Fundamental Disagreement about Monetary Policy and the Term Structure of Interest Rates*

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Abstract

Using a unique dataset of individual professional forecasts we document disagreement about the future path of monetary policy particularly at longer horizons. The stark differences in short rate forecasts imply strong disagreement about the risk-return trade-off of longer-term bonds. Longer-horizon short rate disagreement co-moves with term premiums. We endow investors with heterogeneous beliefs about the short rate path with an affine term structure model with a time-varying long-run mean of the level factor. The model closely fits Treasury yields and the short rate paths predicted by different groups of investors and thus matches the observed differences in expected return profiles. Investors who correctly anticipated the secular decline in rates became increasingly important for the marginal pricing of risk in the Treasury market. Accounting for time-variation in the long-run level of rates and heterogeneity in investment performance eliminates the downward trend in the term premium. Short rate disagreement accounts for about 30 percent of the variation in term premiums.

Keywords: Disagreement, Heterogeneous Beliefs, Survey Forecasts, Yield Curve, Term Premium.

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1 Introduction

Bond yields reflect investors' expectations about the future path of short rates as well as their attitudes toward risk. Most term structure models specify these two components of interest rates for a representative investor. While this provides a reasonable starting point for many analyses, it may mask important dynamics among investors and thus fail to provide a complete account of the driving forces behind bond yields.

In this paper, we first document a number of novel facts about short rate disagreement and term premiums. We start by using a unique and novel dataset of professional forecasters' individual longer-run expectations from Blue Chip Economic Indicators (BCEI) to document the following facts. First, we confirm that expectations about future policy rates are strongly time-varying and that disagreement about their path is substantial, particularly at intermediate to long horizons. Second, the stark differences in short rate forecasts imply sizable disagreement about subjective term premiums capturing the perceived risk-return tradeoff of longer-term bonds. Third, we document that disagreement about short rates comoves strongly with the term premium of the consensus forecaster as well as with estimates of the term premium from a reduced-form no-arbitrage model.

Inspired by these findings, we then propose a term structure model with the following features. There are three pricing factors: level, slope, and curvature. While the slope and curvature factors are stationary, the level factor has a time-varying long-run mean which itself follows a random walk and which acts as an unspanned fourth factor. We show that the model implies a tight link between the expected long-run mean of the level factor and the market price of level risk. As a consequence, it replicates the close connection between the expected path of policy rates and the perceived risk-return tradeoffs of longer-term bonds that we have documented in the first part of the paper. Investors expecting a higher level of future short rates at the same time expect lower excess returns for longer-term bonds, and vice versa. We endow two different investors with this model. Both perfectly observe yields but have polar beliefs about the future path of policy rates. They use observed yields and their short rate expectations to filter the model factors.

We estimate the model using zero coupon Treasury yields as well as the term structure of survey forecasts of the federal funds rate for two different hypothetical investors who represent well the observed range of beliefs about future short rates: the top-10 and bottom-10 average responses of the Blue Chip Financial Forecasts (BCFF) survey. Our model closely fits yields and the two survey forecast paths of the short rate over the sample from February 1986 through May 2023. In the estimated model, investors expecting higher future short rates expect excess bond returns to be negative on average for most maturities. In contrast, investors predicting short rates to be lower perceive average term premiums to vary between about 50 basis points at the one-year to about two percent at the ten-year maturity.

Structural models of the term structure of interest rates with heterogeneous beliefs about short rates, most importantly **?**, suggest that term premium variation is to a large degree driven by disagreement about short rates. In the final part of our analysis, we use the estimated empirical models to quantify the importance of short-rate disagreement for the evolution of term premiums and plausible degrees of policy rate disagreement. We do so in the following way. First, we use the model estimates to compute expected excess returns for bonds of different maturities for both investors. We then let them solve a simple mean-variance problem and allocate their wealth in optimal Treasury portfolios based on the obtained portfolio weights. We show that over the 1986-2003 sample period, the investor with persistently lower expected future short rates would have earned persistently higher returns as she would have tilted her portfolio towards longer maturities. The investor expecting higher future short rates, in turn, would have invested more heavily in short-term bonds and as such earned substantially lower investment returns.

