Commodity Tails and Bond Risk Premia

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Abstract

Commodity tail risk predicts bond excess returns, both theoretically and empirically. Commodity price jumps cause inflation to deviate from the target rate set by the Taylor rule – arousing a central bank response of unknown size. This monetary uncertainty affects long-term but not short-term bonds. As especially large shifts trigger central bank reactions, tail risk predicts bond returns better than volatility. These findings are supported in- and out-of-sample: Commodity up-tail (down-tail) significantly predicts bond returns with out-of-sample R-squares up to 19.07% (5.77%). It is unspanned by the yield curve and existing predictors. Robustness occurs across sub-periods and for international markets.

Keywords: Bond risk premium, Commodity return, Tail risk, Central bank, Return predictability

JEL Classification: G11, G12, G15, P36

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1 Introduction

In the past, many economic recessions have been accompanied or even front-run by large swings in commodity prices. As an input factor for many essential products or as an energy provider for the production process, commodities are closely tied to input and output price levels. Its scarcity in the case of international conflicts or natural disasters initiates changes in the general price level. Thus, commodity price risk implies a risk for changes in consumer prices and inflation. Central banks consider inflation as a key indicator for their monetary policy decisions. In the Taylor rule, deviations of the inflation from the target inflation of 2% as well as macroeconomic variables play a key role for the optimal short-term interest rate. However, the size of the deviation matters. For inflation, the target rate actually is a target range in which small deviations from the target rate do not cause central bank reactions. In contrast, larger deviations affect the economic condition and inflation expectations significantly. They require central bank interventions!

The uncertainty about future inflation, and the central bank reaction to it, is a risk factor for long-term bonds but not for short-term bonds. The latter have anyways expired prior to central bank reaction. This asymmetry in the exposure to interest rate risk causes a higher compensation for long-term bonds, the so-called bond excess returns.

In summary, commodity price risk leads to inflation risk. Through central bank's reaction this creates interest rate uncertainty, which is especially relevant for longer-term bonds. As the effect is predominantly relevant for large swings in commodity prices, commodity tail risk is an especially good predictor for bond excess returns.

We show the aforementioned mechanism in a theoretical model and in an empirical evaluation. In the model we consider an economy with two bonds. The short-term bond is in perfectly elastic supply at a rate set by the central bank. The long-term bond is in fixed supply and priced in equilibrium. The returns required for the long-term bond depend on the variance of the future discount factor, which is altered by the central bank in a noisy way when the inflation is outside a target range. These significant deviations from the target range can occur through fluctuations, but most likely through jumps in the commodity prices. The probability of these jumps is our measure for tail risk. We solve the model analytically and show the results in a static and dynamic model.

We show the theoretical link between commodity and bond markets through a regression of the bond risk premium on the commodity tail risk (CT). We confirm that there is indeed a strong positive relation between the two – both in the uptail as well as in the downtail. Over a one-year horizon, the commodity tail by itself significantly predicts future excess returns for two-, three-, four-, and five-year bonds with an R^2 ranging between 19.88% and 30.12%. A one standard deviation increase in the commodity up-tail (down-tail) indicates an increase of 144 (128) bps in the bond risk premium – compared to an average bond risk premium of 142 bps. Notably, the commodity up-tail (down-tail) strongly predicts bond returns with out-of-sample R-square up to 19.07% (5.77%).

To confirm the model mechanism step by step, we follow it closely in its intermediary stages. The first step of the model is linking commodity tails to inflation. As predicted by the model, commodity uptail increases future inflation and commodity downtail decreases it. The next step in the model mechanism is the linkage between commodity tail risk and interest rate uncertainty. The model connects tail risk on both sides of the distribution to higher uncertainty about monetary policy. Our regression shows that this is indeed correct: Both, uptail and downtail risk increase interest rate uncertainty empirically. The last step in the analysis is then our main finding, that commodity tail risk increases bond excess returns.

As documented theoretically and empirically, the link between commodity markets and bond markets is direct: Changes in the commodity market trigger inflation, which is a direct ingredient of the Taylor rule. This distinguishes commodity markets from other markets, like for example stock markets. In these markets, the link is based on a common macroeconomic factor. Nevertheless, the commodity market is also linked to the overall macroeconomic condition. However, while high stock prices are generally a good sign for the macroeconomic condition, the situation is more complex for commodity prices. High prices can also be supply driven. Supply shortages increase prices and are simultaneously hindering economic progress. Thus, both uptail and downtail commodity prices can be negative news – and empirically are.

Our empirical evaluation is based on recent advances in econometrics and statistics. We construct our tail risk measure based on the methodology developed by Kelly and Jiang (2014).

It estimates a time-varying tail risk from the cross-section of returns. This way, it provides a meaningful measure and does not require extensive sample horizons or high-frequency data. For the construction of one aggregate commodity tail risk measure out of 24 individual commodities, we apply the Partial Least squres (PLS) approach. In comparison with the standard Principal Components Analysis (PCA) the PLS method is more effective in extracting predictive information and filter out noise (Zhao, Zhou, and Zhu, 2021). Due to the variety of the 24 commodities, a great amount of obscured information is inherent in their information set and in this way can be made useful for the prediction.

This paper adds to several strands of literature. First, it contributes to the literature linking commodity and bond markets, both empirically and theoretically. To date, the literature understanding how and why commodity prices impact the bond market is scarce. Only a few studies have investigated their connection. Tang and Xiong (2012), Öztek and Öcal (2017), and Da, Tang, Tao, and Yang (2023) document return synchronization in both market due to the increasing financialization and integration of commodity markets. Also López (2014) and Zhang, Wang, Xiong, and Zou (2021) find an interrelation by recording volatility spillovers. Our paper provides a new strand connecting both markets: Commodity tail risk indicates inflation risk and thus monetary uncertainty – which in turn affects bond market excess returns.

Second, our model strengthens our cognizance of the relevance of commodity prices in the economic network. Among others, Hammoudeh (2007), Silvennoinen and Thorp (2013), and Mensi, Hammoudeh, Shahzad, and Shahbaz (2016) show that commodity prices have also become more closely connected stock markets. Chiang, Hughen, and Sagi, 2015; Chiang and Hughen, 2017 show the influence of commodities on stock returns. Also, commodities serve as an important indicator for the macroeconomic condition. Labys and Maizels (1991) and Browne and Cronin (2010) document a link between oil prices and inflation. Brown, Stephen, Yucel, and Mine (2002), Ferderer (1997), Hamilton (2003), and Ge and Tang (2020) draw connections of commodity prices to GDP growth or more general output growth. Other interrelations of commmodities with macroeconomic factors include political risk (Barsky and Kilian, 2004; Gong and Xu, 2022), business cycles (Leduc and Sill, 2004; Schwark, 2014; Chevallier, Gatumel, and Ielpo, 2014), and production cost (Kilian, 2009). While many of these studies focus on oil prices as the main commodity, we take a variety of commodities into account and find that these indeed matter – some even more than oil. Moreover, we show that tail risk is the better explanatory factor than pure price changes, both theoretically and empirically.

Third, we contribute to a recognition of tail risk as a relevant economic measure for risk. The literature has emphasized the importance of tail risk in influencing economic conditions and asset prices. Starting from Hill (1975), a large body of literature (Jansen and De Vries, 1991; McNeil and Frey, 2000; Hartmann, Straetmans, and Vries, 2004; Poon, Rockinger, and Tawn, 2004; Allen, Bali, and Tang, 2012; Eser and Schwaab, 2016; Chabi-Yo, Ruenzi, and Weigert, 2018; Davydov, Vähämaa, and Yasar, 2021, and Cong, Li, Tang, and Yang (2023)) has advanced the Hill (1975)'s tail estimator on asset returns and investigate its influence on the economy and asset returns. Other researchers start to utilize alternative data, to construct tail risk measures. For example, Gao, Lu, and Song (2019) constructs tail risks based on out-of-the money put option data on various asset classes. Manela and Moreira (2017), and Chen, Yao, Zhang, and Zhu (2023) construct rare disaster index based on text data and option prices. Adrian, Boyarchenko, and Giannone (2019) and Marfè and Pénasse (2024) implement semi-parametric approach and characterizes the tail risk in macroeconomic variables. Our paper employs a panel estimation method, specifically the PLS approach, to model tail risks within a comprehensive selection of commodities. In our framework, commodity tail risks act as important predictors for bond risk premium and future economic condition. Moreover, commodity tail risk outperforms other risk measures like volatility or absolute returns.

Additionally, our paper adds to the literature on the predictability of bond risk premia, especially through factors from outside the bond market. The connection between stock and bond markets has been studied by, among others, Cochrane (2011), Baker, Wurgler, and Yuan (2012), Koijen, Lustig, and Van Nieuwerburgh (2017), Campbell, Pflueger, and Viceira (2020). Schraeder, Sojli, Subrahmanyam, and Tham (2022) show that the volatility-to-volume ratio on the stock market indicates macroeconomic uncertainty and as a result bond excess returns. Other influencing factors include, but are not limited to investors' demand for information(Benamar, Foucault, and Vega, 2021), the combination of the current yield and macroeconomic variables (Moench and Soofi-Siavash, 2022), the intermediary balance sheets (Du, Hébert, and Li, 2023), and the unspanned volatility risks (Bakshi, Crosby, Gao, and Hansen, 2023). Recent studies by Bianchi, Büchner, and Tamoni (2021) and Wan, Fulop, and Li (2022) incorporate machine learning and Bayesian learning approaches to predict bond risk premia.

In comparison with the traditional bond market predictors, we show that our measure of tail risk is adding additional information. Commodity tail risk is unspanned by the Cochrane and Piazzesi (2005) (CP) factor, macroeconomic predictors (the Ludvigson and Ng (2009) (LN) factor), the Cieslak and Povala (2015) (CPo) factor, and the principal components of yields (e.g., Bauer and Hamilton, 2018; Zhao, Zhou, and Zhu, 2021), and the Moench and Soofi-Siavash (2022) (MS) factor. More importantly, CT outperforms these predictors in out-of-sample tests with an out-of-sample R^2 ranging from 5.38% to 27.32%. The empirical results are also robust to accounting for small-sample properties of the data and to employing different statistical testing criterion, such as Bauer and Hamilton (2018)'s bias-corrected p-values (BH p-values).

Finally, our paper shows the stability of commodity tail risk as a predictor also in many other countries, which links it to the literature on risk premia in international bond markets. Dahlquist and Hasseltoft (2013) build a global factor based on the Cochrane and Piazzesi (2005) factor and investigate bond return predictability in four international bond markets – the United States, the United Kingdom, Germany, and Switzerland. Also Zhao, Zhou, and Zhu (2021) forecast bond returns in these four countries by construction of a global macroeconomic factor. In our paper, we find that the CT factor is more pertinent to macroeconomic trends and excels in predicting bond risk premia in a set of major bond countries, including the United States, Canada, France, Germany, Italy, Japan, and the United Kingdom.

The remaining paper proceeds as follows. Section 2 presents our model and its formulates its empirical predictions. Section 3 provides a description of the data and methodology used. Section 4 presents the results of our empirical analysis and demonstrates that the CT factor is an important predictor of bond risk premia. Section 5 follows the model mechanism closely and also documents the validity of the intermediary steps. Section 6 presents additional robustness checks of our empirical findings, confirming the results also for international markets. Finally, section 7 concludes the paper.

2 Theoretical Model

We consider an economy in which shocks to commodity prices drive inflation. When inflation is outside the target interval, the central bank adjusts interest rates according to an inflation-based Taylor rule. Thus, tail risk in commodity prices implies uncertainty about future inflation and the central bank's reaction. This monetary uncertainty affects prices for long-term bonds more than for short-term bonds, driving bond-excess returns. Figure 1 shows the model timeline.

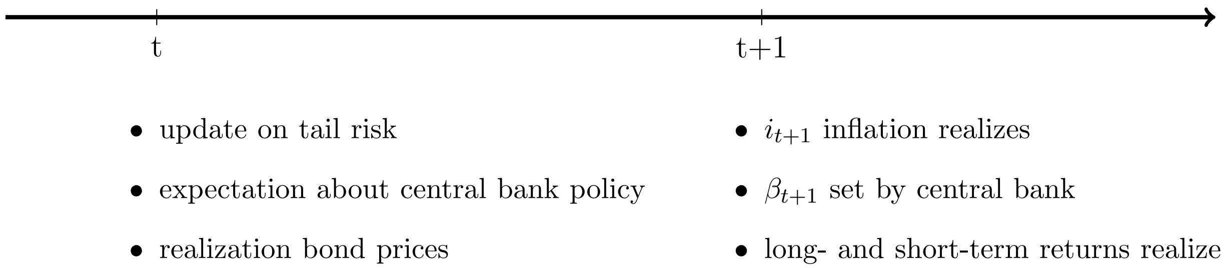

Figure 1: Timeline of events

2.1 Prices and Inflation

Our economy produces goods through the refinement of commodities, which serve as an input factor. Thus, the cost of a good is the product of the commodity price C_t and the cost of its refinement R_t,

$$P_t = R_t \cdot C_t.$$

The growth rate of the refinement cost is normally distributed, $R_{t+1} = R_t \cdot \exp(g_{R,t+1})$, with $g_{R,t+1} \sim N(\mu_R, \sigma_R^2)$. In normal times, the growth rate of the commodity price, $C_{t+1} = C_t \cdot \exp(g_{C,t+1})$, also follows an independent normal distribution. However, in rare tail events, Bernoulli-distributed demand or supply shocks affect the growth rate. The probability of an uptail shock (downtail shock) is π_H (π_L). The corresponding size of the uptail (downtail) shock is normally distributed with mean $\mu_H > 0$ ($\mu_L < 0$) and volatility σ_H (σ_L). In summary, the commodity growth rate follows the distribution

$$g_{C,t+1} \sim N(\mu_C, \sigma_C^2) + B(\pi_H) \cdot N(\mu_H, \sigma_H^2) + B(\pi_L) \cdot N(\mu_L, \sigma_L^2).$$

Thus, on days with a high jump probability (tail risk) in the commodity growth rate, we face two additional sources of uncertainty: First, the mean growth rate in the case of a jump deviates from the mean growth in normal times. Thus, a higher jump risk implies a higher uncertainty about the mean growth rate. Second, in the case of a jump we face uncertainty about the size of the jump.

Aggregating the refinement cost and the commodity price, the total price of a consumption good in the next period is

$$P_{t+1} = R_{t+1} \cdot C_{t+1} = C_t \cdot \exp(g_{C,t+1}) \cdot R_t \cdot \exp(g_{R,t+1}) = P_t \cdot \exp(g_{C,t+1} + g_{R,t+1}) = P_t \cdot \exp(i_{t+1}).$$

Thus, inflation is the sum of the two growth rates, $i_{t+1} = g_{C,t+1} + g_{R,t+1}$. It is directly affected by the Bernoulli distributed upward and downward tail events in commodity prices. Every combination of no/one upward shock and no/one downward shock can occur in this economy. Table 1 summarizes the probabilities of the different scenarios and the corresponding inflation distributions.

		Uptail shock	
		No	Yes
Downtail shock	No	Scenario: N $\pi_N = (1-\pi_H)(1-\pi_L)$ $\mu_N = \mu_C + \mu_R$ $\sigma_N = \sqrt{\sigma_C^2 + \sigma_R^2}$	Seario: U $\pi_U = \pi_H(1-\pi_L)$ $\mu_U = \mu_C + \mu_R + \mu_H$ $\sigma_U = \sqrt{\sigma_C^2 + \sigma_R^2 + \sigma_H^2}$
	Yes	Scenario: D $\pi_D = \pi_L(1-\pi_H)$ $\mu_D = \mu_C + \mu_R + \mu_L$ $\sigma_D = \sqrt{\sigma_C^2 + \sigma_R^2 + \sigma_L^2}$	Scenario: B $\pi_B = \pi_L \cdot \pi_H$ $\mu_B = \mu_C + \mu_R + \mu_H + \mu_L$ $\sigma_B = \sqrt{\sigma_C^2 + \sigma_R^2 + \sigma_H^2 + \sigma_L^2}$

Table 1: Distribution characteristics for different scenarios

2.2 Central Bank Reaction

The central bank reacts to the inflation according to a simplified version of the Taylor rule, which is only based on inflation: When inflation is in the target range around the target rate of two percent, $|i_{t+1} - 2\%| < \alpha$, the central bank does not change its monetary policy. It leaves the interest rate at the mean-reverted (with mean-reversion speed m) previous level $\beta_{t+1} = m \cdot 0.98 + (1-m) \cdot \beta_t$. When inflation is outside the target range, $i_{t+1} - 2\% > \alpha$ or $i_{t+1} - 2\% < -\alpha$, the central bank changes the one-period discount factor. It fights high inflation with high interest rates and tries to prevent deflation through low interest rates. Thus, it sets the discount factor for the next period to $\beta_{t+1} = m \cdot 0.98 + (1-m) \cdot \beta_t - (i_{t+1} - 2\%) \cdot \gamma + \epsilon_{t+1}$, with $\gamma \in [0,1]$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$ being a noise term.

The aggregate distribution of the discount factor set in the next period is a mixture of a discrete and continuous distribution. When the inflation stays inside the target range, the discount factor set in the next period is certain. This event occurs with probability

$$P_t(|i_{t+1} - 2\%| < \alpha) = \sum_{i \in \{N,U,D,B\}} \pi_i \cdot \left[\Phi\left(\frac{2\% + \alpha - \mu_i}{\sigma_i}\right) - \Phi\left(\frac{2\% - \alpha - \mu_i}{\sigma_i}\right) \right].$$

When inflation is outside the target range, the discount factor has the probability density function $\int_{-\infty}^{2\%-\alpha} p(\beta_{t+1}|i_{t+1} = I) \cdot p(i_{t+1} = I) dI + \int_{2\%+\alpha}^{\infty} p(\beta_{t+1}|i_{t+1} = I) \cdot p(i_{t+1} = I) dI$, with

$$p(\beta_t - m \cdot 0.98 + (1-m) \cdot \beta_{t-1} | i_t = I) = \frac{1}{\sigma_\epsilon \sqrt{2\pi}} \exp\left(-0.5 \cdot \left(\frac{x - \gamma \cdot (I - 2\%)}{\sigma_\epsilon}\right)^2\right) \cdot \mathbb{1}_{I \notin [2\%-\alpha, 2\%+\alpha]},$$

$$p(i_t = I) = \sum_{i \in \{N,U,D,B\}} \pi_i \cdot \frac{1}{2\pi\sigma_i} \cdot \exp\left(-\frac{(I - \mu_i)^2}{2 \cdot \sigma_i^2}\right).$$

Thus, everything else equal, higher tail-risk parameters π_L and π_H decrease the probability that the interest rate remains unchanged and consequently increases the interest rate uncertainty.

