer pays regime).*

2. *When only IS_I is in the market, all firms have a price that is a noisy signal of fundamentals. The market for such firms' securities displays finite depth (Investor pays regime).*

3. *When both information sellers are in the market, the information market is segmented: the equilibrium prices of disclosing firms (of mass $(1 - \phi(X^*))$) are fully revealing and the associated secondary markets are infinitely liquid; equilibrium prices of non-disclosing firms (of mass $\phi(X^*)$) are noisy signals of firms' fundamentals, and the associated secondary markets display finite depth (Investor and issuer pays regime).*

With multiple equilibria, either (1) or (2) above obtain.

4 Monopolistic Information Seller

In this section, we consider the case where information sellers vertically integrate. That is, we assume that there is only one information seller (IS) acting as both IS_F and IS_I. The order of the events is unchanged but IS only needs to pay for the information acquisition cost c once at time $t = 0$. In addition, we assume that the monopolistic IS cannot credibly commit not to sell information in the secondary market on the fundamentals of firms that choose not to disclose their reports at $t = 1$. Conditional on the entry of the IS, the equilibrium is characterized by four choice variables in this scenario: the flat disclosure fee q_F that is announced before offering firms a report, the precision of the report that is sent to firms τ_F, the precision of the signal sold to investors on firms with disclosure $\tau_I(s_F)$, and the precision of the signal sold to investors on firms without disclosure of the report $\tau_I(\emptyset)$. We maintain Assumption 1 that investors do not update their belief when there is no disclosure.

As in Section 3, we stipulate and verify later that firms use a threshold strategy such that they only ask the IS to disclose the report if the signal they observe satisfies the condition of being higher than the disclosure threshold $T : s_F \geq T$. Sequential rationality then implies that the optimal precision chosen by the information seller in the secondary market is still given by equation (6),

$$\tau_I^M(r) = \rho\sqrt{\frac{\tau_v(r)}{\tau_u}}, \text{ with } q_I^M(r) = \frac{1}{2\rho}\ln\left(1 + \frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v(r)}}\right),$$

where $\tau_v(r) = \tau_v + \tau_F^M$ and τ_F^M denotes the optimal certification precision to be determined; for a firm without disclosure (i.e. $r = \emptyset$), we have instead:

$$\tau_I^M(\emptyset) = \rho\sqrt{\frac{\tau_v}{\tau_u}} \text{ with } q_I^M(\emptyset) = \frac{1}{2\rho}\ln\left(1 + \frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right).$$

Since IS is always active in the secondary information market, the maximum fee it can charge to firms is the same as IS_F can charge when IS_I enters the market, i.e.

$$\begin{aligned} q_F^M &= \frac{\tau_v}{2\tau_v + \tau_I(\emptyset)}\frac{\tau_F}{\tau_v + \tau_F}(T - \bar{v}) \\ &= \frac{\tau_v}{2\tau_v + \rho\sqrt{\tau_v/\tau_u}}\frac{\tau_F}{\tau_v + \tau_F}(T - \bar{v}). \end{aligned} \tag{26}$$

Note that there is a one-to-one mapping between q_F and T for any given τ_F. Therefore, maximizing profit by choosing (q_F, τ_F) is equivalent to doing it by choosing (T, τ_F). Hence, when acting as both a certification provider and an analyst in the secondary market, IS faces the following optimization problem

$$\begin{aligned} \max_{T,\tau_F} \Pi \equiv \Pr(\tilde{s}_F \geq T)\left[\frac{\tau_v}{2\tau_v + \rho\sqrt{\frac{\tau_v}{\tau_u}}}\frac{\tau_F}{\tau_v + \tau_F}(T - \bar{v}) + \frac{1}{2\rho}\ln\left(1 + \frac{\rho}{2}\sqrt{\frac{1}{\tau_u(\tau_v + \tau_F)}}\right)\right] \\ + \Pr(\tilde{s}_F < T)\frac{1}{2\rho}\ln\left(1 + \frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right). \end{aligned} \tag{27}$$

Denoting by

$$X = \frac{T - \bar{v}}{\sqrt{1/\tau_v + 1/\tau_F}}, \quad s = \sqrt{\frac{\tau_v}{\tau_v + \tau_F}}. \tag{28}$$

and noticing that $((\tau_v + \tau_F)/\tau_F)^{-1/2} = \sqrt{1 - s^2}$, the optimization problem can be reformulated as follows:

$$\max_{X,s}[1-\Phi(X)]\left[\frac{\sqrt{\tau_u}}{\rho+2\sqrt{\tau_u\tau_v}}\sqrt{1-s^2}X+\frac{1}{2\rho}\ln\left(1+\frac{\rho}{2\sqrt{\tau_u\tau_v}}s\right)\right] \qquad (29)$$
$$+\Phi(X)\frac{1}{2\rho}\ln\left(1+\frac{\rho}{2\sqrt{\tau_u\tau_v}}\right).$$