We then use the realized excess returns of both investors to compute the hypothetical evolution of wealth in an economy that only features these two investors. We corroborate our estimated paths of the relative wealth ratio using a nonparametric approach that relies on two assumptions. First one can write the expected returns for the marginal investor in the economy as a weighted average of the top 10 and bottom-10 investor. Second, these weights vary smoothly over time. We show that with these two assumptions, one can use realized returns to estimate the corresponding weights on each of our investors. Both implied paths for the relative wealth ratio between the two investors track each other closely.

In the spirit of **?**, we finally compute a wealth-weighted term premium using the wealth weights resulting from the mean-variance portfolio optimization. We compare this term premium with two benchmarks. The first is the term premium by **?** (ACM) which relies on a model with five pricing factors extracted from Treasury yields but constant parameters. The second is the term premium obtained from our model with a shifting long-run mean of the level factor, fitted to the consensus (average) forecaster from the BCFF survey. We show that the wealth-weighted term premium features no discernable trend over the 1986-2023 sample. In contrast, the ACM term premium and to a lesser degree the consensus term premium feature some downward trend over this period. To quantify the importance of short rate disagreement for term premium variation, we compute a simple variance decomposition of the consensus term premium. In our baseline specification, deviations from a consensus model where both investors hold constant equal shares of the wealth, explain a little less than 30% of the variation of the consensus term premium. We also document that this share is lower when we vary the assumed degree of relative risk aversion and the initial wealth ratio.

Our paper contributes to the small but growing literature on bond pricing with heterogeneous beliefs. For an excellent recent discussion of this literature, see **?**. Our approach is inspired by **?** who theoretically show that heterogeneous beliefs about the long-run mean of fundamentals can be an important driver of bond returns. While in their model individual expected bond returns are constant the economy-wide term premium fluctuates because of time-varying disagreement. In contrast, we propose an affine term structure model with time-varying prices of risk which we use to fit the observed term structure of policy rates for different investors. We further deviate from their analysis by modeling disagreement about the nominal short rate instead of the inflation target. Indeed, as shown in **?**, disagreement about long-run inflation is not sufficient to account for the sizable long-run disagreement about the short rate.

Other authors have also studied bond pricing with heterogeneous beliefs. **?** consider a model in which investors with habit formation utility disagree about the distribution of inflation, not just expected inflation. This disagreement induces heterogeneity in investors' consumption and investment decisions and, on average, raises real and nominal bond yields. They further document empirically that inflation disagreement has a strong effect on real and nominal bond yields over and above the impact of expected inflation, consistent with their theoretical model. **?** study the interactions between risk aversion and disagreement. In their model heterogeneous beliefs arise because agents have different views about the (constant) long-run growth rate of consumption and because their perceptions of the correlation of shocks differs. They find that disagreement has larger effects on equilibrium bond prices when risk aversion is low. More recently, **?** aggregate individual expected excess bond returns based on forecasters' past accuracy in predicting interest rates. In line with our findings, they document that disagreement about bond risk premia is time-varying and persistent. While they show that their measure of aggregate expected bond returns is correlated with disagreement about future real growth and inflation, they do not study its comovement with disagreement about nominal short rates that is the focus of our analysis.

? build a model of the term structure in which investors with heterogeneous information sets form higher-order expectations about the beliefs of all other investors. Equilibrium bond prices then reflect a speculative component which depends on investors' beliefs about the error that the average investor makes when predicting future short rates. Their model suggests that the speculative component explains a sizable fraction of the variation in U.S. Treasury yields. **?** generalize this model to allow for richer price of risk specifications as used in the empirical term structure literature. In their model, investors observe heterogeneous signals of the state variables driving bond yields. They forecast the forecasts of other investors and engage in speculative trading. In equilibrium, individual investors' prices of risk then reflect idiosyncratic signals, higher order expectations of the true state variables, as well as investor-specific expectations of maturity-specific shocks. Importantly, in their model the pricing factors follow stationary vector autoregressions under both the risk-neutral and the physical measure implying that investors do not disagree about short rates in the long-run, in contrast with our empirical evidence.