2.3 Impact on Bond Returns

In the economy, investors have the choice to invest in long-term (2-period) bonds and/or short-term (1-period) bonds. Agents determine their demand by maximizing their mean-variance utility out of next period wealth

$$\max E_t[w_{t+1}] - \frac{\nu}{2} Var_t(w_{t+1})$$

subject to the budget constraint

$$w_{t+1} = x_{B,t}(\beta_{t+1} - \frac{1}{\beta_t} B_t) + \frac{1}{\beta_t} w_t.$$

The first order condition for the maximization problem determines the optimal portfolio holdings in the long-term bond

$$x_{B,t} = \frac{E_t[\beta_{t+1}] - \frac{1}{\beta_t} B_t}{\nu Var_t(\beta_{t+1})}.$$

The short-term bond is in perfectly elastic supply and yields the interest rate set by the central bank. The price of the long-term bond forms in equilibrium such that the market clears: Investors' optimal demand for the two-period bond, $x_{B,t}$ equals the fixed normalized one-unit supply, $x_{B,t} = 1$. Thus, we obtain the following two-period bond price

$$B_t = \beta_t \left\{ E_t[\beta_{t+1}] - \nu Var_t(\beta_{t+1}) \right\}.$$

The bond price depends on the expected change in central bank policy and the uncertainty about the central bank reaction. Appendix B derives the expectation and variance of the next period discount factor β_{t+1}.

2.4 Simulation

To illustrate the effects of tail risk on bond excess returns, we simulate the model. We take the model parameters π_H as a measure for upside tail risk and π_L as a measure for downside tail risk.

[Please insert Figure 2 here]

When considering upside and downside tail risk together (Subfigure a)), an increase in both tail risks leads to an increase in bond excess returns. These bond excess returns are a compensation for risk, coming from two sources. First, in the case of a tail event, the size of the tail shock is uncertain. Thus, in addition to the normal volatility σ_C we are also facing the uncertainty about the size of the tail shock σ_H or σ_L. Second, an increase in the probability of a tail event also increases the uncertainty about the future state of the economy, with or without tail event. As the mean of the commodity growth rate and inflation differ in the tail scenarios (adjusted by μ_H or μ_L), this generates additional uncertainty. Subfigures b) and c) show that the positive relationship between tail risk and bond excess returns holds for both tails, the upside tail risk and the downside tail risk.

The additional uncertainty about the next period interest rate, which affects long term bonds returns, is only elevated when the central bank has incentives to alter the monetary policy. This is the case when the inflation rate deviates significantly (by more than α). The changes in inflation rate are caused by volatility and the realization of tail risk. While the first is most of the time leading to small, negligible deviations from the target rate, the realizations of tail risk cause changes in the monetary policy.

Subfigure d) shows that for normal volatility levels an increase in volatility does not increase monetary policy risk a lot. Thus, bond excess returns do not increase perceptibly when volatility increases. Only when the volatility exceeds a certain level, the volatility itself causes significant deviations (of more than α) of the inflation rate from the target rate. In these cases volatility alone triggers central bank interventions and no tail shocks are necessary to cause interest rate risk.

In a next step, we consider the uptail and the downtail risk to be time-variant. Both follow a Markov process with N states and transition probability matrix Ω, with

$$\Omega = \begin{pmatrix} \pi_{1,1} & \pi_{1,2} & \cdots & \pi_{1,N} \\ \pi_{2,1} & \pi_{2,2} & \cdots & \pi_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N,1} & \pi_{N,2} & \cdots & \pi_{N,N} \end{pmatrix} \tag{1}$$

In the simulation of the model, we consider the transition probability matrix for the uptail risk Ω_U and the transition probability matrix for the downtail risk Ω_D to be the same tridiagonal matrix with $H = 5$ equally distant tail-risk states $(0.02, 0.04, 0.06, 0.08, 0.10)$. We assume the probability of jumping to a neighboring tail-risk state is 0.025. The probability of staying in the current tail-risk state is 0.95 (for the highest and lowest states the probability of staying in the current probability state is 0.975). The tail-risk probability itself moves gradual over time, such that for a non-neighboring state, the probability of switching to it is zero. Even though the uptail and the downtail distributions are kept identical, the Markov chain itself is independent between the uptail risk and the downtail risk process.

Table 2: **Tail-risk regression**

This table displays the regression of bond excess returns on uptail risk and/or downtail risk.

	(1)	(2)	(3)
	rx_B	rx_B	rx_B
Intercept	0.001	0.003	0.003
Uptail	0.048	0.048	
Downtail	0.048		0.048

3 Data and Methodology

In this section, we present our data and methodology. First, we describe our measurement of commodity tail risk, which relies on the tail index introduced by Kelly and Jiang (2014). Second, we outline the computation of Treasury bond risk premia as proposed by Cochrane and Piazzesi (2005). Finally, we discuss the Partial Least Squares (PLS) regression approach, employed to discern the most relevant predictive information from potentially noisy commodity information sets.

3.1 Measure of Commodity Tail risk

We estimate the time-varying component of return tails, π_t, month-by-month by applying Hill (1975)'s power law estimator to the set of daily return observations for a given commodity in month t. With each commodity's nearest- or next-nearest-to-delivery futures contract,[1] we

[1]Following Hill (1975), a large body of literature (McNeil and Frey, 2000; Hartmann, Straetmans, and Vries, 2004; Poon, Rockinger, and Tawn, 2004; Allen, Bali, and Tang, 2012; Kelly and Jiang, 2014; Eser and Schwaab, 2016; Davydov, Vähämaa, and Yasar, 2021; Cong, Li, Tang, and Yang, 2023) has advanced Hill (1975)'s power law estimator to construct tail risk measures, and finds that tail risks are important factors in influencing economy and asset returns.

calculate commodity up- and down-tail risk as follows:

$$\pi_u = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{R_{k,u}}{R_{max}}, \tag{2}$$

$$\pi_d = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{R_{k,d}}{R_{min}}. \tag{3}$$

We denote $R_{k,u}$ or $R_{k,d}$ as the kth daily return that is higher than R_{max} or lower than R_{min}, and n being their total number. Following Kelly and Jiang (2014), we set R_{max} or R_{min} such that it captures the highest or lowest five percent of the futures returns for a given commodity.

For the time-series of up- and down-tail risk we consider the past 6-month daily excess returns of a set of 24 commodities. The corresponding data is sourced from Bloomberg and spans more than 15 years. The commodities are categorized into energy products, livestock, precious metals, industrial metals, and agricultural products (refer to Table 3). This selection aligns with the commodity category of the Standard & Poor's Goldman Sachs Commodity Index (GSCI) and encompasses the most liquid commodity futures contracts.[2] It includes commodities actively traded in prominent exchanges such as the Chicago Board of Trade, New York Mercantile Exchange, New York Board of Trade, London Metal Exchange, and International Petroleum Exchange.[3]

Table 3 presents the summary statistics for the up- and down-tail risk for the 24 commodities. The tail risks exhibit considerable volatility, ranging from 9.52% to 18.30% for the up-tail and from 10.15% to 22.44% for the down-tail. Additionally, the indexes show high autocorrelation, with serial correlation persisting for up to 12 months.

[Please insert Table 3 here]

[2] This comprehensive coverage reinforces the representativeness of our commodities set, as discussed in Moskowitz, Yao, and Pedersen, 2011; Huang, Li, Wang, and Zhou, 2020).

[3] The Chicago Board of Trade exchanges agricultural products and livestock such as wheat, soybean, and lean hog. The New York Mercantile Exchange trades energy products and precious metals. The New York Board of Trade deals with other agricultural products, including sugar, cocoa, and coffee. The London Metal Exchange trades industrial metals and the International Petroleum Exchange deals with energy products like Brent oil.

3.2 Bond Risk Premia

Following existing literature on bond prediction, we focus on the U.S. Treasury bond marke, with price data sourced from the Fama-Bliss dataset of the Center for Research in Security Prices (CRSP). As a supplement to our primary findings, we also incorporate zero-coupon bond yields constructed by Gürkaynak, Sack, and Wright (2007). Following Cochrane and Piazzesi (2005), we define the (log) yield of an n-year bond as:

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}, \tag{4}$$

with $p_t^{(n)} = \log\left(P_t^{(n)}\right)$ being the log bond price of the n-year zero-coupon bond at the time t. The log return of an n-year bond at time t, which is sold as an $(n-1)$-year bond at time $t+1$, is

$$r_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)}. \tag{5}$$

The bond risk premium of an n-year bond is the difference between the n-year bond return and the 1-year interest rate,

$$rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^{(1)}. \tag{6}$$

Table 4 provides a summary of 2-year to 5-year Treasury bond risk premia (excess returns). Notably, these bond risk premia exhibit an increasing trend with longer maturities. For example, the average value of the 5-year monthly bond risk premium is 2.151, while the corresponding figure for the 2-year bond is 0.575. The volatility of bond risk premiums increases with the maturity of the contract – ranging from 1.159% to 4.180%. The autocorrelation coefficients show persistence at 1-month lags, which dissipates for the 12-month lags.

Figure 4 illustrates the time series of bond risk premia between January 1993 and June 2022. Shaded bars in the figure denote economic recessions as identified by the National Bureau of Economic Research (NBER). It is evident that bonds with different maturities exhibit similar trends, with excess returns generally increasing during recessionary periods.

[Please insert Table 4 and Figure 4 here]

3.3 PLS Approach

To consolidate the tail risk of the 24 commodities and extract the information relevant for forecasting bond risk premia, we utilize the Partial Least Squares (PLS) approach. As it is deemed to be more efficient compared to Principal Components Analysis (PCA) (Zhao, Zhou, and Zhu, 2021), we can effectively extract predictive information from high-dimensional return data and filter out noise. The PLS method has been widely adopted in finance and economic literature, particularly in the prediction of stock returns (Huang, Jiang, Tu, and Zhou, 2015; Light, Maslov, and Rytchkov, 2017; Chen, Yao, Zhang, and Zhu, 2023).

Following Kelly and Pruitt (2013), Zhao, Zhou, and Zhu (2021) and Chen, Yao, Zhang, and Zhu (2023), we denote $X_t = (x_{1,t}, x_{2,t}, \ldots, x_{N,t})^T$ as the 24×1 vector of the individual up- or down-tail risk of the 24 commodity futures returns at time t.

In a first step, we regress X_t on a constant and the bond risk premium rx_{t+1}:

$$x_{i,t} = \omega_{i,0} + \omega_{i,1} rx_{t+1} + u_{i,t}, \tag{7}$$

In this equation, the loading ω_i captures the sensitivity of $x_{i,t}$ in the prediction of rx_{t+1}.

In the second step, we run a cross-sectional regression of $x_{i,t}$ on the corresponding loading $\widehat{\omega}_i$, which we obtained in the first step regression,

$$x_{i,t} = c_t + Tail_t \widehat{\omega}_i + v_{i,t}. \tag{8}$$

The regression coefficient $Tail_t$ represents the commodity up- or down-tail factor (CT), which we use to predict bond risk premium rx_{t+1}.

Kelly and Pruitt (2013) show that the two-stage regression can be replaced by an one-step procedure.[4] In the latter the tail risk $Tail_t$ is a linear combination of each individual commodity's up-tail or down-tail risk $x_{i,t}$. The corresponding weights depend on the covariance

[4] We define $\hat{T} = [\hat{Tail}_1 \; \hat{Tail}_2 \; ... \; \hat{Tail}_T]'$ as the commodity up- or down-tail factor, $X = [x'_1 \; x'_2 \; ... \; x'_T]$ as the $L \times T$ matrix of individual commodity tail risk, and $R = [rx_{1+h} \; rx_{2+h} \; ... \; rx_{T+h}]'$ as the future bond risk premium. Then the up- or down-tail factors are

$$\hat{T} = XJ_N X' J_T R (R' J_T X J_N X' J_T R)^{-1} R' J_T R. \tag{9}$$

In this equation $J_L = I_L - \frac{1}{L} l_L l'_L$, I_L is the T-dimensional identity matrix, and l_L is a T-vector of ones.

between the individual tail risk $x_{i,t}$ and future bond risk premia (e.g. Lin, Wu, and Zhou, 2018; Zhao, Zhou, and Zhu, 2021; Chen, Yao, Zhang, and Zhu, 2023). Those with a higher covariance bear a greater weight in the aggregate tail factor.

Table 5 displays summary statistics of the CT. Compared with the up-tail factors (Panel A), the down-tail factors (Panel B) are negative and more volatile. Both factors are positively skewed and highly persistent at the 1-month lag. Owing to a concomitant reduction in the effective sample size, heteroscedasticity- and autocorrelation-consistent (HAC) standard errors exhibit reduced reliability. Consequently, we address this concern by employing the bias-corrected p-values proposed by Bauer and Hamilton (2018).

According to Panel C of Table 5, the factors are positively correlated. This positive correlation also follows from Figure 4, which plots the time series of both tail factors and their rolling correlation.

[Please insert Table 5 and Figure 4 here]

4 Empirical Analysis

This section presents our empirical findings. First, we delve into the in-sample predictive power of commodity up- and down-tail factors in Section 4.1. In Section 4.2, we provide details on the yield-spanning and macro-spanning tests of CT. Section 4.3 compares the forecasting power of our tail factor with existing bond predictors. Finally, Section 4.4 provides the out-of-sample predictive regressions.

4.1 In-Sample Predictive Regression

We use CT to predict bond risk premia based on the following predictive regression,

$$rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \varepsilon_{t+1}, \tag{10}$$

where $rx_{t+1}^{(n)}$ are 2- to 5-year Treasury bond risk premia, $Tail_t$ is the commodity up-tail factor (UT) or the commodity down-tail factor (DT), and ε_{t+1} is an error term. We test the significance of the predictive coefficient by calculating Newey and West (1987) p-value based on the heteroscedasticity- and autocorrelation-consistent (HAC) standard errors. Moreover, we calculate Bauer and Hamilton (2018) p-values based on the parametric bootstrap approach.

Table 6 shows the corresponding regression results. Both, the UT and DT factor serve as significant predictors of bond risk premia (Panels A and B) across various maturities. An increase in the tail factor corresponds to an increase of the average bond risk premium of 0.289 (for the uptail) and 0.229 (for the downtail), with an adjusted R^2 of 28.29% and 22.18%, respectively. Also at the individual maturity levels, we observe a similar predictive power, with an adjusted R^2 ranging from 19.88% to 30.12%. Moreover, the UT and DT are statistically significant at the 1% level, as indicated by the HAC p-values and the bootstrapped p-values.

4.2 Commodity Tails and the Spanning Tests

We test whether our CT contains additional information in forecasting bond returns beyond the observed yields and a large set of macroeconomic and time-series variables. Specifically, we consider the first five principal components (PCs) of the Gürkaynak, Sack, and Wright (2007) month-end yield data,[5] which represents the yield curve level, slope, curvature and 4th and 5th yield factors (for instance, Nelson and Siegel, 1987; Cochrane and Piazzesi, 2005; Bauer and Hamilton, 2018; Moench and Soofi-Siavash, 2022). Also, we reconstruct the eight macroeconomic factors of Moench and Soofi-Siavash (2022) based on our sample period to implement the macro-spanning tests.

Here, we regress bond risk premia on the commodity tail factors (UT and DT) and the first three PCs, the first five PCs of the observed yields, or the macro-yields factors based on the following regression:

$$rx_{t+1}^{(n)} = \alpha + \beta' X_t + \varepsilon_{t+1}, \tag{11}$$

where $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, X_t represents different combinations of factors. Table 7 reports the results of the spanning test for Treasury bond risk premia across various maturities. In Table 7, PC1-PC5 are the five principal

[5] The Fama-Bliss dataset is limited for researching bond excess returns on account of restricted maturities (only up to five years). (e.g., Le and Singleton, 2013; Schraeder, Sojli, Subrahmanyam, and Tham, 2022)

components of Gürkaynak, Sack, and Wright (2007) yields, and M1-M8 are eight macroeconomic factors of Moench and Soofi-Siavash (2022)[6], and UT and DT denote our constructed commodity up- and down-tail factor.

We respectively present the parameter estimates, the adjusted R^2, and the gain in the adjusted R^2 relative to the regression without the CT. From Panel A of Table 7, we find that UT is still highly statistically significant after controlling for the first three PCs, the first five PCs and the macro-yields factors. Moreover, the Bauer and Hamilton (2018) p-values do not change the significance. A similar result is also seen in DT, which is significantly positive in predicting bond returns across different maturities shown in Panel B. With regards to R^2, we find UT (DT) has enormously significant marginal predictive power for future bond returns changes, for instance, UT markedly increases predictability concerning the first three principal components (column 7), the first five principal components (column 9), and the macro-yields model (column 11), with the $\bar{R}^2$ increasing from 15.15% to 23.13% for the two-year maturity reported in Panel A, and the figures for DT are from 11.60% to 18.68%[7].

[Please insert Table 7 and Table A1 here]

4.3 Commodity Tails and Existing Bond Predictors

The essential point is to test whether CT is robust in forecasting Treasury bond risk premia after controlling other competitors. In this subsection, we control for the Cochrane and Piazzesi (2005) (CP) factor, the Ludvigson and Ng (2009) (LN) factor and the Cieslak and Povala (2015) (CPo) factor in our analysis. [8] The construction of these factors is shown in Appendix C.

[6]The eight macroeconomic factors are estimated by regressing 135 macroeconomic series on the PC1-PC5 and then extracting principal components from the residuals. The macroeconomic series covers the most important categories of U.S. economic activity, including the FRED-MD database compiled by McCracken and Ng (2016), the average weekly hours of production and non-supervisory employees, the Philadelphia Fed leading indicator for the U.S. economy, the VXO index, the measure of realized stock market volatility from Berger, Dew-Becker, and Giglio (2020), the Bank of America Merrill Lynch MOVE bond volatility index, the measure of financial uncertainty from Ludvigson, Ma, and Ng (2021); the excess bond premium from Gilchrist and Zakrajšek (2012); and the three-month Treasury bill forecast from the Consensus Economics Survey of Professional Forecasters.

[7]We report the result of regression of bond risk premia on the first three PCs, the first five PCs of the observed yields, or the macro-yields factors alone in Table A1.

[8]Cochrane and Piazzesi (2005) construct a tent-shaped factor and explain more than 30% of the variation in Treasury bond risk premia. Ludvigson and Ng (2009) estimate the 8 principal component factors from the 132 monthly macroeconomic indicators, and significantly predict bond returns in- and out-of-sample. Cieslak and Povala (2015) propose risk premia can be implied by the yield curve and trend inflation.

Here, we regress bond risk premia on the commodity tail factors (UT and DT) and the competitors based on the following regression:

$$rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \gamma F_t + \varepsilon_{t+1}, \tag{12}$$

where $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, $Tail_t$ denotes the commodity up-tail factor (UT) or down-tail factor (DT) based on the PLS method, F_t denotes the bond predictors (the CP factor, the LN factor, and the CPo factor), and ε_{t+1} is an error term. Table 8 reports these predictive results.

Concerning the CP factor, Panel A reveals that UT explains 29.42%–32.57% of the variation in bond risk premia after controlling the CP factor, with the coefficients all significantly positive according to the NW p-value and the BC p-value. Similar significance for DT is also shown in Panel B, whereas the adjusted R^2 is small. Likewise, to test whether the CT contains any predictive information beyond those already subsumed by the CP factor, we also calculate the gain of the $\bar{R}^2$ compared with the model with the CP factor alone. As shown on the first part of two panels, both factors (UT and DT) have a substantial $\Delta\bar{R}^2$ for all bond maturities, which implies CT captures more information on the dynamics of bond risk premia.