The first order conditions with respect to X and s are respectively given by

$$0=\frac{\partial\Pi}{\partial X}=\phi(X)\left\{\frac{\sqrt{\tau_u}}{\rho+2\sqrt{\tau_u\tau_v}}\sqrt{1-s^2}\left[\frac{1-\Phi(X)}{\phi(X)}-X\right]+\frac{1}{2\rho}\ln\left(\frac{1+\frac{\rho}{2\sqrt{\tau_u\tau_v}}}{1+\frac{\rho}{2\sqrt{\tau_u\tau_v}}s}\right)\right\} \qquad (30a)$$

$$0=\frac{\partial\Pi}{\partial s}=[1-\Phi(X)]\left[\frac{1}{2}\frac{1}{2\sqrt{\tau_u\tau_v}+\rho s}-\frac{\sqrt{\tau_u}}{\rho+2\sqrt{\tau_u\tau_v}}X\frac{s}{\sqrt{1-s^2}}\right] \qquad (30b)$$

It turns out that the above conditions are also sufficient for the optimization problem. The results are summarized below.

Proposition 4. *The solution to the monopolistic information seller's problem* (27) *is given by:*

$$T^M=\bar{v}+\sqrt{\frac{\tau_v+\tau_F^M}{\tau_v\tau_F^M}}X^M \qquad (31a)$$

$$\tau_F^M=\frac{\tau_v}{(s^M)^2}-\tau_v, \qquad (31b)$$

where $X^M>0$ *and* $s^M\in(0,1)$ *solve the equations* (30a) *and* (30b). *The optimal precision of signals in the secondary market is as follows:* $\tau_I^M(\emptyset)=\rho\sqrt{\tau_v/\tau_u}$, *and* $\tau_I^M(s_F)=\rho\sqrt{(\tau_v+\tau_F^M)/\tau_u}$. *The optimal disclosure fee is given by* (26).

Proof. The proof is in the Appendix. □

Importantly, and differently from the competitive case, the monopolistic IS always sells information to *all investors* no matter what certification information if offers to the corresponding security issuers.

We can now compare the strategies of the monopolistic information seller with those of competing information sellers. As we have shown in Section 3.3, since the precision chosen by IS_F is $\tau_F^* = +\infty$, information from IS_I on firms that disclose s_F becomes redundant. Therefore $\tau_I^*(s_F)$ does not affect the equilibrium. For ease of comparison, we assume IS_I chooses not to sell information on firms that decide to have s_F disclosed: $\tau_I^*(s_F) = 0$.

Corollary 3. Compared with the case with competing information sellers, we have:

1. $\tau_F^M < \tau_F^* = \infty$;
2. $X^M > X^*$, $T^M > T^*$;
3. $\tau_I^M(\emptyset) = \tau_I^*(\emptyset)$;
4. $\tau_I^M(s_F) > \tau_I^*(s_F) = 0$.

Proof. See the Appendix. □

Compared to the scenario with competing information sellers, a monopolistic IS no longer discloses a fully revealing revealing signal on disclosing firms ($\tau_F^M < \tau_F^*$). This is because IS internalizes the negative externality generated by the competition between IS_I and IS_F. A lower precision τ_F^M leaves room for profit in the secondary market by selling information on firms with disclosure.

The normalized disclosure threshold X affects both the revenue from providing certification and the revenue from selling information to investors. When the normalized threshold is at its optimum with competing information sellers (i.e., $X = X^*$), the effect of a marginal increase it its value has a null impact on the revenue from selling s_F (Lemma 5). However, when information sellers are integrated, increasing X has a positive impact on the revenue from selling information to investors. This is because such information is more valuable to investors on firms without disclosure than on firms with disclosure. A higher X guarantees that a larger fraction of firms chooses not to disclose, which increases the revenue IS makes by selling secondary market information. Hence IS chooses a higher normalized threshold

($X^M > X^*$) to boost its total revenue. This, together with the lower precision τ_F^M, ensures that the disclosure threshold is also higher than in the market with competing information sellers ($T^M > T^*$).

Finally, compared with the scenario with competing information sellers, the cumulative information precision revealed by IS is more homogeneous because no firm's payoff is fully revealed via the certification signal s_F. However, cumulative information precision is also weakly smaller than the corresponding cumulative precision revealed by IS_F and IS_I. This is because, when $s_F < T^*$, total information precision coincides in the two cases. However, when $s_F \geq T^*$, in the integrated information market the cumulative precision is given by