Our paper is also related to the term structure literature using survey information in the model estimation. For example, **?** and **?** use consensus survey forecasts to discipline the time-series dynamics under the physical measure. **?** shows that term premiums implied by a standard affine model and model-free term premiums implied by consensus survey expectations of future short rates have seen a secular decline across ten developed economies since the 1990s. He documents that survey-based measures of inflation disagreement show similar dynamics as these estimated term premiums. **?** build a dynamic term structure model in which a representative investor updates her beliefs about future bond yields. They find that when this updating is conditioned on the dispersion in bond yield forecasts, the model produces substantially smaller forecast errors. We provide a structural interpretation to their findings by explicitly relating term premium dynamics to investors' differences in beliefs and resulting relative wealth fluctuations. Finally, **?** use the universe of surveys of professional forecasters to infer the consensus expected path of future Treasury bill rates. They show that although these short rate expectations show sizable variation, term premiums obtained as the simple difference between yields and expected short rates account for the bulk of yield variation at high and medium-term frequencies.

Finally, our paper is related to the literature on heterogeneous beliefs about future macroeconomic outcomes. Disagreement about the future path of the economy has been documented not only for households and firms, but also for professional forecasters which we also rely on in this paper and who are, arguably, among the best informed economic agents.[1] As argued above, investors need to make forecasts of short rates far into the future to determine the fair value of longer-term bonds. **?** document that professional forecasters hold strongly different beliefs about the path of short rates, particularly at longer horizons. In this paper, we show that such fundamental disagreement about short rates implies stark differences in the perceived risk-return tradeoff of longer-term bonds which, in turn, affect the marginal pricing of risk in the economy.

The remainder of this paper is structured as follows. Section **??** documents some novel facts about short rate disagreement and term premiums. In Section **??**, we describe our affine term structure model with heterogeneous beliefs about the long-run level of rates. Section **??** presents the estimation results and model fit. In Section **??**, we solve the investor portfolio problem and derive its implications for relative wealth dynamics and the aggregate term premium. Section **??** concludes.

2 Disagreement and Expected Returns: Stylized Facts

In this section, we motivate our subsequent analysis by providing some novel stylized facts on disagreement about future policy rates and expected Treasury returns. Our results are based on the Blue Chip Economic Indicators (BCEI) and the Blue Chip Financial Forecasts (BCFF) surveys. The Blue Chip (BC) surveys have been conducted monthly since the early 1980s. They ask two partly overlapping panels, of about 40 professional forecasters, to provide forecasts of the quarterly average of a variety of economic and financial variables. Since the mid 1980s, the surveys have also biannually been collecting forecasts from two years as far as 7-to-11 years ahead. While the BCFF survey publishes the individual forecasts for horizons up to six quarters into the future at a monthly frequency, they only report three quantities for the biannual forecasts of horizons of two years and above. These are the average across all forecasters, which we label the "consensus forecast," as

[1] See, e.g., **?**, **?**, **?**, **?**, and **?**.

well as the average of the top-10 and bottom-10 responses for a given forecasted variable at a given horizon. While the estimated term structure model with disagreement in Section **??** relies on these three forecast series for the federal funds rate from the BCFF survey, we set the stage in this section by providing information also on individual longer term three-month Treasury bill forecasts from the BCEI survey. To the best of our knowledge, no such individual longer-term forecast data has previously been studied in the literature.