In the middle part of the two Panels, we present results from running predictive regressions for the LN factor and the commodity up- or down-tail factor. UT and DT respectively explain 31.74%–35.28% and 26.85%-33.11% of the variation in bond risk premia of various maturities when we add the LN factor to the model. Meanwhile, the coefficients of two types of tail factors are all significantly positive. The inclusion of CT increases the adjusted R^2 by about 15.94 to 22.95 percentage points for all bond maturities, indicating that our constructed factors provide an important complementary information source to the macroeconomic predictor.

In addition, it is clear from the third part of the panels that CT still has high predictive power for bond risk premia, using the CPo factor as a benchmark. In Panel A, we can find that UT is statistically and economically significant with the inclusion of the CPo factor. The adjusted R^2 for average bond risk premia is 30.15%, and the figure for DT is relatively low (24.42% only). Moreover, augmenting the predictive regression with UT or DT increases the explanatory power of about 20.76 or 15.03 percentage points for average excess bond returns, where the most ken-speckle improvement happens for the bond with shorter maturities.

Overall, CT we constructed remains economically and statistically significant even in the presence of the prevailing bond predictors, and accounting for small sample biases by Bauer and Hamilton (2018) p-values do not affect our conclusion. In the meantime, The incremental of the adjusted R^2 is highly substantial across various maturities. By comparison, we also regress bond risk premia on the CP factor, the LN factor, or the CPo factor alone, and report the results in Table A2. Although these factors remain significant at the 5% level for all bonds, the explanatory powers are lower than those reported in other studies (e.g., Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009; Cieslak and Povala, 2015), which can be attributed to the difference in the sample period.

[Please insert Table 8 and Table A2 here]

4.4 Out-of-Sample Predictive Regression

To test the out-of-sampel predictive power of the CT, we follow Goyal and Welch (2008). Using a rolling window to forecast bond risk premia, We split our sample into two halves. We use the first half of the sample to predict the return of the first month of the second half.[9] Then, we update the estimates with the data of this month for the prediction of the next month and continue till the end of the sample.

To judge whether CT contains relevant information, we compare our model to a benchmark model. One traditional benchmark model is the historical average (e.g., Goyal and Welch, 2008; Campbell and Thompson, 2008). We define the historical average bond risk premium as $\overline{rx}_{t+1}^{(n)} = \frac{1}{t}\sum_{j=1}^{t} rx_j^{(n)}$. In addition, we also consider benchmark models with the first three principal components and other bond predictors.

When we define benchmark model forecast as $\widetilde{rx}_t^{(n)}$ and the forecast from the predictive

[9]The first window runs from January 1993 to September 2007 (177 monthly observations) and forecasts the bond risk premium in October 2007.

regression with CT as $\widehat{rx}_t^{(n)}$, the out of sample R^2_{oos} is

$$R^2_{oos} = 1 - \frac{\sum_{t=s_0+1}^{T}\left(rx_t^{(n)} - \widehat{rx}_t^{(n)}\right)^2}{\sum_{t=s_0+1}^{T}\left(rx_t^{(n)} - \widetilde{rx}_t^{(n)}\right)^2}, \tag{13}$$

In this equation, s_0 denotes the sample size that we choose to train the model for the first time. If the out-of-sample R^2 is positive, the mean squared prediction errors (MSPE) of the predictive model with CT is lower than that of the benchmark model, indicating higher predictive accuracy. To examine whether the predictive model with CT and the benchmark model are statistically different, we use the Clark and West (2007) (CW) test and the Giacomini and White (2006) (GW) test. The null hypothesis of the CW test is that two forecasting models have equal MSPE (R^2_{oos}=0), and it is suitable for the nested model. Also the GW test check whether both models have equal predictive ability, but they consider the parameter uncertainty in their null hypothesis.

Table 9 reports the out-of-sample predictive performance of the CT for Treasury excess bond returns of various maturities when the historical average is the benchmark. The empirical results show that both UT and DT have significantly positive R^2_{OOS}s over the bond maturity spectrum (17.03%-19.07% for UT and 4.61%-5.77% for DT). Moreover, in line with our model predictions, the commodity up-tail factor performs better than the downtail factor. The uptail factor possesses a substantial out-of-sample R^2 and remains significant at the 1% level for all bonds according to the CW test and the GW test.

Figure 5 plots the differences in cumulative squared prediction errors (DCSPE) between the benchmark model and the predictive model to assess the persistence of the CT's forecasting performance over time. The DCSPEs of the benchmark model and the uptail model tend to increase steadily over time. The DCSPE of the downtail model rises after a remarkable initial decline.

Table 10 reports the out-of-sample results using various benchmark models. These include the first three PCs of current yields, the CP factor, the LN factor, and the CPo factor. Panel A shows that the addition of the uptail factor to the respective benchmark model increases the out-of-sample R^2s significantly across bond maturities (9.80%-18.76% for the three PCs, 16.68%-20.60% for CP, 9.89%-17.92% for LN, and 17.81%-18.80% for CPo). Regarding the CW test and the GW test, we find that most out-of-sample R^2 values are significant at the 5% level. Therefore, the uptail factor contains complementary information and generates significant out-of-sample predictive power. Also for the downtail factor (Panel B), its addition to the benchmark models generates substantial R^2s in forecasting bond risk premia across various maturities. In addition, Figure 6 presents the time series of the DCSPE without and with the commodity tail factor (UT or DT) as one predictor. When the three PCs, CP, and CPo serve as the benchmark, the differences are positive and increase steadily over time, but for LN, the DCSPE starts rising after several years.

To identify which individual commodity matters most, we follow Next, we follow Lin, Wu, and Zhou (2018), Zhao, Zhou, and Zhu (2021) and Chen, Yao, Zhang, and Zhu (2023): We determine the average covariance between individual commodity up or down tail and future bond risk premia in the out-of-sample regression. Figure 7 displays the rank of the commodities. According to Panel A gold, lean hog and unleaded gasoline are the top three. For downtail, aluminum, corn and sugar play the most significant roles. Considering commodities, which are important for bot tails we find six commodities located in the top ten for bot tails: gold, unleaded gasoline, gas oil, soy oil, coffee, and corn.

[Please insert Table 9, Table 10, Figure 5, Figure 6 and Figure 7 here]

5 Economic Mechanism

To empirically document the economic mechanism, we check the intermediate steps that follow from the model. The first model implication is the link between commodity tail risk and future inflation. We document this link in Subsection 5.1. The higher inflation uncertainty in turn generates higher uncertainty about the central bank reaction. It generates interest rate uncertainty, as we empirically document in Subsection 5.2.

While downtail risk is a negative sign for the economy both in stock markets and commodity markets, the uptail can be different. In contrast to the stock market, where high stock prices indicate a good economic condition, high commodity prices can also initiated by supply shortages or interruptions of the supply chain. In Subsection 5.3, we show that actually the latter is the case and that both, uptail and downtail risk, is a negative predictor for indicators of macroeconomic prosperity.

5.1 Commodity Tails and Inflation

In a first step, we link commodity tail risk to future inflation[10] based on the following regression:

$$Inflation_{t+1}^{h} = \alpha + \beta Tail_t + \varepsilon_{t+1}, \tag{14}$$

$Tail_t$ represents the commodity up- or down-tail factor extracted based on the PLS method. Following the method of Stock and Watson (1999), we use $I_{t+h}^{h} - I_t$ to measure the inflation between period t and $t+h$, where $I_{t+h}^{h} = (1200/h)\log(CPI_{t+h}/CPI_t)$ and $I_t = (1200)\log(CPI_t/CPI_{t-1})$.

Table 11 reports the regression's coefficient estimates and adjusted R^2. Panel A shows that UT is a significantly positive predictor of future 3- to 36-month inflation (using Newey and West (1987) p-values). An increase in UT raises future inflation with a regression slope from 0.022 to 0.072. Moreover, UT factor explains 1.87%-8.52% of the variation in inflation across the horizons. Similar to the forecasting power of the up-tail factor, DT by itself significantly predicts future 3- to 36-month inflation with R^2 ranging from 3.10% to 7.74%. As expected, the coefficients of DT are negative, indicating that an increase in the down-tail factor is followed by a lower inflation level.

Thus, changes in commodity prices are closely linked to inflation levels. This can induce the central bank to change the short-term interest rate – creating interest rate uncertainty.

[Please insert Table 11 here]

[10] We complement previous studies that focus on inflation predictions (e.g., Stock and W Watson, 2003; Stock and Watson, 2007; Wright, 2009; Salisu, Ademuyiwa, and Isah, 2018; Salisu, Swaray, and Sa'id, 2021; Kilian and Zhou, 2022).

5.2 Commodity Tails and Interest Rate Uncertainty

The next step in our model is the connection between commodity tail risk and interest rate uncertainty. To illustrate this link, we regress future interest rate uncertainty on the up- to down-tail risk of 24 commodity returns. To measure

$$IRU_{t+1}^{h} = \alpha + \beta Tail_t + \varepsilon_{t+1}, \tag{15}$$

where interest rate uncertainty (IRU) is the standard deviation of the daily federal funds rate with 1- to 6-month horizons and $Tail_t$ represents our commodity up- or down-tail factor.

Table 12 reports the corresponding regression results. According to Panel A, the commodity up-tail factor is significantly positive with the p-values less than 1% and the adjusted R^2s greater than 10% across various horizons. According to panel B, also the down-tail factor has a significant positive impact on future interest rate uncertainty, with the adjusted R^2s ranging from 21.20% to 24.03%. Thus, an increase in UT or DT triggers higher future interest rate uncertainty. Given that this interest rate risk is priced in long-term bonds, excess returns will be higher.

[Please insert Table 12 here]

5.3 Links to the Macroeconomic Condition

Following Fama and French (1989) and Cochrane (2011), investors with higher risk aversion require a higher risk premium when they face the risk that the economic condition deteriorates. If the increase in the CT implies great economic uncertainty, it should predict a positive risk premium in the bond market. Consistently, our main results of in-sample regressions indicate that the commodity tail factors (UT and DT) increases are in fact associated with subsequent higher bond risk premia. In this subsection, we regress the future economic variables on the current CT constructed based on bond risk premia in order to test whether the CTs are countercyclical variables.

$$MacroVariable_{t+1} = \alpha + \omega Tail_t^{(avg)} + \epsilon_{t+1}, \tag{16}$$

where $MacroVariable_{t+1}$ represents the GDP growth (GDP Growth), the industrial production growth (IPG), the nonfarm payroll growth (Payroll Growth), and the unemployment rate (Unemp Rate) in time $t+1$, $Tail_t^{(avg)}$ represents the commodity up- or down-tail factor extracted from equal-weighted yearly bond risk premia in the PLS method, and ϵ_{t+1} is an error term.

Table 13 documents the parameter estimates and the explanatory power of predictive regressions (16). First, we can see that the commodity up-tail factor has a significantly negative relationship with the future GDP growth, IPG, and Payroll Growth from Panel A, and their $\bar{R}^2$ is 4.59%, 5.07% and 6.00%, respectively. Besides, concerning the future unemployment rate, it exhibits significant predictive ability with a positive coefficient, and the values of $\bar{R}^2$ are higher. Also, similar results are shown in Panel B, suggesting that the commodity up- and down-tail factors serve as a strong signal for future macroeconomic conditions.

To summarize, we find that our CTs are significantly related to the future business cycle, and higher tail risk implies deteriorations in future economic condition. This finding indicates that the link to future economic state is a possible channel through which CTs affect the time-varying bond risk premia.

[Please insert Table 13 here]

6 Robustness Tests

In this section, we assess the robustness of the commodity tail factors' predictability. These questions will be addressed: (1) whether the CT can predict bond risk premium within various subsamples or (2) the different forecasting lags; (3) whether the CT extracted from different moving windows can predict bond risk premium; (4) whether the CT can significantly improve the predictive ability compared with commodity futures returns or volatility; (5) how does CT perform in forecasting other countries' Treasury bond returns; (6) whether CT has a more substantial effect during recessions.

6.1 Subsamples

Our full period covers 30 years, with many macroecoomic changes and alterations in monetary policy. To test the robustness of our results over time, we divide the full sample into three subsamples: 2000-2022, 2005-2022, and 2010-2022.

Table 14 shows that the predictive results of UT and DT are significant in all subsamples, though the coefficients vary across periods. Moreover, our results are robust to the Bauer and Hamilton (2018) test.

[Please insert Table 14 here]

6.2 Varying Forecasting Lags

To show that the CT factor can predict bond risk premia with different forecasting lags, we regress the future 3-, 6-, 9- and 12-month Treasury bond risk premia on the commodity up- or down-tail factor: $rx_{t+i}^{(n)} = \alpha + \beta Tail_t + \varepsilon_{t+i}$, where $rx_{t+i}^{(n)}$ is the future i-month bond risk premium.

Table 15 reports the results of in-sample predictions for excess bond returns with the different forecasting lags. Panel A shows that UT significantly predicts future 3- to 12-month bond risk premia in-sample and out-of-sample. However, the adjusted R^2 values are negative for DT in out-of-sample regressions.

Overall, all in-sample regressions are significantly positive between the CT and the future 3-, 6-, 9- and 12-month excess bond returns. Also, the commodity up-tail factor has persistent out-of-sample forecast performances.

[Please insert Table 15 here]

6.3 Estimation Windows

Up to now, we have constructed the tail ris based on the past 6-month commodity future returns. As a robustness check, we alter the estimation window size between 1-month and 60-month and use it in the standard regression $rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \varepsilon_{t+1}$.

Figure 8 shows that the CT has a substantial in- and out-of-sample adjusted R^2 in predicting bond risk premia for varying estimation window sizes. Interestingly, the highest R^2 values are not obtained in our traditional 6-month moving window. UT generates the greatest R^2 in 12-month or 14-month moving windows for in- or out-of-sample forecasts. For DT, greatest R^2 occurs for an even larger estimation window of two to three years. Moreover, UT performs better than DT in most conditions, especially for out-of-sample forecasts.

[Please insert Figure 8 here]

6.4 Commodity Returns and Volatility

Our model predicts that commodity tail risk is a better predictor than commodity return and volatility.[11] To compare the effect of tail risk and return or volatility empirically, we regress bond risk premia on the commodity return factor or the commodity volatility factor, which we have estimated based on the PLS method

$$rx_{t+1}^{(n)} = \alpha + \beta Return_t + \epsilon_{t+1}, \tag{17}$$

$$rx_{t+1}^{(n)} = \alpha + \beta Volatility_t + \epsilon_{t+1}, \tag{18}$$

where $rx_{t+1}^{(n)}$ is the equal-weighted yearly and two- to five-year Treasury bond risk premia, $Return_t$ and $Volatility_t$ represent the commodity returns factor and the commodity volatility factor. Individual commodity future return is the average value of the past 6-month daily returns, and individual commodity future volatility is calculated by

$$V_{i,t}^M = \sqrt[2]{\frac{\sum_{d=1}^{D_t}(R_{i,d}^D - \bar{R}_{i,t})^2}{n}}, \tag{19}$$

where $V_{i,t}^M$ is the past 6-month daily return volatility of commodity future i in the past 6-month, $R_{i,d}^D$ is daily return of i in day d and $\bar{R}_{i,t}$ is the average daily return of i in the past 6-month.

[11] Existing literature has confirmed that commodity return (e.g., Black, Klinkowska, McMillan, and McMillan, 2014; Jacobsen, Marshall, and Visaltanachoti, 2019; Wang, Pan, Liu, and Wu, 2019; Iyke and Ho, 2021; Li, Wu, and Zhou, 2021) and volatility (e.g., Arouri, Lahiani, and Nguyen, 2011; Creti, Joëts, and Mignon, 2013; Mensi, Beljid, Boubaker, and Managi, 2013; Christoffersen and Pan, 2018; Xiao and Wang, 2022) are notable for predicting the future stock market performance. However, similar studies for the bond market are rare.

Panel A in Table 16 shows that the commodity return factor positively predict in-sample Treasury bond risk premia across various maturities with an average adjusted R^2 of 7.8%. However, the factor performs poorly in out-of-sample prediction with a negative $\bar{R}^2_{OOS}$ for all maturities. Similar to the commodity return factor, the commodity volatility factor performs well only in in-sample regressions, but out-of-sample R^2 values are negative and insignificant.

Thus, the commodity tail factors (UT and DT) have a higher in-sample and out-of-sample R^2 and a more substantial predictive power than return or volatility – especially when considering out-of-sample performances.

[Please insert Table 16 here]

6.5 Stock Tail Risk and Treasury Bond Tail Risk

While our primary focus lies in examining commodity tail risk, we conduct a comparative analysis of the predictive efficacy of stock tail risk and treasury bond tail risk in forecasting treasury bond risk premia. Specifically, for stocks, we adhere to the methodology outlined by Kelly and Jiang (2014) to compute time-varying tail risk, which can be directly estimated from the cross-section of stock returns utilizing daily CRSP data for NYSE/AMEX/NASDAQ stocks with share codes 10 and 11. Conversely, for treasury bonds, we calculate the tail risk of 1- to 30-year treasury bond yields as documented in Gürkaynak, Sack, and Wright (2007) with the equation (3). Subsequently, we utilize the PLS method to forecast future treasury bond risk premia.

Panel A in Table 17 reveals that the predictive power of the stock tail risk factor in forecasting future Treasury bond risk premia is limited, with an average adjusted R^2 of 0.5% and $\bar{R}^2_{OOS}$ close to 0. Conversely, the treasury bond tail risk factor exhibits favorable performance in both in-sample and out-of-sample regressions. However, it is noteworthy that its predictive capacity remains inferior to that of commodity tail risk factors.

[Please insert Table 17 here]

6.6 Global Markets

The described link between commodity tail risk and bond excess returns is not only prevalent in the US market. To show this link, we test the CT's predictive ability in other G7 countries in this subsection using the regression

$$rx^{(5)}_{c,t+1} = \alpha + \beta Tail_{c,t} + \varepsilon_{t+1}, \tag{20}$$

where $rx^{(Avg)}_{c,t+1}$ is the five-year excess bond returns on country c at the time $t+1$, $Tail_{c,t}$ is the PLS commodity up- or down-tail factor for country c at the time t, c represents country: Canada, France, Germany, Italy, Japan and the U.K.[12].

Table 18 reports the results of in-sample and out-of-sample predictive regression of excess bond returns on the CT. The sample covers the period from January 1996 to June 2022. According to the in-sample results, UT and DT are significant predictors of bond risk premia according to the NW p-values. The average coefficient is 0.251 for UT and 0.265 for DT, indicating that the increase in commodity tail factors tend to be followed by a higher bond premium in the next period. Besides, the adjusted R^2 is substantial for most countries. As for the our-of-sample predictive regression, R^2_{OOS} of UT is significantly positive in all countries, and for DT, there are five countries generating the significantly positive R^2_{OOS}.