$$\tau_F^M + \tau_I^M(s_F) < \tau_F^* = \infty.$$

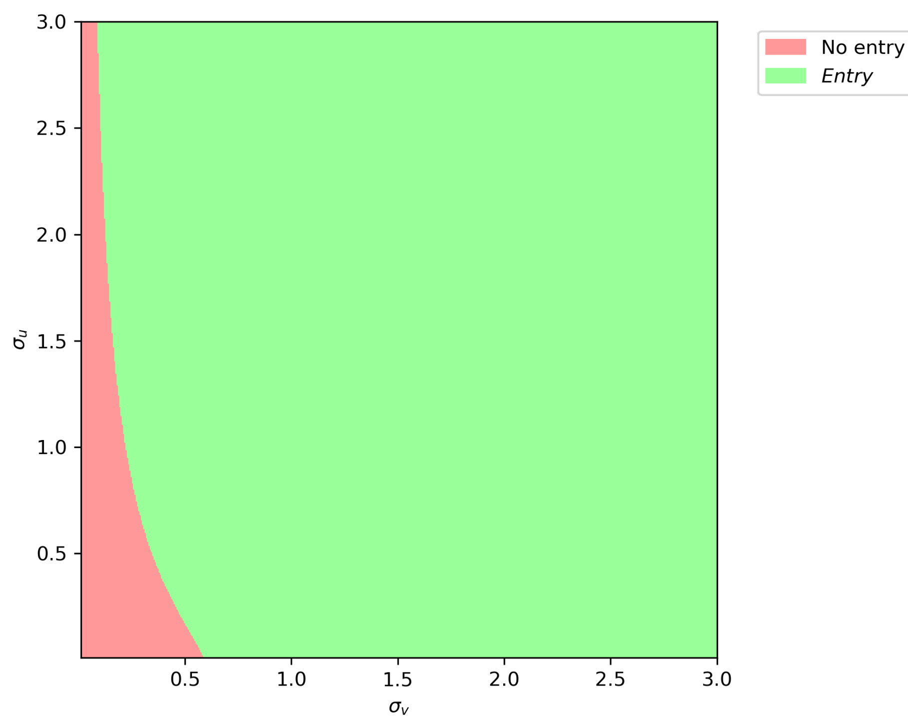

Figure 3: The monopolistic information seller's entry decision as a function of σ_v and σ_u. Parameter values: $\rho = 5$, $c = 0.05$. $\sigma_v^2 = 1/\tau_v$ $\sigma_u^2 = 1/\tau_u$.

Figure 3 partitions the space of parameter values (σ_v, σ_u) in two regions: in the orange (green) region IS enters (does not enter) the market.

In Figure 4, we compare entry decisions across the case with competing and vertically integrated information sellers. The shaded area in the figure reflects the parameter values (σ_v, σ_u) for which the information monopolist enters the market according to Figure 3 (that is, the orange region in the figure).