[Please insert Table 18 here]

6.7 Stock Market

Interest rate risk also affects the stock market. To investigate this link, we determine monthly S&P 500 index excess returns over 1-, 3-, 6-, and 12-month horizons using the standard methodology outlined in Goyal and Welch (2008). We use the following predictive regression for stock returns,

$$R^{(n)}_{t+1} = \alpha + \beta Tail_t + \varepsilon_{t+1}. \tag{21}$$

[12] The bond prices for Canada, Germany, Japan, the United Kingdom are from their central banks. The bond prices for the other countries are from Bloomberg.

In this regression, $R_{t+1}^{(n)}$ represents the monthly S&P 500 index excess returns over the n-month horizon, $Tail_t$ denotes either the commodity up-tail factor (UT) or the commodity down-tail factor (DT), and ε_{t+1} is the error term.

Table 19 shows that UT demonstrates significant and positive predictive power for S&P 500 index excess returns, particularly over longer horizons. DT exhibits significance in predicting stock excess returns in in-sample regressions. However, its predictive performance is insignificant in out-of-sample predictions.

[Please insert Table 19 here]

In summary, commodity tail risk also predicts stock excess returns, but the link is not as clear as for bond excess returns. Moreover, it is only relevant for longer-term horizons.

7 Conclusion

In this paper, we show (theoretically and empirically) that commodity tail risk predicts bond excess returns. The driving force in this connection is the uncertainty about the central bank's reaction to price changes. Even though a rough guidance through the Taylor rule is known, the predictions are still noisy. This uncertainty affects long-term bonds more than short-term bonds and results in bond excess returns.

To study this relationship empirically, we construct the commodity tail factor (CT). It provides a new source of information that is not spanned by the current yield curve. Our study contributes to the literature that examines the effect of the tail distribution of 24 commodity returns on bond returns and adds complementary evidence to cross-asset pricing literature. Our findings also suggest that the commodity tails can significantly predict future economic fundamentals and exhibit counter-cyclical patterns.

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Table 3: **Summary Statistics on the Up Tail Risk and Down Tail Risk of Commodities Futures Returns**

This table displays summary statistics on the individual up and down tail risk of 24 commodities futures returns. We report the average (Mean), the maximum value (Max.), the minimum value (Min.), the standard deviation (Std dev), the skewness and autocorrelations ($\rho 1$ and $\rho 12$ denote the first and twelfth order autocorrelations). The tail risk of commodity futures returns is calculated based on the method that Kelly and Jiang (2014) proposed. The data start in January 1992 and end in June 2022.

	Panel A: Up Tail Risk of Commodities Futures Returns							**Panel B: Down Tail Risk of Commodities Futures Returns**						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	Mean(%)	Max.(%)	Min.(%)	Std dev.(%)	Skewness	ρ_1	ρ_{12}	Mean(%)	Max.(%)	Min.(%)	Std dev.(%)	Skewness	ρ_1	ρ_{12}
Brent oil	25.02	69.19	4.24	11.30	1.03	0.75	0.10	24.44	62.30	4.60	10.15	0.85	0.70	-0.07
Crude oil	27.13	81.76	6.07	12.93	1.23	0.73	-0.06	27.81	162.90	3.07	21.10	4.22	0.84	0.14
Gas oil	26.06	74.40	8.02	10.91	1.22	0.69	0.05	24.83	77.76	5.29	11.46	1.11	0.71	0.06
Unleaded gasonline	33.13	82.29	8.21	14.23	0.81	0.70	0.01	30.77	74.70	7.19	13.85	1.09	0.68	0.00
Heating oil	25.86	76.82	6.80	11.97	1.55	0.73	-0.05	33.85	179.47	7.66	22.44	3.08	0.81	0.01
Natgas	33.72	83.20	9.97	14.02	0.79	0.67	0.00	27.64	90.63	4.43	11.91	0.93	0.61	0.07
Cattle	23.98	66.33	3.94	13.51	0.91	0.76	0.01	33.89	84.56	3.54	17.45	0.45	0.80	-0.02
Lean hog	41.63	99.06	4.51	18.30	0.54	0.74	0.33	39.05	94.80	4.02	20.16	0.59	0.78	0.40
Gold	28.88	99.26	5.75	13.18	1.30	0.65	0.03	30.81	74.18	8.44	11.22	0.92	0.59	0.07
Platinum	27.16	67.30	1.15	11.72	0.82	0.64	0.02	31.76	77.82	5.97	13.09	0.75	0.75	-0.06
Silver	29.88	87.34	5.42	13.10	0.99	0.68	-0.03	33.10	94.72	7.52	14.06	0.90	0.71	-0.08
Aluminum	24.86	55.51	7.86	9.64	0.54	0.70	0.08	24.76	82.09	5.02	12.10	1.65	0.78	0.24
Copper	25.25	55.76	6.88	9.52	0.45	0.69	0.20	26.70	62.94	5.25	11.34	0.71	0.74	0.12
Nickel	26.66	81.59	6.74	12.01	1.40	0.74	0.07	28.39	90.28	5.20	12.71	1.09	0.74	0.09
Zinc	27.14	88.91	6.67	13.68	1.16	0.78	0.09	28.11	78.70	6.22	14.26	0.84	0.81	0.23
Cocoa	25.40	56.08	5.00	10.78	0.39	0.65	0.08	25.98	80.67	5.24	11.42	1.07	0.60	0.12
Coffee	28.78	81.19	7.04	14.15	1.25	0.72	0.20	28.68	78.01	7.03	11.53	0.79	0.67	0.10
Corn	33.09	71.22	9.61	13.68	0.38	0.71	0.10	30.77	92.07	5.42	14.37	1.40	0.76	0.01
Cotton	30.30	69.86	5.78	12.10	0.63	0.74	0.09	24.94	93.47	4.21	12.25	1.74	0.76	-0.07
Soybean	29.10	69.87	8.92	11.38	0.74	0.63	-0.11	33.82	91.27	7.47	16.17	0.87	0.70	0.13
Soymeal	29.56	74.02	4.29	12.54	0.45	0.71	0.16	35.27	94.82	6.39	17.98	1.05	0.78	-0.03
Soyoil	25.31	80.60	2.83	11.14	0.91	0.68	0.19	24.47	79.26	5.23	10.81	1.38	0.77	-0.04
Sugar	29.90	71.20	4.33	11.86	0.46	0.71	-0.04	33.22	104.62	6.40	15.64	1.14	0.74	0.24
Wheat	29.64	64.81	7.91	11.47	0.74	0.72	0.06	26.36	101.95	6.20	14.59	2.22	0.81	0.06

Table 4: **Summary Statistics of Bond Risk Premia**

This table displays summary statistics on Treasury bond risk premium. We report the average (Mean), the maximum value (Max.), the minimum value (Min.), the standard deviation (Std dev), the skewness and autocorrelations ($\rho 1$ and $\rho 12$ denote the first and twelfth order autocorrelations). The data start in January 1992 and end in June 2022.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Mean(%)	Max.(%)	Min.(%)	Std dev.(%)	Skewness	ρ_1	ρ_{12}
Average	1.420	8.436	-5.727	2.699	-0.093	0.928	-0.062
2-year	0.575	3.537	-2.468	1.159	0.186	0.937	0.087
3-year	1.182	7.334	-5.254	2.301	0.004	0.934	0.003
4-year	1.771	10.317	-6.899	3.281	-0.101	0.926	-0.075
5-year	2.151	12.556	-8.389	4.180	-0.156	0.922	-0.124

Table 5: **Summary Statistics of Commodity Tail Factors**

This table displays summary statistics on the commodity up-tail factor (UT) and down-tail factor (DT). In Panel A or B, we respectively report the average (Mean), the maximum value (Max.), the minimum value (Min.), the standard deviation (Std dev), the skewness and autocorrelations ($\rho1$ and $\rho12$ denote the first and twelfth order autocorrelations) of UT or DT based on bond risk premia across various maturities. In panel C, we report pairwise correlations of UT and DT. All series start from January 1992 and end in June 2022.

Panel A: Descriptive Statistics of Up-tail Factor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Mean(%)	Max.(%)	Min.(%)	Std dev.(%)	Skewness	ρ_1	ρ_{12}
$UT^{(Avg)}$	1.32	15.98	-9.13	4.98	0.35	0.79	0.12
$UT^{(2)}$	0.35	7.13	-4.37	2.14	0.40	0.80	0.15
$UT^{(3)}$	0.94	13.71	-8.02	4.22	0.37	0.79	0.13
$UT^{(4)}$	1.67	19.62	-11.26	6.12	0.33	0.79	0.12
$UT^{(5)}$	2.39	25.01	-14.34	7.77	0.32	0.79	0.10

Panel B: Descriptive Statistics of Down-tail Factor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Mean(%)	Max.(%)	Min.(%)	Std dev.(%)	Skewness	ρ_1	ρ_{12}
$DT^{(Avg)}$	-0.92	18.11	-14.71	5.59	0.35	0.81	0.16
$DT^{(2)}$	-0.57	6.45	-6.64	2.20	0.16	0.82	0.24
$DT^{(3)}$	-1.01	12.85	-13.25	4.62	0.28	0.81	0.21
$DT^{(4)}$	-1.08	23.20	-17.70	6.90	0.36	0.81	0.16
$DT^{(5)}$	-0.95	32.64	-22.10	9.11	0.35	0.80	0.11

Panel C: : Correlations matrix

	$UT^{(Avg)}$	$UT^{(2)}$	$UT^{(3)}$	$UT^{(4)}$	$UT^{(5)}$
$DT^{(Avg)}$	0.461	0.463	0.459	0.457	0.456
$DT^{(2)}$	0.422	0.435	0.425	0.417	0.413
$DT^{(3)}$	0.451	0.459	0.451	0.446	0.444
$DT^{(4)}$	0.456	0.458	0.453	0.452	0.452
$DT^{(5)}$	0.464	0.461	0.459	0.461	0.463

Table 6: **Regressions of Bond Risk Premia on Commodity Tail Factors**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \varepsilon_{t+1}$, where $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, $Tail_t$ denotes the commodity up-tail factor (UT) or commodity down-tail factor (DT) based on the PLS method and ε_{t+1} is an error term. In round brackets, we report the p-value based on the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). Simultaneously, we report the p-values based on the parametric bootstrap approach of Bauer and Hamilton (2018) in square brackets. ***, **, and * represent significance level at 1%, 5%, and 10%. The sample period is January 1992 to June 2022.

Panel A: Regressions on Commodity Up-tail Factor

	(1) Average	(2) 2-year	(3) 3-year	(4) 4-year	(5) 5-year
Constant	0.010***	0.005***	0.009***	0.013***	0.015***
	(0.001)	(0.000)	(0.001)	(0.001)	(0.003)
UT	0.289***	0.298***	0.297***	0.282***	0.282***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\bar{R}^2(\%)$	28.29	30.12	29.47	27.39	27.17

Panel B: Regressions on Commodity Down-tail Factor

	(1) Average	(2) 2-year	(3) 3-year	(4) 4-year	(5) 5-year
Constant	0.016***	0.007***	0.014***	0.020***	0.023***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
DT	0.229***	0.271***	0.243***	0.221***	0.206***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.002]	[0.000]	[0.000]	[0.000]
$\bar{R}^2(\%)$	22.18	26.24	23.58	21.44	19.88

Panel C: Regressions on Commodity Up- and Down-tail Factor

	(1) Average	(2) 2-year	(3) 3-year	(4) 4-year	(5) 5-year
Constant	0.013***	0.006***	0.011***	0.016***	0.018***
	(0.002)	(0.001)	(0.001)	(0.002)	(0.007)
UT	0.217***	0.218***	0.223***	0.212***	0.216***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.002]	[0.000]	[0.000]	[0.000]
DT	0.139***	0.178***	0.151***	0.136***	0.120***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.002]	[0.000]	[0.000]	[0.000]
$\bar{R}^2$	34.7	39.3	36.7	33.8	32.4

Table 7: **Commodity Tail Factors, Yield-spanning Tests and Macro-spanning tests**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $rx_{t+1}^{(n)} = \alpha + \beta' X_t + \varepsilon_{t+1}$. $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, PC1-PC5 are the five principal components of Gürkaynak, Sack, and Wright (2007) yields, and $M1$-$M8$ are eight macroeconomic factors of Moench and Soofi-Siavash (2022), which are estimated by regressing 135 macroeconomic series on the PC1-PC5 and then extracting principal components from the residuals. UT and DT denote the commodity up- and down-tail factor. In round brackets, we report the p-value based on the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). And in square brackets, we report the bias-corrected (BC) p-values based on the parametric bootstrap approach of Bauer and Hamilton (2018). We also report the gain in $\bar{R}^2$ relative to the model without the commodity tail factors. ***, **, and * represent significance level at 1%, 5%, and 10% based on the Newey and West (1987) p-value. The sample period is January 1992 to June 2022.

Panel A: Commodity Up-tail Factor and Spanning Hypothesis Tests

	(1) Average						(2) 2-year						(3) 3-year						(4) 4-year						(5) 5-year					
PC1	0.05	(0.31)	0.05	(0.26)	0.05	(0.19)	0.03	(0.11)	0.03*	(0.08)	0.04**	(0.04)	0.05	(0.20)	0.05	(0.16)	0.06	(0.10)	0.06	(0.32)	0.06	(0.27)	0.06	(0.20)	0.05	(0.47)	0.06	(0.42)	0.06	(0.37)
PC2	-0.26	(0.41)	-0.27	(0.30)	-0.27	(0.26)	-0.03	(0.83)	-0.03	(0.77)	-0.04	(0.69)	-0.13	(0.61)	-0.14	(0.52)	-0.15	(0.46)	-0.32	(0.40)	-0.33	(0.30)	-0.34	(0.26)	-0.55	(0.26)	-0.57	(0.17)	-0.56	(0.14)
PC3	-0.84	(0.50)	-0.81	(0.52)	-0.81	(0.47)	-0.19	(0.71)	-0.17	(0.74)	-0.14	(0.76)	-0.62	(0.55)	-0.59	(0.58)	-0.56	(0.56)	-1.29	(0.39)	-1.26	(0.42)	-1.26	(0.36)	-1.30	(0.51)	-1.27	(0.53)	-1.29	(0.46)
PC4			-9.78*	(0.06)	-9.80**	(0.02)			-3.96	(0.13)	-4.08*	(0.06)			-7.93	(0.10)	-8.04**	(0.04)			-11.74*	(0.06)	-11.75**	(0.02)			-15.33**	(0.03)	-15.25**	(0.01)
PC5			-38.46**	(0.02)	-38.54***	(<0.01)			-12.89**	(0.04)	-13.26**	(0.03)			-28.84**	(0.03)	-29.25**	(0.01)			-44.50**	(0.03)	-44.55***	(<0.01)			-67.24***	(0.00)	-66.74***	(0.00)
$M1$					-0.01	(0.28)					-0.01***	(<0.01)					-0.02**	(0.03)					-0.01	(0.39)					0.00	(0.86)
$M2$					-0.12***	(0.00)					-0.03*	(0.08)					-0.08**	(0.03)					-0.15***	(0.00)					-0.21***	(0.00)
$M3$					-0.12***	(<0.01)					-0.08***	(0.00)					-0.12***	(0.00)					-0.15***	(<0.01)					-0.15**	(0.03)
$M4$					-0.21***	(0.00)					-0.10***	(0.00)					-0.19***	(0.00)					-0.25***	(0.00)					-0.29***	(0.00)
$M5$					0.04	(0.32)					0.01	(0.45)					0.04	(0.36)					0.05	(0.33)					0.07	(0.28)
$M6$					0.00	(0.98)					-0.01	(0.78)					-0.02	(0.75)					0.00	(0.99)					0.03	(0.74)
$M7$					0.07	(0.25)					0.05**	(<0.05)					0.08	(0.11)					0.08	(0.27)					0.05	(0.56)
$M8$					0.03	(0.72)					-0.01	(0.83)					0.00	(0.94)					0.04	(0.69)					0.09	(0.50)
UT	0.27***	(0.00)	0.25***	(0.00)	0.25***	(0.00)	0.27***	(0.00)	0.26***	(0.00)	0.23***	(0.00)	0.28***	(0.00)	0.26***	(0.00)	0.24***	(0.00)	0.26***	(0.00)	0.24***	(0.00)	0.24***	(0.00)	0.26***	(0.00)	0.24***	(0.00)	25.02***	(0.00)
BC p-val.		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		[0.01]		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]		[0.00]
$\bar{R}^2$%	30.42		36.42		45.60		32.43		36.83		48.21		31.38		36.34		46.41		29.82		35.44		44.77		29.80		36.75		44.97	
$(\Delta\bar{R}^2)$	(21.57)		(18.86)		(17.57)		(23.13)		(20.10)		(15.15)		(23.03)		(20.15)		(16.91)		(21.03)		(18.48)		(17.33)		(20.34)		(17.72)		(18.25)	

Panel B: Commodity Down-tail Factor and Spanning Hypothesis Tests

	(1) Average						(2) 2-year						(3) 3-year						(4) 4-year						(5) 5-year					
PC1	0.03	(0.51)	0.031	(0.51)	0.043	(0.35)	0.03	(0.14)	0.03	(0.13)	0.03*	(0.05)	0.04	(0.33)	0.04	(0.33)	0.05	(0.18)	0.04	(0.54)	0.04	(0.55)	0.05	(0.38)	0.03	(0.73)	0.03	(0.74)	0.04	(0.59)
PC2	-0.35	(0.26)	-0.345	(0.19)	-0.360	(0.17)	-0.06	(0.62)	-0.06	(0.55)	-0.07	(0.49)	-0.21	(0.42)	-0.21	(0.33)	-0.22	(0.30)	-0.43	(0.27)	-0.42	(0.19)	-0.44	(0.17)	-0.70	(0.16)	-0.69	(0.10)	-0.71	(0.09)
PC3	-0.81	(0.60)	-0.821	(0.57)	-0.753	(0.57)	-0.08	(0.89)	-0.09	(0.88)	-0.05	(0.92)	-0.50	(0.70)	-0.51	(0.68)	-0.44	(0.69)	-1.23	(0.51)	-1.25	(0.49)	-1.17	(0.48)	-1.42	(0.56)	-1.43	(0.53)	-1.35	(0.53)
PC4			-10.67**	(0.03)	-10.798**	(0.01)			-5.32**	(0.02)	-5.27**	(0.01)			-9.81**	(0.03)	-9.81**	(0.01)			-12.71**	(0.04)	-12.89**	(0.02)			-14.89**	(0.04)	-15.20**	(0.02)
PC5			-53.01***	(0.00)	-52.037***	(0.00)			-18.18***	(0.00)	-17.85***	(0.00)			-40.53***	(0.00)	-39.72***	(0.00)			-62.47***	(0.00)	-61.13***	(0.00)			-91.04***	(0.00)	-89.69***	(0.00)
$M1$					-0.01	(0.42)					-0.01**	(0.04)					-0.02	(0.11)					-0.01	(0.49)					0.00	(0.85)
$M2$					-0.08	(0.18)					-0.02	(0.29)					-0.05	(0.28)					-0.10	(0.15)					-0.13	(0.17)
$M3$					-0.14**	(0.02)					-0.08***	(0.00)					-0.13**	(0.01)					-0.17**	(0.01)					-0.17*	(0.06)
$M4$					-0.14**	(0.04)					-0.07***	(<0.01)					-0.14**	(0.02)					-0.17*	(0.06)					-0.20*	(0.08)
$M5$					0.05	(0.40)					0.02	(0.35)					0.04	(0.35)					0.06	(0.42)					0.07	(0.42)
$M6$					0.01	(0.95)					0.00	(0.94)					0.00	(0.96)					0.00	(0.99)					0.02	(0.89)
$M7$					0.07	(0.26)					0.06**	(0.04)					0.09	(0.10)					0.08	(0.28)					0.06	(0.55)
$M8$					0.03	(0.82)					0.00	(0.97)					0.00	(0.98)					0.03	(0.80)					0.06	(0.72)
DT	0.21***	(0.00)	0.22***	(0.00)	0.18***	(0.00)	0.24***	(0.00)	0.25***	(0.00)	0.20***	(0.00)	0.22***	(0.00)	0.23***	(0.00)	0.19***	(0.00)	0.20***	(0.00)	0.21***	(0.00)	0.18***	(0.00)	0.19***	(0.00)	0.20***	(0.00)	0.17***	(0.00)
BC p-val.		[0.01]		[0.00]		[0.01]		[<0.01]		[0.00]		[0.01]		[<0.01]		[0.00]		[0.01]		[0.01]		[<0.01]		[0.02]		[<0.01]		[<0.01]		[0.02]
$\bar{R}^2$%	24.56		34.10		38.93		27.98		36.68		44.65		25.21		34.20		40.32		24.03		33.04		37.95		23.51		33.71		37.27	
$(\Delta\bar{R}^2)$	(15.71)		(16.53)		(10.90)		(18.68)		(19.95)		(11.60)		(16.87)		(18.01)		(10.82)		(15.24)		(16.09)		(10.51)		(14.05)		(14.68)		(10.55)	

Table 8: **Regressions of Bond Risk Premia on Commodity Tails Factors and Competitors**

This table displays the parameter estimates and the adjusted R^2 values of the following predictive regressions: (1) "Commodity tails and the CP factors": $rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \gamma CP_t + \varepsilon_{t+1}$; (2) "Commodity tails and the LN factors": $rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \gamma LN_t + \varepsilon_{t+1}$; and (3) "Commodity tails and the CPo factors": $rx_{t+1}^{(n)} = \alpha + \beta Tail_t + \gamma CPo_t + \varepsilon_{t+1}$, where $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, $Tail_t$ denotes the commodity up-tail factor (UT) or commodity down-tail factor (DT) based on the PLS method, CP_t, LN_t and CPo_t respectively denote the CP factor of Cochrane and Piazzesi (2005), the LN factor of Ludvigson and Ng (2009), the CPo factor of Cieslak and Povala (2015) and ε_{t+1} is an error term. In square brackets, we report the NW p-value at the left and the bias-corrected (BC) p-values at the right. We also report the gain in $\bar{R}^2$ relative to the model without the commodity tails factors. ***, **, and * represent significance level at 1%, 5%, and 10% based on the NW p-value. The sample period is January 1992 to June 2022.

Panel A: Regressions of Bond Risk Premia on Commodity Up-Tail Factor and Competitors

	(1) UT and CP factors			(2) UT and LN factors			(3) UT and CPo factors		
	UT [p-value]	CP (p-value)	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)	UT [p-value]	LN (p-value)	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)	UT [p-value]	CPo (p-value)	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)
Average	0.259***	0.530	30.81	0.259***	0.678***	33.50	0.262***	0.488	30.15
	[0.000,0.000]	(0.188)	(20.31)	[0.000,0.000]	(0.005)	(21.32)	[0.000,0.001]	(0.263)	(20.76)
2-year	0.270***	0.223	32.57	0.268***	0.289***	35.28	0.277***	0.189	31.63
	[0.000,0.000]	(0.180)	(22.42)	[0.000,0.000]	(0.005)	(22.95)	[0.000,0.000]	(0.306)	(23.75)
3-year	0.268***	0.455	32.06	0.264***	0.596***	35.02	0.274***	0.382	31.03
	[0.000,0.000]	(0.172)	(21.63)	[0.000,0.000]	(0.003)	(21.94)	[0.000,0.000]	(0.297)	(22.71)
4-year	0.251***	0.659	30.04	0.251***	0.829***	32.67	0.255***	0.610	29.37
	[0.000,0.001]	(0.182)	(19.56)	[0.000,0.000]	(0.005)	(20.42)	[0.000,0.001]	(0.252)	(20.03)
5-year	0.252***	0.782	29.42	0.254***	0.985**	31.74	0.253***	0.778	29.12
	[0.000,0.002]	(0.211)	(19.42)	[0.000,0.000]	(0.010)	(20.84)	[0.000,0.002]	(0.243)	(19.22)

Panel B: Regressions of Bond Risk Premia on Commodity Down-tail Factor and Competitors

	(1) DT and CP factors			(2) DT and LN factors			(3) DT and CPo factors		
	DT [p-value]	CP [p-value]	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)	DT [p-value]	LN [p-value]	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)	DT [p-value]	CPo [p-value]	$\bar{R}^2(\%)$ ($\Delta\bar{R}^2(\%)$)
Average	0.197***	0.556	24.87	0.205***	0.794***	29.60	0.201***	0.537	24.42
	[0.000,0.005]	(0.146)	(14.37)	[0.000,0.002]	(0.001)	(17.42)	[0.000,0.007]	(0.199)	(15.03)
2-year	0.242***	0.244*	29.22	0.245***	0.329***	33.11	0.248***	0.203	28.01
	[0.000,0.005]	(0.085)	(19.06)	[0.000,0.002]	(0.000)	(20.78)	[0.000,0.005]	(0.210)	(20.13)
3-year	0.212***	0.489	26.55	0.217***	0.690***	31.28	0.219***	0.419	25.47
	[0.000,0.004]	(0.113)	(16.11)	[0.000,0.002]	(0.000)	(18.20)	[0.000,0.005]	(0.222)	(17.15)
4-year	0.190***	0.680	24.15	0.199***	0.977***	29.06	0.193***	0.659	23.72
	[0.000,0.009]	(0.152)	(13.67)	[0.000,0.003]	(0.001)	(16.81)	[0.000,0.010]	(0.204)	(14.38)
5-year	0.175***	0.828	22.30	0.187***	1.190***	26.85	0.176***	0.888	22.41
	[0.000,0.012]	(0.179)	(12.29)	[0.000,0.002]	(0.002)	(15.94)	[0.000,0.013]	(0.181)	(12.52)

Table 9: **Out-of-sample Regressions Using Commodity Tails Factors**

The table displays out-of-sample statistics for the predictive regressions of bond risk premia over expanding windows. R^2_{00S} denotes the out-of-sample R^2 statistic relative to the historical average model. CW denotes the Clark and West (2007) out-of-sample statistic, and GW denotes the Giacomini and White (2006) out-of-sample MSPE-adjusted statistic. Out-of-sample tests are conducted using recursive estimation forecasts that are generated using all past observations. Forecasts begin $T/2+1$ observations after the start of the sample, where T is the total number of observations. ***, **, and * represent significance level at 1%, 5%, and 10%. All series end in June 2022.

Panel A: Out-of-sample Regression Using Commodity Up-tail Factor

	(1) Average	(2) 2-year	(3) 3-year	(4) 4-year	(5) 5-year
$R^2_{OOS}(\%)$	19.03	17.03	17.41	18.08	19.07
$(\bar{R}^2_{OOS}(\%))$	(18.57)	(16.55)	(16.94)	(17.61)	(18.61)
CW	3.583***	3.362***	3.383***	3.583***	3.779***
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
GW	10.130***	10.021***	9.151***	9.637***	10.702***
(p-value)	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)

Panel B: Out-of-sample Regression Using Commodity Down-tail Factor

	(1) Average	(2) 2-year	(3) 3-year	(4) 4-year	(5) 5-year
$R^2_{OOS}(\%)$	5.77	4.61	5.67	5.01	5.36
$(\bar{R}^2_{OOS}(\%))$	(5.23)	(4.07)	(5.13)	(4.47)	(4.82)
CW	1.392*	1.420*	1.375*	1.337*	1.439*
(p-value)	(0.082)	(0.078)	(0.085)	(0.091)	(0.075)
GW	3.866**	2.585	3.658*	3.506*	4.048**
(p-value)	(0.049)	(0.108)	(0.056)	(0.061)	(0.044)

Table 10: **Out-of-sample Regressions Using Commodity Tails Factors and Competitors**

The table displays out-of-sample statistics for the predictive regressions of bond risk premia over expanding windows. R^2_{00S} or $\bar{R}^2_{00S}$ respectively denotes the out-of-sample R^2 and the out-of-sample adjusted R^2 statistic relative to the benchmark models, and the benchmark models include the three PCs of the yield curve, the CP factor, the LN factor and the CPo factor. *CW* denotes the Clark and West (2007) out-of-sample statistic, and *GW* denotes the Giacomini and White (2006) out-of-sample MSPE-adjusted statistic. Out-of-sample tests are conducted using recursive estimation forecasts that are generated using all past observations. Forecasts begin $T/2+1$ observations after the start of the sample, where T is the total number of observations. ***, **, and * represent significance level at 1%, 5%, and 10%. All series end in June 2022.

Panel A: Out-of-sample Regression Using Commodity Up-tail Factor and Competitors

	(1) (UT + PCs) vs. PCs			(2) (UT + CP) vs. CP			(3) (UT + LN) vs. LN			(4) (UT + CPo) vs. CPo		
	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)
Average	11.85	1.463*	4.075**	18.42	1.936**	15.390***	16.22	2.792***	4.112**	18.80	2.723***	20.095***
	(11.34)	(0.072)	(0.044)	(17.95)	(0.026)	(0.000)	(15.74)	(0.003)	(0.043)	(18.33)	(0.003)	(0.000)
2-year	18.76	1.463*	3.702*	20.60	1.977**	17.948***	9.89	2.364***	1.167	18.62	2.611***	19.104***
	(18.29)	(0.072)	(0.054)	(20.15)	(0.024)	(0.000)	(9.37)	(0.009)	(0.280)	(18.15)	(0.005)	(0.000)
3-year	10.96	1.455*	3.866**	19.07	1.912**	15.512***	12.82	2.499***	2.057	17.81	2.587***	18.421***
	(12.34)	(0.073)	(0.049)	(18.61)	(0.028)	(0.000)	(12.32)	(0.006)	(0.152)	(17.34)	(0.005)	(0.000)
4-year	9.80	1.469*	4.189**	18.13	1.929**	15.418***	15.49	2.737***	3.973**	17.95	2.629***	18.505***
	(10.44)	(0.071)	(0.041)	(17.66)	(0.027)	(0.000)	(15.00)	(0.003)	(0.046)	(17.48)	(0.004)	(0.000)
5-year	11.85	1.517*	4.164**	16.68	2.002**	14.399***	17.92	3.098***	6.389**	18.61	2.610***	19.071***
	(9.28)	(0.065)	(0.041)	(16.21)	(0.023)	(0.000)	(17.45)	(0.001)	(0.011)	(18.15)	(0.005)	(0.000)

Panel B: Out-of-sample Regression Using Commodity Down-tail Factor and Competitors

	(1) (DT + PCs) vs. PCs			(2) (DT + CP) vs. CP			(3) (DT + LN) vs. LN			(4) (DT + CPo) vs. CPo		
	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)	$R^2_{OOS}(\%)$ ($\bar{R}^2_{OOS}(\%)$)	*CW* (*p*-value)	*GW* (*p*-value)
Average	18.14	2.271**	12.968***	19.57	2.524***	29.435***	8.14	1.751**	0.953	18.66	2.594***	18.901***
	(17.67)	(0.012)	(0.000)	(19.11)	(0.006)	(0.000)	(7.62)	(0.040)	(0.329)	(18.19)	(0.005)	(0.000)
2-year	27.32	2.204**	7.892***	21.45	2.659***	29.464***	5.38	1.682**	0.101	19.04	2.564***	21.100***
	(26.90)	(0.014)	(0.000)	(21.01)	(0.004)	(0.000)	(4.84)	(0.046)	(0.751)	(18.58)	(0.005)	(0.000)
3-year	20.98	2.288**	11.185***	20.29	2.546***	30.034***	8.31	1.790**	0.693	19.11	2.550***	21.620***
	(20.52)	(0.011)	(0.000)	(19.83)	(0.005)	(0.000)	(7.79)	(0.037)	(0.405)	(18.64)	(0.005)	(0.000)
4-year	16.88	2.268**	13.688***	18.76	2.529***	29.262***	7.83	1.750**	0.909	17.82	2.633***	21.123***
	(16.40)	(0.012)	(0.000)	(18.30)	(0.006)	(0.000)	(7.30)	(0.040)	(0.340)	(17.35)	(0.004)	(0.000)
5-year	13.95	2.098**	14.463***	17.73	2.483***	27.359***	7.45	1.750**	1.284	17.20	2.587***	21.681***
	(13.46)	(0.018)	(0.000)	(17.26)	(0.007)	(0.000)	(6.92)	(0.040)	(0.257)	(16.72)	(0.005)	(0.000)

Table 11: **Predictive Regressions of Inflation Based on the Commodity Tail Factors**

This table displays parameter estimates and adjusted R^2 values with commodity tail factors for regression: $Inflation^h_{t+1} = \alpha + \beta Tail_t + \varepsilon_{t+1}$, where $Inflation^h_{t+1}$ represents the inflation level based on the CPI index for various horizons h=3, 6, ..., 36 months, $Tail_t$ represents the commodity up-tail factor or down-tail factor extracted from the inflation in PLS method, and ε_{t+1} is an error term. Following the method of Stock and Watson (1999), we use $I^h_{t+h} - I_t$ to measure the inflation , where $I^h_{t+h} = (1200/h)\log(CPI_{t+h}/CPI_t)$ and $I_t = (1200)\log(CPI_t/CPI_{t-1})$. We also report the p-values for the Newey and West (1987) standard errors. ***, **, and * represent significance level at 1%, 5%, and 10%. All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on UT

Horizon	UT	p-value	$\bar{R}^2(\%)$
6	0.054**	0.026	5.10
12	0.073**	0.023	6.99
18	0.085***	0.009	8.17
24	0.089**	0.011	8.47
30	0.083**	0.015	8.00
36	0.072**	0.033	6.84

Panel B: Predictive Regressions Based on DT

Horizon	DT	p-value	$\bar{R}^2(\%)$
6	-0.047***	0.001	4.41
12	-0.056***	0.000	5.30
18	-0.065***	0.000	6.27
24	-0.080***	0.000	7.74
30	-0.047***	0.013	4.37
36	-0.047***	0.012	4.37

Table 12: **Predictive Regressions of Interest Rate Uncertainty Based on the Commodity Tail Factors**

This table displays parameter estimates and adjusted R^2 values with commodity tail factors for regression: $IRU^h_{t+1} = \alpha + \beta Tail_t + \varepsilon_{t+1}$, where IRU^h_{t+1} represents the interest uncertainty constructed based on the federal funds rate for various horizons h=1, 2, ..., 6 months, which is the central interest rate in the U.S. financial market, $Tail_t$ represents the commodity up-tail factor or down-tail factor extracted from IU in PLS method, and ε_{t+1} is an error term. Here, we use the volatility to measure interest uncertainty. p-values based on the Newey and West (1987) standard errors are reported, and ***, **, and * represent significance level at 1%, 5%, and 10%. All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on UT

Horizon	UT	p-value	$\bar{R}^2(\%)$
1	0.127***	(0.000)	12.50
2	0.104***	(0.000)	10.88
3	0.101***	(0.000)	10.91
4	0.119***	(0.000)	12.82
5	0.139***	(0.000)	14.84
6	0.153***	(0.000)	16.28

Panel B: Predictive Regressions Based on DT

Horizon	DT	p-value	$\bar{R}^2(\%)$
1	0.216***	(0.000)	21.20
2	0.236***	(0.000)	23.46
3	0.229***	(0.000)	22.95
4	0.239***	(0.000)	23.94
5	0.241***	(0.000)	24.03
6	0.244***	(0.000)	23.41

Table 13: **Predictive Regressions of Macro Variables Based on the Commodity Tail Factors**

This table displays parameter estimates and adjusted R^2 values with commodity tails factors for regression: $MacroVariable_{t+1} = \alpha + \omega Tail_t^{(avg)} + \epsilon_{t+1}$, where $MacroVariable_{t+1}$ represents the GDP growth (GDP Growth), the industrial production growth (IPG), the nonfarm payroll growth (Payroll Growth), and the unemployment rate (Unemp Rate) in time $t+1$, $Tail_t^{(avg)}$ represents the commodity up- or down-tail factor extracted from equal-weighted yearly bond risk premia in PLS method, and ϵ_{t+1} is an error term. We also report the p-values for the Newey and West (1987) standard errors in the brackets. ***, **, and * represent significance level at 1%, 5%, and 10%. All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on UT

	UT	p-value	$\bar{R}^2(\%)$
GDP Growth	-0.130**	0.003	4.59
IPG	-0.213***	(0.000)	5.07
Payroll Growth	-0.124***	(0.000)	6.00
Unemp Rate	1.293***	(0.000)	6.54

Panel B: Predictive Regressions Based on DT

	DT	p-value	$\bar{R}^2(\%)$
GDP Growth	-0.072*	(0.052)	1.23
IPG	-0.105**	(0.017)	1.36
Payroll Growth	-0.114***	(0.000)	6.46
Unemp Rate	1.452***	(0.000)	10.57

Table 14: **The Results of Predictions within Subsamples**

This table displays parameter estimates and adjusted R^2 values for predictive regression: $rx_{t+1}^{(Avg)} = \alpha + \beta Tail_t + \varepsilon_{t+i}$, where $rx_{t+1}^{(Avg)}$ is equal-weighted yearly bond risk premia, $Tail_t$ is the current commodity tail factors (UT or DT), and ε_{t+1} is an error term. Column (1) covers the period of 2000 to 2022, column (2) covers the period of 2005 to 2022, and column (3) covers the period of 2010 to 2022. In round brackets, we report the p-value based on the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). Simultaneously, we report the p-values based on the parametric bootstrap approach of Bauer and Hamilton (2018) in square brackets. ***, **, and * represent significance level at 1%, 5%, and 10%.