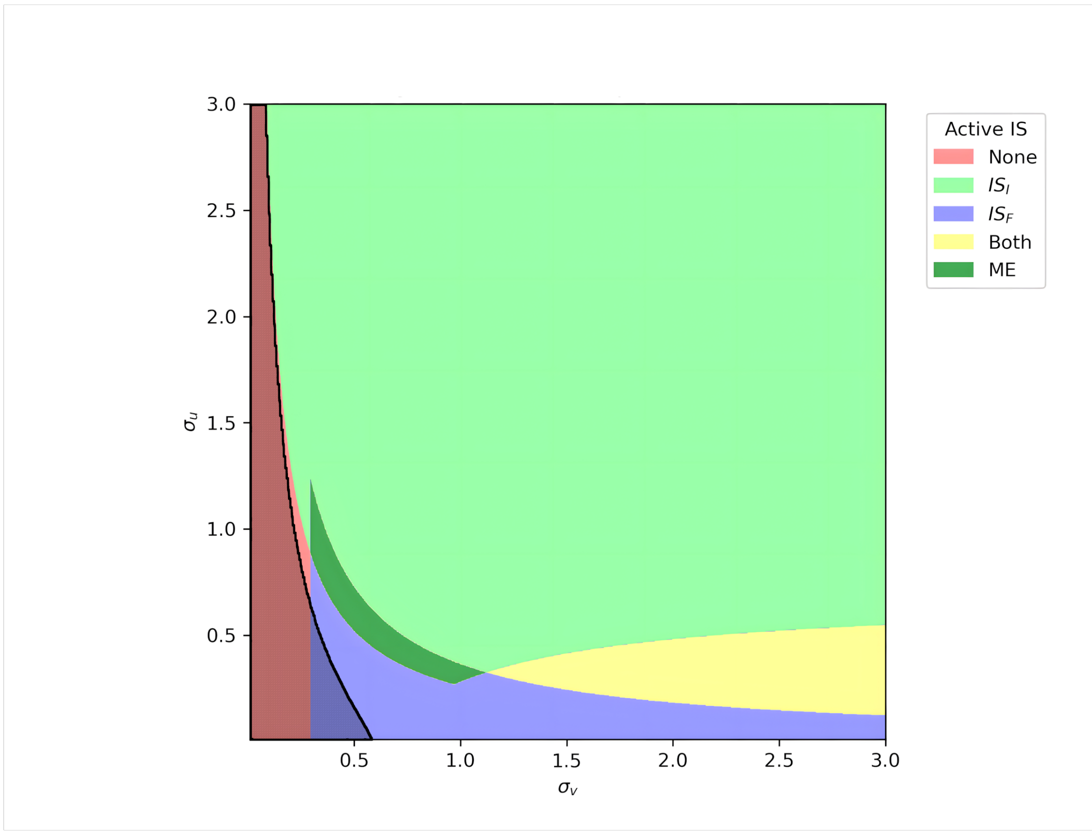

Figure 4: Entry decisions in the monopolistic information seller regime and in the case with competing information sellers as a function of σ_v and σ_u. Parameter values: $\rho = 5$, $c = 0.05$. $\sigma_v^2 = 1/\tau_v$ $\sigma_u^2 = 1/\tau_u$. The dark shaded area corresponds to the values of σ_v, σ_u for which the monopolist enters the market.

Corollary 4 (Comparative Statics). A higher τ_v or τ_u yields:

1. A decrease in X^M.

2. An increase in τ_F^M and $\tau_I^M(s_F)$.

Proof. See the Appendix. □

This result shows that as uncertainty (or liquidity trading volumes) in asset payoff decreases, the monopolistic information seller sells to a larger fraction of firms ($1 - \Phi(X)$ increases). In addition, both the disclosed information and additional information on the disclosed firms become more precise. The reason is that as σ_u (σ_v) decreases, investors value information less, and the monopolistic information seller caters more to the firms' demand by increasing the precision of s_F and consequently $\tau_I^M(s_F)$.

5 Empirical implications

In this section, we discuss how our model relates to observed institutional arrangements and empirical literature. After highlighting several testable empirical implications, we revisit the payment regime switch in the credit rating industry in the 1970s through the lens of our model.

Our results on competing information sellers yield a prediction on the payment regimes.

Prediction 1.

1. *The investor-pay (issuer-pay) regime dominates the information market when liquidity demand volumes are large (small).*
2. *Investor-pay and issuer-pay regimes coexist only for assets with large payoff uncertainty and for liquidity demand volumes in an intermediate range.*

This prediction is consistent with the evidence in Kirk (2011), who finds that paid-for research is typically purchased by companies with low share turnover and high uncertainty over future earnings. Additionally, the prediction is also in line with the observed arrangements in the credit rating market and the equity research market for large, liquid companies. With trading dominated by institutional investors in the over-the-counter market (OTC), corporate and municipal bonds are on the lower liquidity side compared to exchange-traded equities.[10] Consistent with our prediction, the issuer-pay model dominates the information market for these bonds. Our model predicts that information for risky assets with high liquidity trading volumes, such as stocks with large market capitalization, is paid by investors, which is in line with the practice in the equity research market. Notably, the information market on small or mid-sized stocks, which tends to be on the riskier side with limited liquidity trading volumes, is also consistent with our prediction: investor-paid research coexists with firm-sponsored research.[11]

Our analysis of a monopolistic information seller provides us with a more complete picture of the information seller's strategy. Our model predicts that a monopolistic information seller reaps revenue from both firms and investors. Consistent with our model, although CRAs largely rely on selling ratings to issuers, they also sell additional information to investors on top of their ratings.[12] Similarly, investment banks also charge IPO firms for analyst coverage while selling information to investors (Cliff and Denis, 2004). In addition, it is a common practice for sell-side equity analysts to disseminate their recommendations publicly while generating revenue by selling more precise information to their paying clients.[13] This two-tier information strategy seems puzzling.[14] Given the empirical evidence that analyst coverage increases the probability of investment banks winning deals from the covered firms (Ljungqvist et al., 2008), public recommendations can be interpreted as a strategy for selling information to firms. In addition, under this interpretation, analysts naturally issue more "buy" recommendations than "sell"[15], as they sell to firms with relatively better fundamentals.

Regarding information quality provided by different information sellers, our model has

[10]Edwards et al. (2007) document that individual bond issues did not trade on 48 percent of days in their 2003 sample, and that the average number of daily trades in an issue, conditional on trading, is just 2.4.

[11]See, e.g., a Wall Street Journal article on firms with medium market capitalization use sponsored equity research.

[12]For example, Moody's investment research.

[13]For evidence that investors pay for valuable information, see, e.g., Di Maggio et al. (2022), Goldstein et al. (2009).

[14]Garcia and Sangiorgi (2011) obtains the "public recommendations" and "private tipping" as two corner solutions in equilibrium. However, the information seller does not apply the two strategies simultaneously to the same asset.

[15]For a recent distribution, see, Factset data.

the following prediction.