Panel A: Predictive Regressions Based on Commodity Up-tail Factor

	(1) 2000-2022	(2) 2005-2022	(3) 2010-2022
Constant	0.012***	0.008**	0.010***
	(0.000)	(0.012)	(0.005)
UT	0.420***	0.387***	0.343***
	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.000]	[0.000]
$\bar{R}^2$	42.62	37.21	33.82
$\bar{R}^2_{OOS}$	32.38	12.24	2.05
CW	3.638	3.174	1.670

Panel B: Predictive Regressions Based on Commodity Down-tail Factor

	(1) 2000-2022	(2) 2005-2022	(3) 2010-2022
Constant	0.022***	0.019***	0.017***
	(0.000)	(0.000)	(0.005)
DT	0.340***	0.335***	0.486***
	(0.000)	(0.000)	(0.000)
Bootstrap p-value	[0.000]	[0.000]	[0.000]
$\bar{R}^2$	33.78	32.92	48.25
$\bar{R}^2_{OOS}$	2.97	-0.91	2.93
CW	1.790	1.474	1.851

Table 15: **The Results of Predictions with the Different Forecasting Lags**

This table displays parameter estimates and adjusted R^2 values for predictive regression: $rx^{(n)}_{t+i} = \alpha + \beta Tail_t + \varepsilon_{t+i}$, where $rx^{(n)}_{t+i}$ is equal-weighted yearly or 2- to 5-year bond risk premia of the future i months, and i ranges from 3 to 12 here. $Tail_t$ is the current commodity tail factors (UT or DT). All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on Commodity Up-tail Factor

	Average			2-year			3-year			4-year			5-year		
Lag	UT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	UT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	UT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	UT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	UT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$
3	0.345***	33.55	16.64	0.358***	35.95	10.91	0.356***	35.14	13.83	0.336***	32.60	16.12	0.332***	31.85	18.46
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
6	0.304***	29.07	14.72	0.357***	35.02	8.90	0.327***	31.59	12.46	0.296***	28.28	13.87	0.275***	26.02	15.46
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
9	0.236***	21.63	6.05	0.275***	25.77	1.61	0.256***	23.70	5.87	0.230***	21.18	6.09	0.214***	19.56	7.83
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
12	0.198***	17.93	2.34	0.202***	18.42	6.30	0.211***	19.07	5.75	0.196***	17.83	2.61	0.190***	17.28	2.31
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		

Panel B: Predictive Regressions Based on Commodity Down-tail Factor

	Average			2-year			3-year			4-year			5-year		
Lag	DT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	DT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	DT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	DT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$	DT	$\bar{R}^2(\%)$	$\bar{R}^2_{OOS}(\%)$
3	0.223***	22.61	-6.78	0.269***	27.08	-7.45	0.241***	24.46	-7.84	0.214***	21.78	-6.80	0.195***	19.50	-4.98
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
6	0.184***	17.92	-4.48	0.210***	21.05	-10.06	0.201***	19.82	-7.63	0.179***	17.34	-4.66	0.166***	15.67	-0.30
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
9	0.216***	20.18	-1.02	0.242***	23.06	-4.55	0.234***	21.80	-0.53	0.212***	19.74	-0.63	0.196***	18.37	-1.64
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		
12	0.214***	21.55	-5.83	0.231***	23.54	-8.61	0.227***	22.98	-6.40	0.210***	21.06	-5.06	0.204***	20.28	-2.65
	(0.000)			(0.000)			(0.000)			(0.000)			(0.000)		

Table 16: **Regressions of Treasury Bond Risk Premia on Lagged Commodity Return Factor or Commodity Volatility Factor**

This table displays the parameter estimates and the adjusted R^2 values of the following predictive regression: Pane A: "Commodity returns factor": $rx_{t+1}^{(n)} = \alpha + \beta Return_t + \epsilon_{t+1}$; and Panel B: "Commodity volatility factor": $rx_{t+1}^{(n)} = \alpha + \beta Volatility_t + \epsilon_{t+1}$, where $rx_{t+1}^{(n)}$ is the equal-weighted yearly and two- to five-year Treasury bond risk premia, $Return_t$ and $Volatility_t$ represent the commodity returns factor and the commodity volatility factor, which are based on 24 commodity futures daily returns through the PLS method. Each commodity future return is defined as the average value of the past 6-month daily returns, and each commodity future volatility is calculated by $V_{i,t}^{M} = \sqrt[2]{\frac{\sum_{d=1}^{D_t}(R_{i,d}^{D} - \bar{R}_{i,t})^2}{n}}$, where $V_{i,t}^{M}$ is the past 6-month daily return volatility of commodity future i in the past 6-month, $R_{i,d}^{D}$ is daily return of i in day d and $\bar{R}_{i,t}$ is the average daily return of i in the past 6-month. We respectively report the coefficients, the Newey-West p-values (in parentheses), the adjusted in-sample R^2 ($\bar{R}^2$) and out-of-sample R^2 (R_{OOS}^2). ***, **, and * represent significance level at 1%, 5%, and 10%. All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on Commodity Return Factor

	Average	2-year	3-year	4-year	5-year
Constant	0.015***	0.006***	0.013***	0.019***	0.023***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Return	0.069 **	0.109***	0.101***	0.070**	0.064***
	(0.012)	(0.002)	(0.007)	(0.010)	(0.007)
$\bar{R}^2(\%)$	6.47	10.38	9.63	6.56	5.96
$\bar{R}_{OOS}^2(\%)$	-2.61	-0.26	-3.23	-2.43	-0.23

Panel B: Predictive Regressions Based on Commodity Volatility Factor

	Average	2-year	3-year	4-year	5-year
Constant	0.015***	0.005***	0.012***	0.018***	0.023***
	(0.001)	(0.006)	(0.001)	(0.000)	(0.000)
Volatility	0.038**	0.050	0.039*	0.039**	0.041***
	(0.020)	(0.106)	(0.059)	(0.020)	(0.006)
$\bar{R}^2(\%)$	3.10	4.70	3.35	3.10	3.22
$\bar{R}_{OOS}^2(\%)$	-70.71	-109.21	-92.39	-65.09	-51.96

Table 17: **Regressions of Treasury Bond Risk Premia on Lagged Stock Tail Risk or Treasury Bond Tail Risk**

This table displays the parameter estimates and the adjusted R^2 values of the following predictive regression: Pane A: "Stock tail risk factor": $rx_{t+1}^{(n)} = \alpha + \beta ST_t + \epsilon_{t+1}$; and Panel B: "Treasury bond tail risk factor": $rx_{t+1}^{(n)} = \alpha + \beta BT_t + \epsilon_{t+1}$, where $rx_{t+1}^{(n)}$ is the equal-weighted yearly and two- to five-year Treasury bond risk premia, ST_t and BT_t represent the stock tail risk factor and the treasury bond tail risk factor. The stock tail risk factor can be directly estimated from the cross-section of stock returns utilizing daily CRSP data for NYSE/AMEX/NASDAQ stocks with share codes 10 and 11, as described by Kelly and Jiang (2014). Additionally, the tail risk associated with each Treasury bond yield is computed as outlined in equation 3, and then we employ the PLS method to amalgamate the tail risk of 1- to 30-year Treasury bond yields to construct the treasury bond tail risk factor. We respectively report the coefficients, the Newey-West p-values (in parentheses), the adjusted in-sample R^2 ($\bar{R}^2$) and out-of-sample R^2 (R^2_{OOS}). ***, **, and * represent significance level at 1%, 5%, and 10%. All series start in January 1992 and end in June 2022.

Panel A: Predictive Regressions Based on Stock Tail Risk Factor

	Average	2-year	3-year	4-year	5-year
Constant	0.055	0.024	0.049	0.061	0.086
	(0.361)	(0.433)	(0.381)	(0.397)	(0.319)
ST	-0.100	-0.045	-0.091	-0.107	-0.158
	(0.472)	(0.524)	(0.481)	(0.523)	(0.430)
$\bar{R}^2(\%)$	0.51	0.58	0.61	0.33	0.55
$\bar{R}^2_{OOS}(\%)$	0.11	-1.30	-0.74	0.02	0.71

Panel B: Predictive Regressions Based on Treasury Bond Tail Risk Factor

	Average	2-year	3-year	4-year	5-year
Constant	0.016***	0.007***	0.014***	0.020***	0.024***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
BT	0.055*	0.062**	0.064**	0.070**	0.066**
	(0.059)	(0.028)	(0.022)	(0.015)	(0.021)
$\bar{R}^2(\%)$	6.36	5.21	5.94	6.17	6.73
$\bar{R}^2_{OOS}(\%)$	6.55	8.29	7.07	6.05	5.53

Table 18: **Predictive Regression with Commodity Tails across Different Markets**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $rx_{t+1}^{(5)} = \alpha + \beta Tail_t + \varepsilon_{t+1}$, where $rx_{t+1}^{(5)}$ denotes the five-year Treasury bond risk premia, $Tail_t$ denotes the commodity up-tail factor (UT) or commodity down-tail factor (DT) based on the PLS method and ε_{t+1} is an error term. We respectively report the coefficients, p-values, the adjusted in-sample R^2 ($\bar{R}^2$) and out-of-sample R^2 (R^2_{OOS}). ***, **, and * represent significance level at 1%, 5%, and 10% according to the NW p-value. All series start in January 1996 and end in June 2022.

Panel A: Predictive Regressions Based on Commodity Up-tail Factor

	Constant	p-value	UT	p-value	$\bar{R}^2$	R^2_{OOS}
Canada	0.015***	(0.000)	0.312***	(0.000)	32.33	28.59
France	0.017***	(0.000)	0.204***	(0.000)	20.07	8.52
Germany	0.016***	(0.000)	0.221***	(0.000)	21.38	15.03
Italy	0.033***	(0.000)	0.292***	(0.000)	29.47	19.49
Japan	0.014***	(0.000)	0.299***	(0.000)	28.55	11.51
U.K.	0.013***	(0.001)	0.180***	(0.000)	16.15	11.54

Panel B: Predictive Regressions Based on Commodity Down-tail Factor

	Constant	p-value	DT	p-value	$\bar{R}^2(\%)$	$R^2_{OOS}(\%)$
Canada	0.022***	(0.000)	0.326***	(0.000)	32.03	23.04
France	0.018***	(0.000)	0.220***	(0.000)	21.45	-0.95
Germany	0.019***	(0.000)	0.218***	(0.000)	21.32	0.36
Italy	0.025***	(0.000)	0.278***	(0.000)	27.31	15.81
Japan	0.012***	(0.000)	0.395***	(0.000)	38.47	2.21
U.K.	0.015***	(0.002)	0.152***	(0.000)	14.06	10.30

Table 19: **Regressions of S&P 500 Index Excess Returns on Commodity Tail Factors**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $R^{(n)}_{t+1} = \alpha + \beta Tail_t + \varepsilon_{t+1}$, where $R^{(n)}_{t+1}$ denotes 1-, 3-, 6-, and 12-month S&P 500 index excess returns at time $t+1$, $Tail_t$ denotes the commodity up-tail factor (UT) or commodity down-tail factor (DT) based on the PLS method and ε_{t+1} is an error term. We respectively report the coefficients, p-values, the adjusted in-sample R^2 ($\bar{R}^2$) and out-of-sample R^2 (R^2_{OOS}). ***, **, and * represent significance level at 1%, 5%, and 10% according to the NW p-value. All series start in January 1992 and end in June 2022.

Panel A: Regressions on Commodity Up-tail Factor

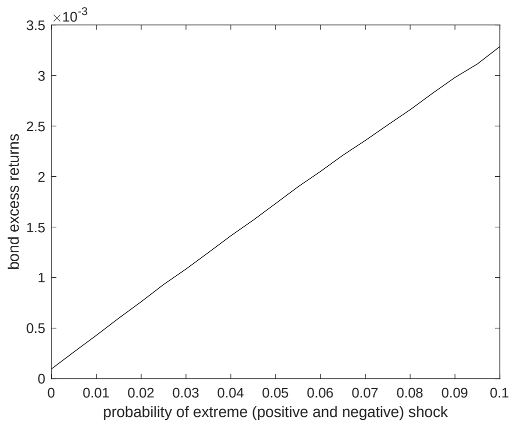
	(1) $R^{(1)}$	(2) $R^{(3)}$	(3) $R^{(6)}$	(4) $R^{(12)}$
Constant	0.006***	0.018***	0.039***	0.074***
	(0.002)	(0.000)	(0.000)	(0.000)
UT	0.056***	0.113***	0.182***	0.263***
	(0.000)	(0.000)	(0.000)	(0.000)
$\bar{R}^2$(%)	5.41	11.36	17.96	25.63
R^2_{OOS}(%)	-3.99	0.79	0.79	23.20

Panel B: Regressions on Commodity Down-tail Factor

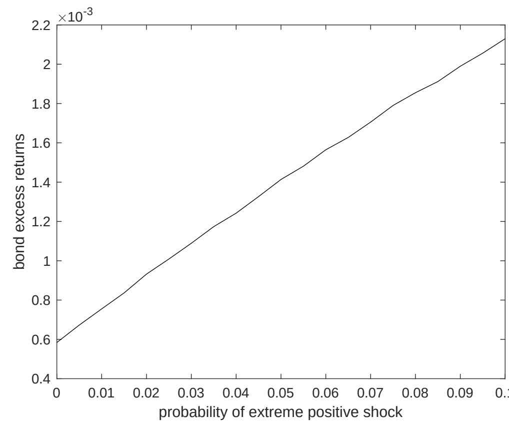
	(1) $R^{(1)}$	(2) $R^{(3)}$	(3) $R^{(6)}$	(4) $R^{(12)}$
Constant	0.002	0.008	0.028	0.063
	(0.369)	(0.255)	(0.018)	(0.005)
DT	0.032	0.084	0.164	0.181
	(0.000)	(0.000)	(0.000)	(0.000)
$\bar{R}^2$(%)	2.94	8.14	16.28	18.05
R^2_{OOS}(%)	-3.36	-6.14	-6.14	-2.65

Figure 2: **Impact of Tail Risk on Bond Risk Premia.**

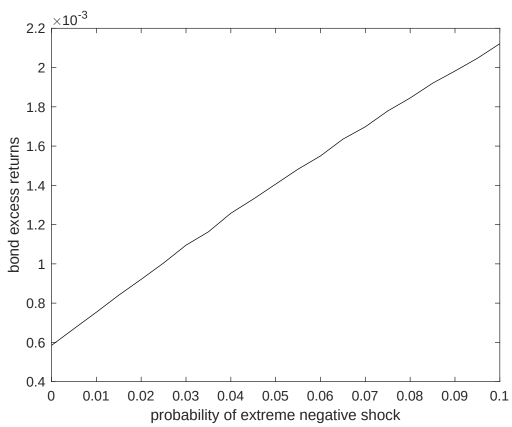
The figures plot the impact of tail risk on bond premia. We measure tail risk through the probability of an extreme uptail event π_u and/or an extreme downtail event π_d. Subfigure a) plots the impact an increase in the risk of both tails together. An increase in tail risk leads to an increase in bond excess returns. Subfigure b) plots the impact of tail risk in the upper tail, measured through π_u, and Subfigure c) considers the downtail risk, measured throguh π_d. Subfigure d) shows the impact of an increase in inflation volatility. For small or medium levels of volatility increases, the impact of volatility is smaller than the impact of tail risk. Only when the volatility increases to a level at which the volatility itself generates extreme realisations, volatility matters.



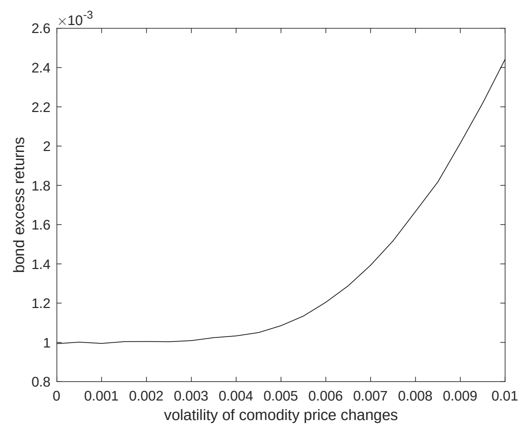
(a) Uptail and downtail risk



(b) Uptail risk



(c) Downtail risk



(d) Volatility

Figure 3: **Time Series of Treasury Bond Risk Premia**

This figure displays monthly bond risk premia (excess bond return) with equal-weighted yearly, two-, three-, four-, five-year maturities, and the shaded bars indicate economic recessions. Recession dates are from NBER website (https://www.nber.org/). All series end in June 2022.

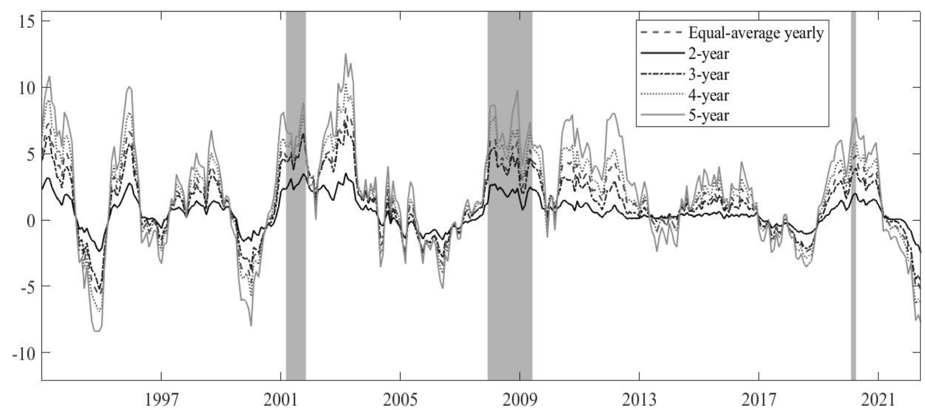
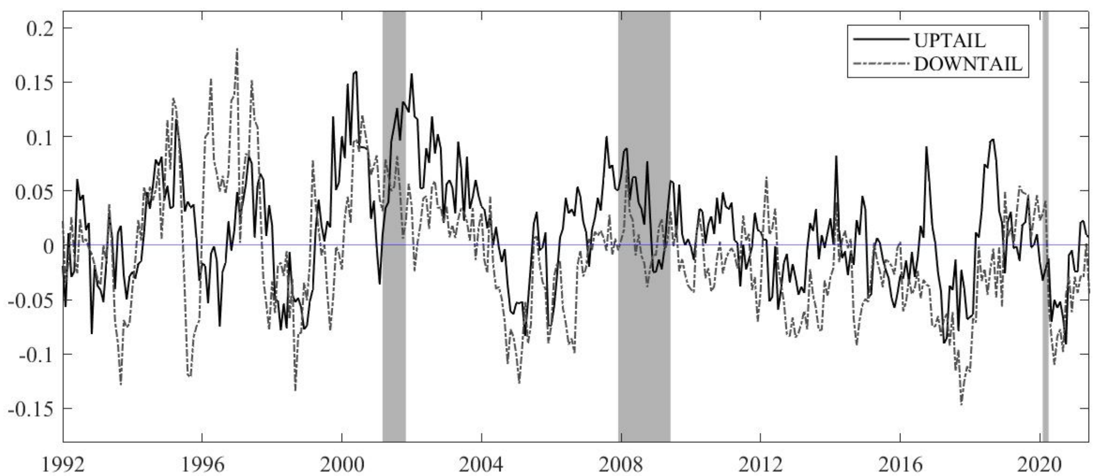


Figure 4: **Time Series and Correlations of Constructed Commodity Tail Factors**

This figure displays time series and correlations of commodity up-tail factor (UT) and commodity down-tail factor (DT) constructed by the partial least square (PLS) method based on equal-weighted yearly bond risk premia, and the shaded bars indicate economic recessions. Recession dates are from NBER website. In Panel A, we plot time series of commodity tail factors, and rolling $UT - LT$ correlations using 3-year window are shown in Panel B. All series end in June 2022.

Panel A: Time Series of Constructed Commodity Tail Factors



Panel B: Correlations of Constructed Commodity Tail Factors

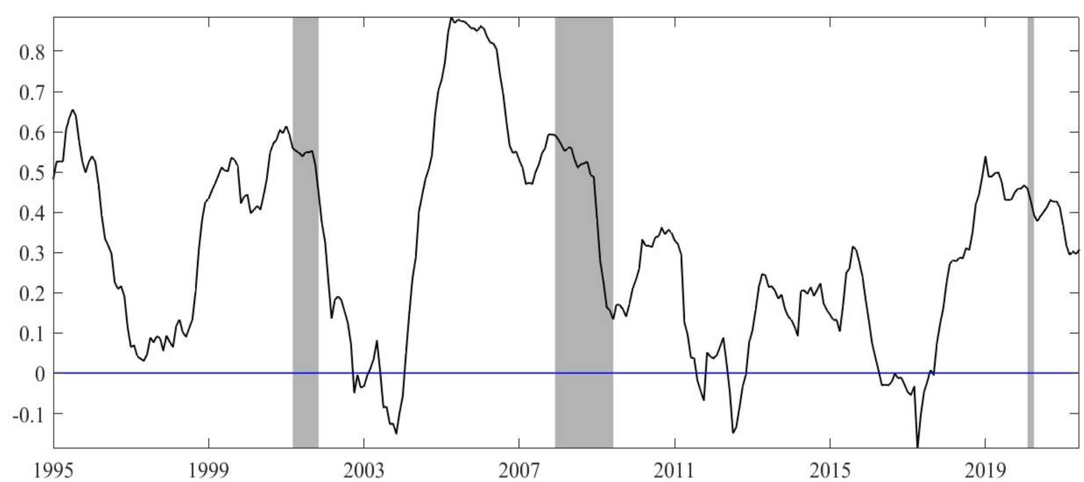
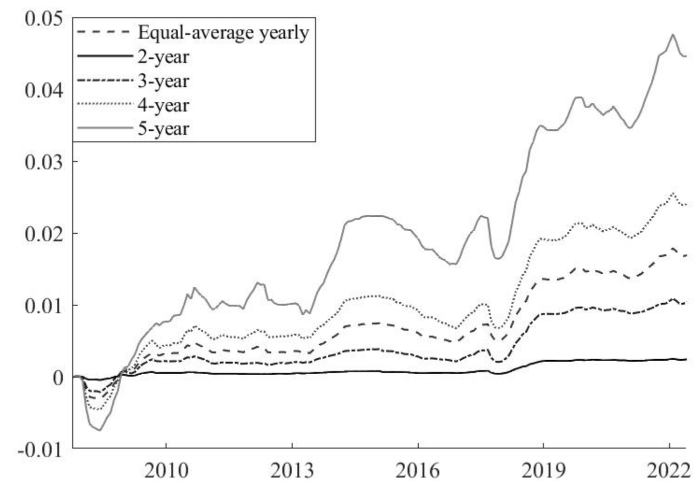


Figure 5: **Difference in Cumulative Squared Error for Commodity Tails Factors**

The figures plot the difference in cumulative squared predictive error between the predictive model using the commodity up-tail factor (UT) or commodity down-tail factor (DT) as the predictor and the benchmark model using historical average value. The out-of-sample period is from October 2007 to June 2022.

Panel A: DCSPE for Commodity Up-tail Factor



Panel B: DCSPE for Commodity Down-tail Factor

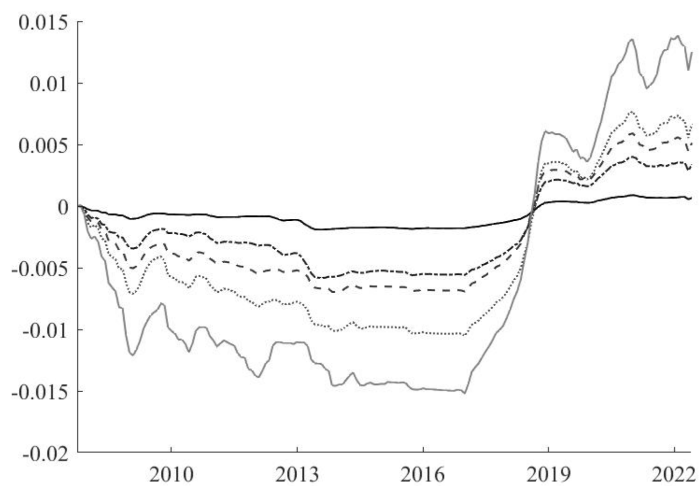
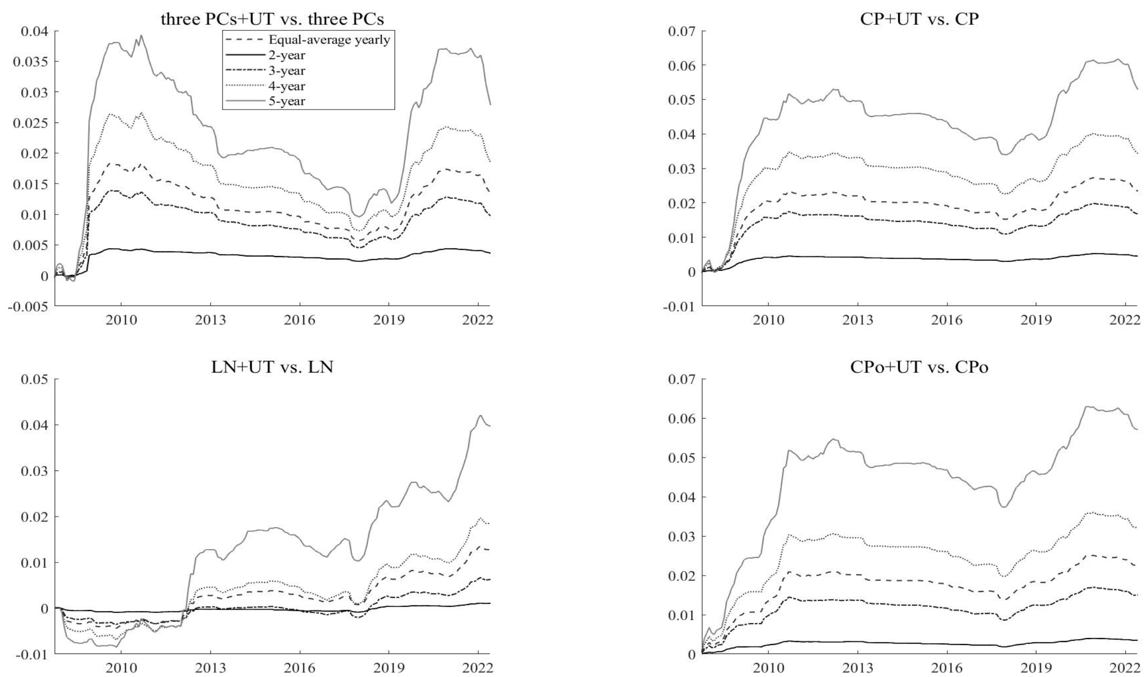


Figure 6: **Difference in Cumulative Squared Error with and without Commodity Tail Factors**

The figures plot the difference in cumulative squared predictive error with or without using the commodity up-tail factor (UT) or commodity down-tail factor (DT) as one predictor. The three PCs, CP, LN, CPo are the first three PCs of current yields, the Cochrane and Piazzesi (2005) factor, the Ludvigson and Ng (2009) factor, and the Cieslak and Povala (2015) factor, respectively. The out-of-sample period is from October 2007 to June 2022.

Panel A: DCSPE for Commodity Up-tail Factor



Panel B: DCSPE for Commodity Down-tail Factor

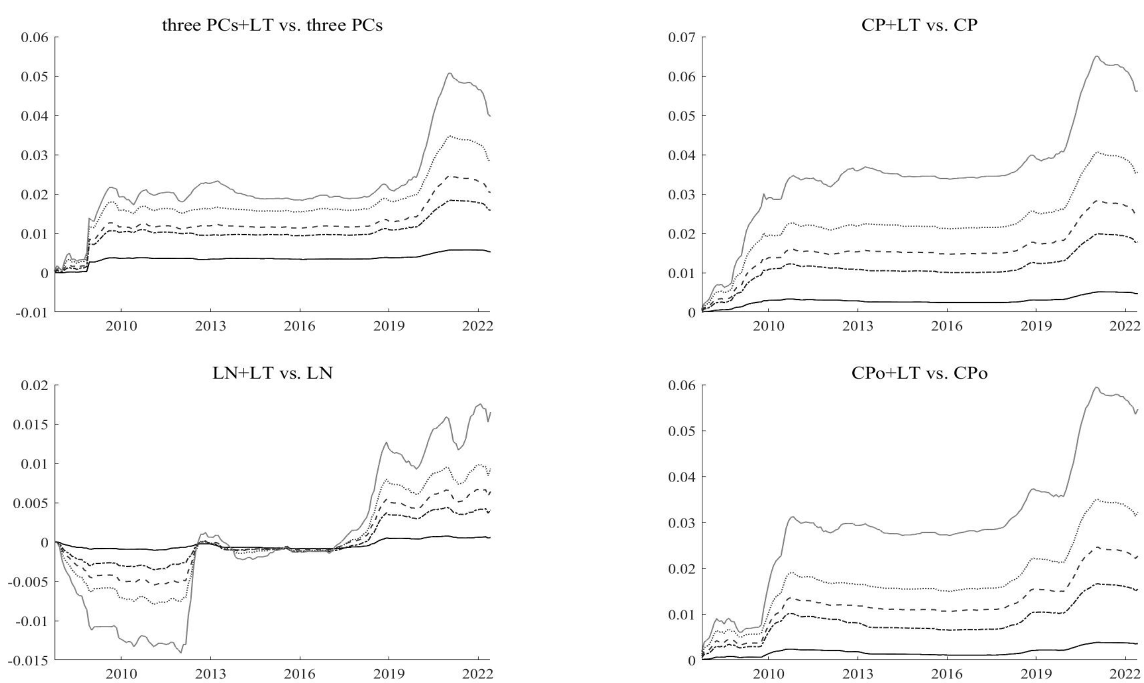


Figure 7: **Commodity Importance for Bond Risk Premia Prediction**

The figures present the importance of the up-tail and down-tail risk of 24 commodity futures returns for equal-weighted yearly bond risk premia prediction. At every window of the out-of-sample PLS regression, we can obtain a group of covariance between individual commodity up- or down-tail risk and bond excess returns, which represents the weight of given commodity tail. Here, we define the importance as the average value of the weights. In the left (right) figure, we show the importance of 24 commodity up (down) tails, and the out-of-sample period is from October 2007 to June 2022.

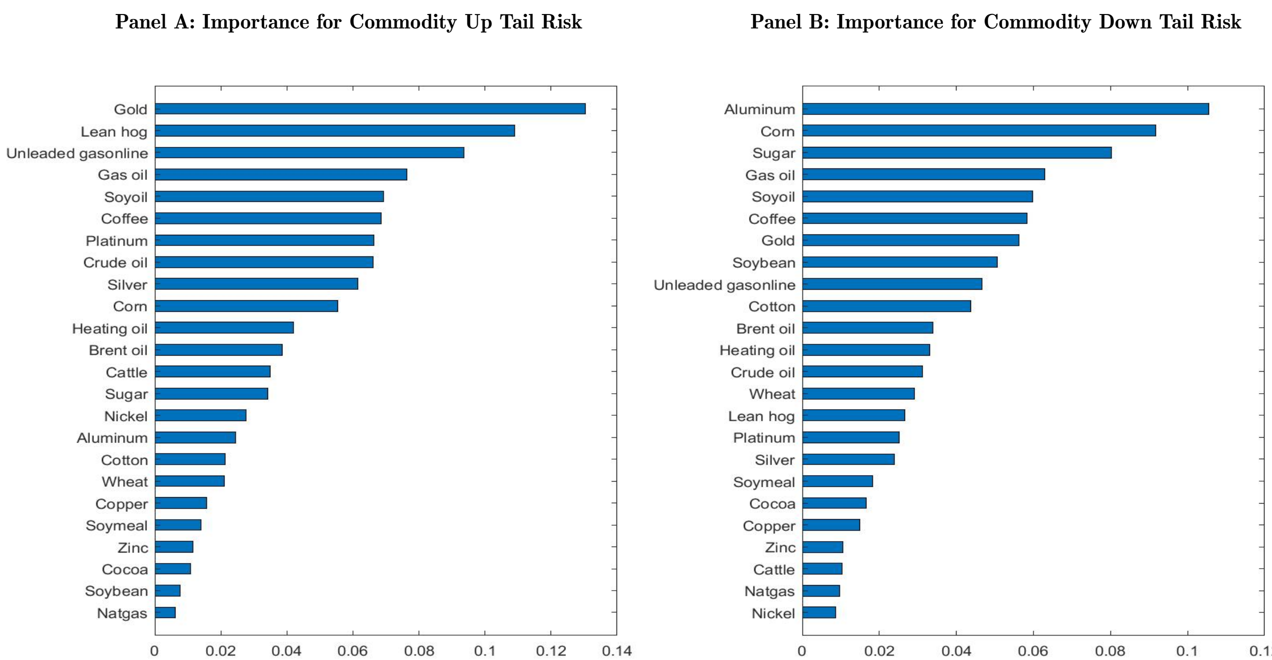
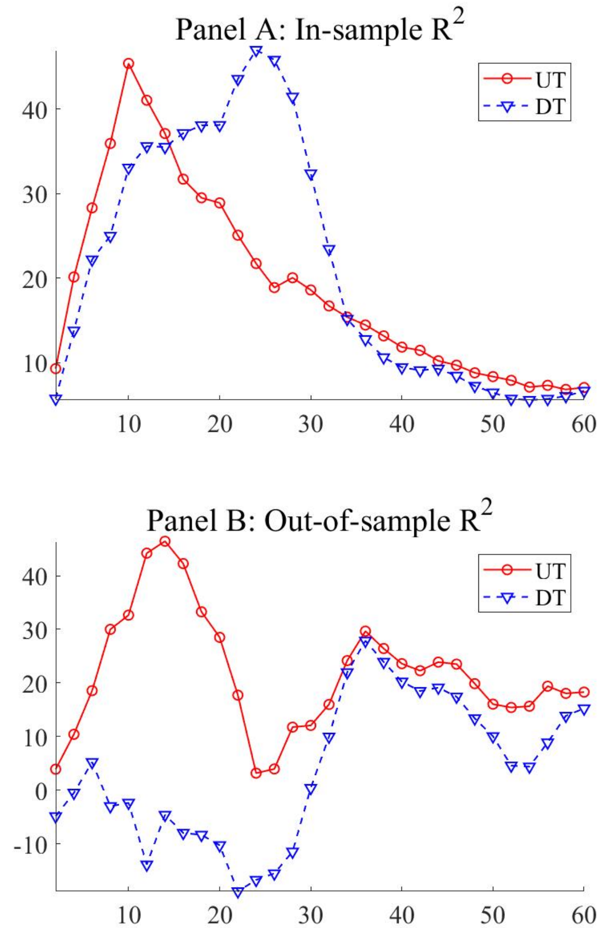


Figure 8: **In-sample and Out-of-sample R^2 for Different Moving Windows**

The figures display the explanatory power of the commodity tail factors (UT and DT) when it is constructed using different moving window lengths. It reports the in-sample and out-of-sample adjusted R^2 of the regression equation $rx_{t+1}^{(Avg)} = \alpha + \beta Tail_t^{(L)} + \epsilon_{t+1}$, where $rx_{t+1}^{(Avg)}$ is the monthly equal-weighted yearly bond risk premia and $Tail_t^{(L)}$ is the commodity tail factors constructed using window length L. The window size ranges between 2 months and 60 months, and the step size is 2 months. All series end in June 2022.



Appendices

A Additional Results

Table A1: **Regressions of Bond Risk Premia on the Current Yield Curve and Macroeconomic factors**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $rx_{t+1}^{(n)} = \alpha + \beta' X_t + \varepsilon_{t+1}$. $rx_{t+1}^{(n)}$ denotes the equally-weighted yearly and 2- to 5-year Treasury bond risk premia, PC1-PC5 are the five principal components of Gürkaynak, Sack, and Wright (2007) yields, and $M1$-$M8$ are eight macroeconomic factors of Moench and Soofi-Siavash (2022), which are estimated by regressing 135 FRED-MD series on the PC1-PC5 and then extracting principal components from the residuals. In round brackets, we report the p-value based on the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). ***, **, and * represent significance level at 1%, 5%, and 10% based on the Newey and West (1987) p-value. The sample period is January 1992 to June 2022.