Prediction 2. *Information in the issuer-pay regime is more precise than in the investor-pay regime.*

With more information revealed by IS_F, investors acquire less information and engage in less speculative trading, resulting in lower trading volumes in equilibrium. Corollary 2 implies that

Prediction 3. *The issuer-pay regime is associated with smaller trading volumes than the investor-pay regime.*

Testing Predictions 2 and 3 requires the econometrician to observe both regimes on the same type of security, a setting that can hardly be observed in reality according to Prediction 1. The payment-regime switch of three large credit rating agencies (Moody's, Standard & Poor's (S&P), and Fitch) in the 1970s provides a nice laboratory to test our predictions. Therefore we revisit below this transition from the investor-pay model to the issuer-pay model in the credit rating industry through the lens of our model.

Using a difference-in-differences setting, Jiang et al. (2012) find that S&P's (treatment group) assigns higher bond ratings after it switches from investor-pay to issuer-pay fees in 1974, compared to Moody's rating (control). Using the same setting, Bonsall IV (2014) shows that more optimistic ratings are associated with stronger future financial performance. He further provides evidence that rating informativeness increases after the CRA switches from the investor-pay model to the issuer-pay model. The finding that S&P issues higher ratings after the switches is also consistent with the result that only issuers with better fundamentals pay for certification.

Prediction 3 is also consistent with empirical data. After the big three CRAs switched from the investor-pay regime to the issuer-pay regime in the 1970s, bond trading volumes in the New York Stock Exchange(NYSE) reversed their increasing trend since the early 1960s even though the number of listed bonds kept increasing (see Biais and Green (2019)).

Our model also sheds new light on the cause of the transition. White (2010) outlines four reasons for the regime switch: (i) the spread of photocopy machines allowed investors to free ride information from their friends; (ii) a shock (bankruptcy of Pen-Central Road) in 1970 in the bond market made debt issuers more conscious of the need to assure investors that they (the issuers) were low-risk; (iii) large credit rating firms belatedly capitalize their special status set by regulations[16]; and (iv) this regime switch is an idiosyncratic change in the two-sided market, in which both issuers and investors can pay for information. Our model supports the two-sided market explanation.[17] Figure 2 and Prediction 1 suggest that the investor-pay regime thrives on liquidity trading volumes. Empirical data hint at a decline in liquidity trading volumes before the switch. Biais and Green (2019) document that since the 1940s bond trading had steadily migrated from the NYSE to the over-the-counter market. The latter is populated by institutional investors and presumably sustains less noise trading. According to Biais and Green (2019), the total number of bond issues listed in the NYSE increased by around 40% between 1965 and 1970 while total bond trading volume in the NYSE increased by about only 20% in the same period. Since most trading volume concentrates on newly issued bonds, these figures in fact suggest a decrease in trading volume per bond, which indicates a decline in liquidity trading volume.[18] The transition may not be very "idiosyncratic".

6 Concluding remarks

We study how the deep parameters of asset markets shape the structure of information markets. We highlight the strategic interaction between paid-for and sell-side analysts, and

[16]Regulations in the 1930s force banks to invest in only "investment grade" securities as determined by "recognized rating manuals", which includes only Standard & Poor's, Moody's, and Fitch. Insurance and pension regulators followed suit. See White (2010).

[17]The rise of photocopy machines seems not decisive, since equity information, which also suffers from information leakage through photocopying, remains investor-paid. Explanation (ii), as noted in White (2010), also implies investors are willing to pay more for information and hence yield unclear predictions.

[18]Goldstein et al. (2021) find that by day 10 after issue, trading of corporate bonds declines to 10% of its peak value at day 2.

show how their competition and the structure of information markets depend on characteristics like asset payoff uncertainty and the volume of liquidity (traders) demand. Our model rationalizes the dichotomy of industry structures in paid-for (and credit rating) and sell-side research markets. Though we focus on the market for financial information, our framework can be easily extended to the market for ESG information, a topic which is left for future research.

References

Admati, A. and P. Pfleiderer (1986). A monopolistic market for information. *Journal of Economic Theory* (39), 400–438.

Admati, A. R. and P. Pfleiderer (1988). Selling and trading on information in financial markets. *The American Economic Review 78*(2), 96–103.

Bergemann, D. and M. Ottaviani (2021). Information markets and nonmarkets. In *Handbook of industrial organization*, Volume 4, pp. 593–672. Elsevier.

Biais, B. and R. Green (2019). The microstructure of the bond market in the 20th century. *Review of Economic Dynamics 33*, 250–271.

Bolton, P., X. Freixas, and J. Shapiro (2012). The credit ratings game. *The Journal of Finance 67*(1), 85–111.

Bonsall IV, S. B. (2014). The impact of issuer-pay on corporate bond rating properties: Evidence from moody' s and s&p' s initial adoptions. *Journal of Accounting and Economics 57*(2-3), 89–109.

Cespa, G. (2008). Information sales and insider trading with long-lived information. *The Journal of Finance 63*(2), 639–672.

Cliff, M. T. and D. J. Denis (2004). Do initial public offering firms purchase analyst coverage with underpricing? *The Journal of Finance 59*(6), 2871–2901.

Di Maggio, M., M. Egan, and F. Franzoni (2022). The value of intermediation in the stock market. *Journal of Financial Economics 145*(2), 208–233.

Diamond, D. W. (1985). Optimal release of information by firms. *The journal of finance 40*(4), 1071–1094.

Diamond, D. W. and R. E. Verrecchia (1991). Disclosure, liquidity, and the cost of capital. *The Journal of Finance 46*(4), 1325–1359.

Edwards, A. K., L. E. Harris, and M. S. Piwowar (2007). Corporate bond market transaction costs and transparency. *The Journal of Finance 62*(3), 1421–1451.

Gao, P. and P. J. Liang (2013). Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research 51*(5), 1133–1158.

Garcia, D. and F. Sangiorgi (2011). Information sales and strategic trading. *The Review of Financial Studies 24*(9), 3069–3104.

Goldstein, I. and C. Huang (2020). Credit rating inflation and firms' investments. *The Journal of Finance 75*(6), 2929–2972.

Goldstein, I. and L. Yang (2017). Information disclosure in financial markets. *Annual Review of Financial Economics 9*, 101–125.

Goldstein, I. and L. Yang (2019). Good disclosure, bad disclosure. *Journal of Financial Economics 131*(1), 118–138.

Goldstein, M. A., E. S. Hotchkiss, and S. S. Nikolova (2021). Dealer behavior and the trading of newly issued corporate bonds. *Available at SSRN 1022356*.

Goldstein, M. A., P. Irvine, E. Kandel, and Z. Wiener (2009). Brokerage commissions and institutional trading patterns. *The Review of Financial Studies 22*(12), 5175–5212.

He, H. and J. Wang (1995). Differential information and dynamic behavior of stock trading volume. *The Review of Financial Studies 8*(4), 919–972.

Jiang, J. X., M. H. Stanford, and Y. Xie (2012). Does it matter who pays for bond ratings? historical evidence. *Journal of Financial Economics 105*(3), 607–621.

Kashyap, A. K. and N. Kovrijnykh (2016). Who should pay for credit ratings and how? *The Review of Financial Studies 29*(2), 420–456.

Kim, O. and R. E. Verrecchia (1991). Trading volume and price reactions to public announcements. *Journal of accounting research 29*(2), 302–321.

Kirk, M. (2011). Research for sale: determinants and consequences of paid-for analyst research. *The Journal of Financial Economics 100*(1), 182–200.

Ljungqvist, A., F. Marston, and W. J. Wilhelm (2008). Scaling the hierarchy: How and why investment banks compete for syndicate co-management appointments. *The Review of Financial Studies 22*(10), 3977–4007.

Piccolo, A. and J. Shapiro (2022). Credit ratings and market information. *The Review of Financial Studies 35*(10), 4425–4473.

S. Nageeb, A., N. Haghpanah, X. Lin, and R. Siegel (2022). How to sell hard information. *Quarterly Journal of Economics 137*, 619–678.

Sangiorgi, F. and C. Spatt (2017). The economics of credit rating agencies. *Foundations and Trends® in Finance 12*(1), 1–116.

Skreta, V. and L. Veldkamp (2009). Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics 56*(5), 678–695.

Verrecchia, R. E. (1982). The use of mathematical models in financial accounting. *Journal of Accounting Research*, 1–42.

Vives, X. (2008). *Information and learning in markets: the impact of market microstructure*. Princeton University Press.

White, L. J. (2010). Markets: The credit rating agencies. *Journal of Economic Perspectives 24*(2), 211–226.

A Proofs

A.1 Proof of Proposition 1

Proof. CARA Informed traders submit a price contingent order

$$X_I(s_{Ii}, p(r)) = \frac{E[v|s_{Ii}, p(r)] - p(r)}{\rho \text{Var}[v|s_{Ii}, p(r)]}, \tag{32}$$

with $E[v|s_{Ii}, p(r)] = (\tau_v(r)\mu(r) + \beta\tau_u z + \tau_I(r)s_{Ii})/(\tau_v(r) + \beta^2\tau_u + \tau_I(r))$, $\beta \equiv \tau_I(r)/\rho$, and $\text{Var}[v|s_{Ii}, p(r)] = (\tau_v(r) + \beta^2\tau_u + \tau_I(r))^{-1}$. Based on this, the market clearing price is given by

$$p(r) = \int_0^1 X_I(s_{Ii}, p(r))di + \frac{\alpha_E}{\beta}u, \tag{33}$$

with $\alpha_E \equiv \tau_I(r)/(\tau_v(r)+\beta^2\tau_u+\tau_I(r))$. Replacing the expressions for conditional expectation and variance and invoking the SLLN, we obtain

$$p(r) = \frac{\tau_v(r)}{\tau_v(r) + \beta^2\tau_u + \tau_I(r)}\mu(r) + \frac{\rho(1 + \beta\tau_u/\rho)}{\tau_v(r) + \beta^2\tau_u + \tau_I(r)}(\beta v + u).$$