	(1) Average					
PC1	0.11**	(0.03)	0.11**	(0.03)	0.11**	(0.01)
PC2	-0.44	(0.21)	-0.44	(0.16)	-0.44	(0.14)
PC3	-0.38	(0.82)	-0.38	(0.81)	-0.38	(0.79)
PC4			−11.51*	(0.06)	−11.51**	(0.02)
PC5			−46.71***	(0.00)	−46.71***	(<0.01)
$M1$					−0.03*	(0.09)
$M2$					-0.07	(0.27)
$M3$					−0.19***	(<0.01)
$M4$					−0.20***	(0.00)
$M5$					0.08	(0.12)
$M6$					0.01	(0.88)
$M7$					0.11	(0.15)
$M8$					-0.01	(0.94)
$\bar{R}^2$	8.85		17.56		28.03	

	(2) 2-year					
PC1	0.06***	(<0.01)	0.06***	(<0.01)	0.06***	(0.00)
PC2	-0.09	(0.53)	-0.09	(0.50)	-0.09	(0.44)
PC3	0.09	(0.89)	0.09	(0.89)	0.09	(0.88)
PC4			-5.07	(0.11)	−5.07**	(0.04)
PC5			−16.48***	(<0.01)	−16.48**	(0.03)
$M1$					−0.02***	(0.00)
$M2$					-0.01	(0.62)
$M3$					−0.10***	(0.00)
$M4$					−0.10***	(0.00)
$M5$					0.03	(0.17)
$M6$					-0.01	(0.89)
$M7$					0.07**	(0.03)
$M8$					-0.02	(0.67)
$\bar{R}^2$	9.30		16.73		33.05	

	(3) 3-year					
PC1	0.10**	(0.02)	0.10**	(0.02)	0.10***	(<0.01)
PC2	-0.28	(0.35)	-0.28	(0.31)	-0.28	(0.27)
PC3	-0.13	(0.93)	-0.13	(0.92)	-0.13	(0.91)
PC4			−9.77*	(0.09)	−9.77**	(0.03)
PC5			−36.16***	(<0.01)	−36.16**	(0.02)
$M1$					−0.04**	(0.01)
$M2$					-0.04	(0.50)
$M3$					−0.18***	(0.00)
$M4$					−0.19***	(0.00)
$M5$					0.06	(0.13)
$M6$					-0.01	(0.90)
$M7$					0.12*	(0.07)
$M8$					-0.04	(0.73)
$\bar{R}^2$	8.34		16.19		29.50	

	(4) 4-year					
PC1	0.13**	(0.04)	0.13**	(0.03)	0.13**	(0.02)
PC2	-0.55	(0.20)	-0.55	(0.16)	-0.55	(0.13)
PC3	-0.73	(0.71)	-0.73	(0.71)	-0.73	(0.68)
PC4			−13.86*	(0.06)	−13.86**	(0.02)
PC5			−53.95***	(<0.01)	−53.95**	(0.01)
$M1$					-0.04	(0.12)
$M2$					-0.10	(0.22)
$M3$					−0.23***	(<0.01)
$M4$					−0.24***	(<0.01)
$M5$					0.09	(0.13)
$M6$					0.01	(0.89)
$M7$					0.13	(0.16)
$M8$					0.00	(0.98)
$\bar{R}^2$	8.79		16.96		27.45	

	(5) 5-year					
PC1	0.14*	(0.06)	0.14*	(0.06)	0.14**	(0.03)
PC2	-0.86	(0.11)	−0.86*	(0.08)	−0.86*	(0.07)
PC3	-0.74	(0.77)	-0.74	(0.77)	-0.74	(0.74)
PC4			−17.36**	(0.04)	−17.36**	(0.01)
PC5			−80.26***	(0.00)	−80.26***	(0.00)
$M1$					-0.03	(0.30)
$M2$					-0.14	(0.18)
$M3$					−0.24**	(0.02)
$M4$					−0.27**	(0.01)
$M5$					0.12	(0.11)
$M6$					0.05	(0.68)
$M7$					0.13	(0.30)
$M8$					0.02	(0.90)
$\bar{R}^2$	9.46		19.03		26.72	

Table 2: **Regressions of Bond Risk Premia on the Existing Predictors Alone**

This table displays the parameter estimates and the adjusted R^2 values of the predictive regression: $rx_{t+1}^{(n)} = \alpha + \gamma F_t + \varepsilon_{t+1}$, where $rx_{t+1}^{(n)}$ denotes 2- to 5-year Treasury bond risk premia, F_t denotes the CP factor, the LN factor, and the CPo factor and ε_{t+1} is an error term. In round brackets, we report the p-value based on the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). ***, **, and * represent significance level at 1%, 5%, and 10% based on the Newey and West (1987) p-value. The sample period is January 1992 to June 2022.

	CP	$\bar{R}^2$	LN	$\bar{R}^2$	CPo	$\bar{R}^2$
2-year	0.422**	10.16	0.432***	12.33	0.394**	7.88
	(0.012)		(0.001)		(0.029)	
3-year	0.850**	10.44	0.883***	13.08	0.804**	8.32
	(0.012)		(0.000)		(0.026)	
4-year	1.215**	10.48	1.219***	12.25	1.213**	9.34
	(0.010)		(0.001)		(0.017)	
5-year	1.513***	10.00	1.467***	10.90	1.589**	9.90
	(0.009)		(0.001)		(0.010)	

B Derivation of $E_t[\beta_{t+1}]$ and $Var_t(\beta_{t+1})$

Thus, the expectation of the next period $(t+1)$ discount factor is

$$
\begin{aligned}
&E_t[\beta_{t+1} - m \cdot 0.98 - (1-m) \cdot \beta_t] \\
&\quad = E\left[\left(-\left(i_{t+1} - 2\%\right) \cdot \gamma + \epsilon_{t+1}\right) \cdot \mathbb{1}_{i_{t+1} < 2\% - \alpha}\right] + E\left[\left(-\left(i_{t+1} - 2\%\right) \cdot \gamma + \epsilon_{t+1}\right) \cdot \mathbb{1}_{i_{t+1} > 2\% + \alpha}\right] \\
&\qquad = E_t\left[-\left(i_{t+1} - 2\%\right) \cdot \gamma \cdot \mathbb{1}_{i_{t+1} < 2\% - \alpha}\right] + E\left[-\left(i_{t+1} - 2\%\right) \cdot \gamma \cdot \mathbb{1}_{i_{t+1} > 2\% + \alpha}\right] \\
&\qquad\quad = \sum_{i \in \{N,U,D,B\}} \pi_i \cdot T\left(\mu_i, \sigma_i\right)
\end{aligned}
$$

The variance of the next period $(t+1)$ discount factor is

$$
\begin{aligned}
Var(\beta_{t+1}) &= Var(\beta_{t+1} - m \cdot 0.98 - (1-m) \cdot \beta_t) \\
&= E[(\beta_{t+1} - m \cdot 0.98 - (1-m) \cdot \beta_t)^2] - E[\beta_{t+1} - m \cdot 0.98 - (1-m) \cdot \beta_t]^2
\end{aligned}
$$

The second term is already determined in the above equations. So, we are left with the first expectation term.

$$
\begin{aligned}
&E[(\beta_{t+1} - m \cdot 0.98 - (1-m) \cdot \beta_t)^2] \\
&\quad = E\left[\left(-\left(i_{t+1} - 2\%\right) \cdot \gamma + \epsilon_{t+1}\right)^2 \cdot \mathbb{1}_{i_{t+1} < 2\% - \alpha}\right] + E\left[\left(-\left(i_{t+1} - 2\%\right) \cdot \gamma + \epsilon_{t+1}\right)^2 \cdot \mathbb{1}_{i_{t+1} > 2\% + \alpha}\right] \\
&\qquad = E\left[\left(i_{t+1} - 2\%\right)^2 \cdot \gamma^2 \cdot \mathbb{1}_{i_{t+1} < 2\% - \alpha}\right] + E\left[\left(i_{t+1} - 2\%\right)^2 \cdot \gamma^2 \cdot \mathbb{1}_{i_{t+1} > 2\% + \alpha}\right] \\
&\qquad\qquad + \sigma_\epsilon^2 \cdot P(|i_{t+1} - 2\%| > \alpha \\
&\qquad\qquad = \sum_{i \in \{N,U,D,B\}} \pi_i \cdot S\left(\mu_i, \sigma_i\right) + \sigma_\epsilon^2 \left(1 - P(|i_{t+1} - 2\%| < \alpha)\right)
\end{aligned}
$$

Derivation of $T(\mu, \sigma)$

For downward deviations of the inflation rate, we get the following integral:

$$
\begin{aligned}
&-\gamma \int_{-\infty}^{2\% - \alpha} (I - 2\%) \cdot \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{I - \mu}{\sigma}\right)^2\right) dI \\
&\qquad = -\gamma \int_{-\infty}^{\frac{2\% - \alpha - \mu}{\sigma}} (\tilde{I} \cdot \sigma + \mu - 2\%) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tilde{I}^2\right) d\tilde{I} \\
&\qquad = \gamma\sigma \left[\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tilde{I}^2\right)\right]_{-\infty}^{\frac{2\% - \alpha - \mu}{\sigma}} - \gamma\left(\mu - 2\%\right)\Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)
\end{aligned}
$$

$$
= \gamma\sigma \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% - \alpha - \mu}{\sigma}\right)^2\right) - \gamma\left(\mu - 2\%\right)\Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)
$$

For upward deviations of the inflation rate, we obtain:

$$-\gamma\int_{2\%+\alpha}^{\infty}(I-2\%)\cdot\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{I-\mu}{\sigma}\right)^{2}\right)dI$$

$$=-\gamma\sigma\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{2\%+\alpha-\mu}{\sigma}\right)^{2}\right)-\gamma\left(\mu-2\%\right)\left(1-\Phi\left(\frac{2\%+\alpha-\mu}{\sigma}\right)\right)$$

Thus,

$$T(\mu,\sigma)=-\gamma\sigma\frac{1}{\sqrt{2\pi}}\left(\exp\left(-\frac{1}{2}\left(\frac{2\%+\alpha-\mu}{\sigma}\right)^{2}\right)-\exp\left(-\frac{1}{2}\left(\frac{2\%-\alpha-\mu}{\sigma}\right)^{2}\right)\right)$$
$$-\gamma\left(\mu-2\%\right)\left(1-\Phi\left(\frac{2\%+\alpha-\mu}{\sigma}\right)+\Phi\left(\frac{2\%-\alpha-\mu}{\sigma}\right)\right)$$

Derivation of the expression for $S(\mu,\sigma)$

$$\gamma^{2}\int_{-\infty}^{2\%-\alpha}(I-2\%)^{2}\cdot\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{I-\mu}{\sigma}\right)^{2}\right)dI$$

Using the rule of substitution with $\tilde{I}=\frac{I-\mu}{\sigma}$, we obtain

$$=\gamma^{2}\int_{-\infty}^{\frac{2\%-\alpha-\mu}{\sigma}}\left(\tilde{I}\cdot\sigma+\mu-2\%\right)^{2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\tilde{I}^{2}\right)d\tilde{I}$$
$$=\gamma^{2}\sigma^{2}\int_{\frac{-2\%+\alpha+\mu}{\sigma}}^{\infty}\tilde{I}^{2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\tilde{I}^{2}\right)d\tilde{I}$$
$$+\gamma^{2}\cdot 2\sigma\cdot(\mu-2\%)\int_{-\infty}^{\frac{2\%-\alpha-\mu}{\sigma}}\tilde{I}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\tilde{I}^{2}\right)d\tilde{I}+\gamma^{2}\cdot(\mu-2\%)^{2}\int_{-\infty}^{\frac{2\%-\alpha-\mu}{\sigma}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\tilde{I}^{2}\right)d\tilde{I}$$
$$=\frac{\gamma^{2}}{2}\sigma^{2}\int_{\frac{-2\%+\alpha+\mu}{\sigma}}^{\infty}\tilde{I}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\tilde{I}^{2}\right)2\tilde{I}d\tilde{I}$$
$$-\gamma^{2}\cdot 2\sigma\cdot(\mu-2\%)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{2\%-\alpha-\mu}{\sigma}\right)^{2}\right)+\gamma^{2}\cdot(\mu-2\%)^{2}\,\Phi\left(\frac{2\%-\alpha-\mu}{\sigma}\right)$$

We distinguish two cases:
Case 1: $\alpha+\mu-2\%\geq 0$

$$=\frac{\gamma^{2}}{2}\sigma^{2}\int_{\frac{(\alpha+\mu-2\%)^{2}}{\sigma^{2}}}^{\infty}\frac{1}{2^{3/2}\Gamma(3/2)}x^{1/2}\exp\left(-\frac{1}{2}x\right)dx$$

$$-\gamma^{2}\cdot 2\sigma\cdot(\mu-2\%)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{2\%-\alpha-\mu}{\sigma}\right)^{2}\right)+\gamma^{2}\cdot(\mu-2\%)^{2}\,\Phi\left(\frac{2\%-\alpha-\mu}{\sigma}\right)$$

$$= \frac{\gamma^2}{2}\sigma^2 \left(1 - C_3\left(\frac{(2\% - \alpha - \mu)^2}{\sigma^2}\right)\right)$$

$$-\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% - \alpha - \mu}{\sigma}\right)^2\right) + \gamma^2 \cdot (\mu - 2\%)^2\, \Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)$$

Case 2: $\alpha + \mu - 2\% \leq 0$

$$= \frac{\gamma^2}{2}\sigma^2 \left(1 + \int_0^{\frac{(\alpha+\mu-2\%)^2}{\sigma^2}} \frac{1}{2^{3/2}\Gamma(3/2)} x^{1/2} \exp\left(-\frac{1}{2}x\right) dx\right)$$

$$-\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% - \alpha - \mu}{\sigma}\right)^2\right) + \gamma^2 \cdot (\mu - 2\%)^2\, \Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)$$

$$= \frac{\gamma^2}{2}\sigma^2 \left(1 + C_3\left(\frac{(2\% - \alpha - \mu)^2}{\sigma^2}\right)\right)$$

$$-\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% - \alpha - \mu}{\sigma}\right)^2\right) + \gamma^2 \cdot (\mu - 2\%)^2\, \Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)$$

Following a similar derivation, we get

$$\gamma^2 \int_{2\%+\alpha}^{\infty} (I - 2\%)^2 \cdot \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{I - \mu}{\sigma}\right)^2\right) dI$$

Case A: $\alpha - \mu + 2\% \geq 0$

$$= \frac{\gamma^2}{2}\sigma^2 \left(1 - C_3\left(\frac{(2\% + \alpha - \mu)^2}{\sigma^2}\right)\right)$$

$$+\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% + \alpha - \mu}{\sigma}\right)^2\right) + \gamma^2 \cdot (\mu - 2\%)^2 \left(1 - \Phi\left(\frac{2\% + \alpha - \mu}{\sigma}\right)\right),$$

Case B: $\alpha - \mu + 2\% \leq 0$

$$= \frac{\gamma^2}{2}\sigma^2 \left(1 + C_3\left(\frac{(2\% + \alpha - \mu)^2}{\sigma^2}\right)\right)$$

$$+\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{2\% + \alpha - \mu}{\sigma}\right)^2\right) + \gamma^2 \cdot (\mu - 2\%)^2 \left(1 - \Phi\left(\frac{2\% + \alpha - \mu}{\sigma}\right)\right),$$

with C_3 being the CDF of a Chi-squared distribution with three degrees of freedom.

Taking both together, we get

$$S(\mu, \sigma) = \frac{\gamma^2}{2}\sigma^2 \left(2 + \left(1 - 2 \cdot \mathbb{1}_{\alpha+\mu-2\%\geq 0}\right) C_3\left(\frac{(2\% - \alpha - \mu)^2}{\sigma^2}\right) + \left(1 - 2 \cdot \mathbb{1}_{\alpha-\mu+2\%\geq 0}\right) C_3\left(\frac{(2\% + \alpha - \mu)^2}{\sigma^2}\right)\right)$$

$$-\gamma^2 \cdot 2\sigma \cdot (\mu - 2\%)\frac{1}{\sqrt{2\pi}} \left[\exp\left(-\frac{1}{2}\left(\frac{2\% - \alpha - \mu}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{2\% + \alpha - \mu}{\sigma}\right)^2\right)\right]$$

$$+\gamma^2 \cdot (\mu - 2\%)^2 \left(1 - \Phi\left(\frac{2\% + \alpha - \mu}{\sigma}\right) + \Phi\left(\frac{2\% - \alpha - \mu}{\sigma}\right)\right).$$

C Construction of the Variables

C.1 Construction of the First Three or Five PCs

We consider a $(T \times M)$ data matrix X of M yields and sample size T. We construct the first three principal components as follows:

1. We calculate the sample covariance matrix of the given data:

$$\Sigma_{(M \times M)} = Cov(X).$$

2. We perform an eigenvalue decomposition of the covariance matrix Σ to calculate the eigenvalues D and the corresponding eigenvectors W:

$$\Sigma = W\ D\ W^{-1},$$

where D is the diagonal matrix containing the eigenvalues (sorted in descending order) on the main diagonal, and W is the matrix whose columns are the corresponding eigenvectors.

3. We denote the first three or five columns of W as $V_{(M \times 3)}$ or $V_{(M \times 5)}$. The first three or five principal components are obtained as:

$$[PC1\ PC2\ PC3] = X\ V,$$

$$[PC1\ PC2\ PC3\ PC4\ PC5] = X\ V. \tag{1}$$

C.2 Construction of the macro-yields model of Moench and Soofi-Siavash (2022)

We construct the Macroeconomic factors as in Moench and Soofi-Siavash (2022):

1. We estimate the yield curve factors as principal components of Gürkaynak, Sack, and Wright (2007) Treasury yields based on Appendices C.1.

2. We regress 135 macroeconomic series on the five yield curve factors:

$$Macro_i = \gamma_0 + \gamma' PC + u,$$

where $Macro_i$ represents the ith macroeconomic series, PC represents the five yield curve factors, and u is the residual.

3. We extract eight principal components from the residuals based on the dynamic factor models (DFM) method described by Stock and Watson (2016), the eight macroeconomic factors are defined as

$$[M1\ M2\ M3\ M4\ M5\ M6\ M7\ M8]. \tag{2}$$

C.3 Construction of the CP Factor

We construct the CP factor as in Cochrane and Piazzesi (2005):

1. We define the forward rate at time t for the period between time $t+n-1$ and $t+n$ as:

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}.$$

2. We regress the average excess log return across the maturity spectrum on the one-year yield and the four subsequent one-year forward rates $\mathbf{f}_t \equiv (y_t^{(1)}\ f_t^{(2)}\ f_t^{(3)}\ f_t^{(4)}\ f_t^{(5)})'$:

$$\overline{rx}_{t+1}^{(n)} = \gamma_0 + \gamma'\mathbf{f}_t + u_{t+1}.$$

   This regression implicitly assumes that the same function of forward rates predicts excess bond returns at all maturities.

3. We compute the CP factor as:

$$CP_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t, \tag{3}$$

   i.e., the predicted part of the regression, where $\hat{\gamma}_0$ and $\hat{\gamma}'$ denote parameter estimates.

C.4 Construction of the LN Factor

We construct the LN factor as in Ludvigson and Ng (2009):

1. We estimate the first eight principal components of 132 monthly economic series, and define the six-factor subset as

$$\mathbf{F}_t = (\hat{F}_{1t}\ \hat{F}_{1t}^3\ \hat{F}_{2t}\ \hat{F}_{4t}\ \hat{F}_{8t})',$$

   to minimize the BIC criterion. Among them, $\hat{F}_{1t}^3$ is the cubic function in the first estimated factor.

2. We regress the average excess log return across the maturity spectrum on the six-factor subset:

$$\overline{rx}_{t+1}^{(n)} = \gamma_0 + \gamma'\mathbf{F}_t + u_{t+1}.$$

3. We compute the LN factor as:

$$LN_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{F}_t, \tag{4}$$

   i.e., the predicted part of the regression, where $\hat{\gamma}_0$ and $\hat{\gamma}'$ denote parameter estimates.

C.5 Construction of the CPo Factor

We construct the CPo factor as in Cieslak and Povala (2015):

1. We construct a discounted moving average of past CPI inflation:

$$\tau_{t,CPI} = (1 - v) \sum_{i=0}^{t-1} v^i \pi_{t-i},$$

where $\tau_{t,CPI}$ denotes the trend inflation, which is calculated by the equation (1), and v equals 0.987 (Cieslak and Povala, 2015), and π is the annual core CPI growth rate.

2. We regress the average excess log return across the maturity spectrum on the average yield, 1-year yield, and trend inflation:

$$\overline{rx}_{t+1}^{(n)} = d_0 + d_1 \overline{y}_t + d_2 y_t^{(1)} + d_3 \tau_{t,CPI} + u_{t+1},$$

3. We compute the CPo factor as:

$$CPo_t = \hat{d}_0 + \hat{d}_1 \overline{y}_t + \hat{d}_2 \overline{y}_t^{(1)} + \hat{d}_3 \tau_{t,CPI}, \tag{5}$$

i.e., the predicted part of the regression, where $\hat{d}_0$, $\hat{d}_1$ and $\hat{d}_2$ denote parameter estimates.