Identifying the price coefficients yields the expressions in the proposition. □

A.2 Proof of Propositions 2

Proof. As argued above, for cases *(i)-(iv)*, there always exists a strictly dominant strategy for either information seller. Therefore, a unique Nash equilibrium in pure strategies exists. In case *(v)*, the strategy profiles that one information seller enters while the other stays outside form a Nash equilibrium: if IS_I enters, the best response of IS_F is to stay out of the market, because $\Pi_F(1, 1) < 0$. Since $\Pi_I(1, 0) > 0$, the best response to $e_F = 0$ for IS_I is entry ($e_I = 1$). Hence, ($e_I = 1, e_F = 0$) forms a Nash equilibrium. A similar argument also obtains for ($e_I = 0, e_F = 1$). There also exists a unique strictly mixed strategy equilibrium with strategy profile α_F and α_I, chosen such that both information sellers are indifferent between the decision to enter and that of staying out of the market. □

A.3 Proof of Corollary 2

Proof. Given that if $x \sim N(0, \sigma^2)$, then $E[|x|] = \sigma\sqrt{\frac{2}{\pi}}$, using (25) and the Fubini's theorem, we thus have

$$E[\int_0^1 |X_i(p(r), r, s_{Ii})|di] = \begin{cases} \sigma_u\sqrt{\frac{2}{\pi}} & \text{if } r = s_F \\ \sqrt{\frac{\tau_I}{\rho^2} + \sigma_u^2}\sqrt{\frac{2}{\pi}} & \text{if } r = \emptyset \end{cases} \tag{34}$$

Since $\sqrt{\frac{\tau_I}{\rho^2} + \sigma_u^2} > \sigma_u$ when $\tau_I > 0$, we thus have that trading volume in issuer-pays regime is smaller than that in the investor-pays regime. Recall that IS_I only produces information for firms without disclosure when both information sellers enter the market, the trading volume of the investor-and-issuer pays regime hence lies in between.

□

A.4 Proof of Proposition 4

Proof. The proof is divided into three steps.

Claim 1. For any $X > 0$, there exists a unique global optimum $s^*(X) \in (0, 1)$ which solves Equation (30b).

Proof. The partial derivative $\partial\Pi/\partial s$ is

$$\frac{\partial \Pi}{\partial s} = \frac{1}{2}\frac{1}{2\sqrt{\tau_u\tau_v} + \rho s} - \frac{Xs}{\sqrt{\tau_v(1 - s^2)}}.$$

One can easily verify that the term $\partial\Pi/\partial s$ is strictly decreasing in s. Besides, the term $(\partial\Pi/\partial s)|_{s=0} = 1/(4\sqrt{\tau_u\tau_v}) > 0$, and as s converges to 1, the term $\partial\Pi/\partial s$ converges to $-\infty$. A unique interior solution exists. □

Claim 2. There exists $\bar{X} > 0$ such that, for any $s \in [0, 1)$, there exists a unique global optimum $X^*(s) \in (0, \bar{X}]$, which solves Equation (30a).

Proof. If $X \leq 0$, then

$$\Pi(X,s) < \Pi(\infty,s) = \frac{1}{2\rho}\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right).$$

We then focus on $X > 0$. Transform Equation (30a) as

$$\frac{\partial\Pi}{\partial X} = \phi(X)\left\{\sqrt{\frac{1-s^2}{\tau_v}}\left[\frac{1-\Phi(X)}{\phi(X)} - X\right] + \frac{1}{2\rho}\left[\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right) - \ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}s\right)\right]\right\}.$$

We then prove that the term

$$\frac{\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right) - \ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}s\right)}{\sqrt{1-s^2}}$$

is bounded above for $s \in [0,1)$. This is true since

$$\lim_{s\to1^+}\frac{\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right) - \ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}s\right)}{\sqrt{1-s^2}} = 0$$

by L'Hopital's rule. Define $\bar{X}$ implicitly as the solution to

$$\frac{1-\Phi(\bar{X})}{\phi(\bar{X})} - \bar{X} = -\frac{\sqrt{\tau_v}}{\rho}\max_{s\in[0,1)}\frac{\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right) - \ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}s\right)}{\sqrt{1-s^2}}$$

For any s, define $X^*(s)$ implicitly as the solution to

$$\frac{1-\Phi(X(s))}{\phi(X(s))} - X(s) = -\frac{\sqrt{\tau_v}}{\rho}\frac{\ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}\right) - \ln\left(1+\frac{\rho}{2}\sqrt{\frac{1}{\tau_u\tau_v}}s\right)}{\sqrt{1-s^2}}$$

The Mill's ratio is strictly decreasing, we have that $X^*(s) \leq \bar{X}$. $X^*(s) > 0$ for any $s \in [0,1)$. Moreover, for all $X \in (0, X^*(s))$, $\partial\Pi/\partial X > 0$, and vice versa. Henceforth, $X^*(s)$ is the unique global maximum for any $X \in (0, \infty]$. Accordingly, $X^*(s)$ is the unique global maximum for any X. $\square$

For any $(X_0, s_0) \in [-\infty, \infty] \times [0, 1]$, construct the following series recursively. If $s_0 = 1$, then define X_1 any fixed real value in $(0, \bar{X}]$. Otherwise, define $X_1 := X^*(s_0)$. Define $s_1 := s^*(X_1)$. Given well-defined series $(X_t, s_t)_{t=0}^k$, where $k \geq 1$, define $X_{k+1} := X^*(s_k)$ and $s_1 := s^*(X_{k+1})$. By Claim 1 and 2, the sequence $(X_t, s_t)_{t=0}^\infty$ is well-defined, bounded for $t \geq 1$ and $\Pi(X_t, s_t) \geq \Pi(X_{t-1}, s_{t-1}), \forall t \geq 1$. There exists a subsequence $(X_{t_k}, s_{t_k})_{k=0}^\infty$ which converges to $(\tilde{X}, \tilde{s})$. Henceforth, we have $\Pi(\tilde{X}, \tilde{s}) \geq \Pi(X_0, s_0)$ for all $(X_0, s_0) \in [-\infty, \infty] \times [0, 1]$. $\square$

A.5 Proof of Corollary 3

Proof. Equation (30b) implies that $0 < s < 1$, which therefore implies that $0 < \tau_F^M < \infty$. Equation (30a) implies that

$$\frac{1 - \Phi(X^M)}{\phi(X^M)} - X^M < 0 = \frac{1 - \Phi(X^*)}{\phi(X^*)} - X^*.$$

Since the Mills ratio $(1 - \Phi(X))/\phi(X)$ is a decreasing function, so is the function $(1 - \Phi(X))/\phi(X) - X$. Therefore, we have that $X^M > X^*$ or $(T^M - \bar{v})/\sqrt{1/\tau_v + 1/\tau_F^M} \geq (T^* - \bar{v})/\sqrt{1/\tau_v}$. Since $\tau_F^M < \infty$, we have $T^M > T^*$. The rest then follows. $\square$

A.6 Proof of Corollary 4

Proof. By reformulating Equation (30a) and (30b), we have that (X^M, s^M) is a solution to

$$X - \frac{1 - \Phi(X)}{\phi(X)} = \frac{\frac{2\sqrt{\tau_u \tau_v} + \rho}{2\sqrt{\tau_u}} \ln\left(\frac{2\sqrt{\tau_u \tau_v} + \rho}{2\sqrt{\tau_u \tau_v} + \rho s}\right)}{\sqrt{1 - s^2}} \tag{35}$$

$$\frac{s}{\sqrt{1 - s^2}} = \frac{1}{X} \frac{2\sqrt{\tau_u \tau_v} + \rho}{2\sqrt{\tau_u}(2\sqrt{\tau_u \tau_v} + \rho s)} \tag{36}$$

Besides, denote $X(s)$ the solution to Equation (35), fixing s. Denote $s(X)$ the solution to Equation (36), fixing X.

In the first step, we show that $X(s)$ is strictly decreasing in both τ_u and τ_v, fixing s. Proving the first part is easy and therefore omitted. Proving the second part involves taking partial derivative of the term $\frac{2\sqrt{\tau_u\tau_v}+\rho}{2\sqrt{\tau_u}}\ln\left(\frac{2\sqrt{\tau_u\tau_v}+\rho}{2\sqrt{\tau_u\tau_v}+\rho s}\right)$ with respect to τ_v and then using the fact that $\ln(1+x)\leq x$.

In the second step, we show that $s(X)$ is strictly decreasing in τ_u and τ_v. This proof also involves simple transformations and therefore omitted.

In the third step, we show that $s(X)$ is strictly decreasing in X while $X(s)$ is strictly decreasing in s. The proof of the first part is trivial and therefore omitted. For the second part, we take derivative of the term $\frac{\ln\left(\frac{2\sqrt{\tau_u\tau_v}+\rho}{2\sqrt{\tau_u\tau_v}+\rho s}\right)}{\sqrt{1-s^2}}$ with respect to s and using the fact that $\ln(1+x)<x$, we have that the term is strictly decreasing in s.

Finally, increasing τ_u (or τ_v) moves the curve $X(s)$ leftwards and the curve $s(X)$ downwards. Therefore the solution (X^M, s^M) both decreases in τ_u and τ_v. □