Our analysis is motivated by **?** who show that the term structure of disagreement about future policy rates is upward sloping. While forecasters agree to a large extent about monetary policy in the near-term, they have strongly opposing views about the medium and longer-term policy outlook, which at least partially reflects disagreement about the fundamentals of the economy. Here, we expand on these results by showing that fundamental or long-term disagreement about short term interest rates *i*) is not driven by outlier predictions; *ii*) is a persistent phenomenon in the sense that individual forecasters tend to see high or low future short rates across all forecast horizons; *iii*) implies sizable fundamental disagreement about bond risk premia; and *iv*) is strongly correlated with the term premium of the consensus forecaster and that implied by an affine term structure model.

Figure **??** shows the individual predictions for the three-month Treasury bill at horizons of two and 7-11 years into the future. The left-hand chart documents that while individual longer-term forecasts broadly move together there is a considerable degree of disagreement among forecasters already at the two-year horizion. They disagree by as much as six percentage points about the level of the three-month TBill. The strong disagreement is particularly pronounced just after the start of the large-scale asset purchase programs by the Federal Reserve in 2009, but drops considerably when calendar-based forward guidance was introduced in the summer of 2011 (**?**). The chart also shows that the width of the forecast distribution as measured by the difference between the top-10 and bottom-10 average responses is wide and varies considerably over time.

The right-hand chart of Figure **??** provides the predictions of the three-month TBill of the same individuals in the long-run. As expected, there is less of a cyclical element in these forecasts. That said, the chart also shows that the entire distribution of long-run forecasts of the short rate has trended down over the sample. This strongly suggests that the long-run level of the short rate is perceived to vary over time.

This feature of forecasters' beliefs will be a central element of our modeling strategy. Interestingly, while there clearly is a strong common element in the individual forecasts, the distribution at this very long horizon is also quite wide. This indicates that forecasters disagree to a considerable extent about the long-run value of the short rate. Quite strikingly, at the end of our sample some forecasters believe the long-run value of the Treasury Bill will remain below two percent while others see it go back to a level of around four percent. These heterogeneous assessments likely reveal sharply different views of the steady state of the economy.

As it is inherently difficult to predict far into the future, one might worry that individual forecasters' responses are to some extent arbitrary and do not necessarily reflect their views of the world. While we do not observe the names of individual forecasters in our sample of long-term predictions, we are able to trace their forecast paths at any given point in time. We can thus check whether the individual medium to long-run predictions are consistent in the sense that they reveal a particular forecaster expecting higher or lower future short rates. We rank the individual forecasts at all medium to long-run horizons and compute the rank correlation between adjacent horizons. This measures the probability that a forecaster who expects low rates at, say, the four-year ahead horizon also expects low rates at the five-year horizon.

Figure **??** displays these rank correlations across forecasters along with their 90 percent confidence interval for adjacent medium to long-term forecast horizons. At all horizons, the correlations are large and precisely estimated. The rank correlations are somewhat lower, around 70 percent, at medium-term horizons suggesting that individual forecasts are to some degree driven by different views about the state of the business cycle and the corresponding monetary stance at these horizons. That said, for longer forecast horizons the rank correlations reach almost 100 percent at the six year and 7-11 year ahead horizon. This implies that individual forecasts are highly consistent across horizons and likely reflect fundamentally different views about monetary policy in the long run.

The term premium is defined as the difference between the yield on a government bond and the average short rate expected to prevail over the life of the bond. It is a measure of the compensation investors demand for bearing interest rate risk. Since we observe survey participants' individual forecast paths for the short rate, we can compute the term premiums consistent with their short-rate beliefs for various forward horizons. Figure **??** displays the evolution of forward term premiums implied by the individual TBill forecasts for the one-to-two year and 7-11 year forward horizons.

The figure shows that the assessment of the compensation that long-term bond investors command differs widely across forecasters. Moreover, especially in the latter part of the sample quite a few survey participants see term premiums in negative territory, possibly suggesting that they view longer term Treasuries as hedges against adverse states of the economy. While individuals' views about term premiums are quite heterogeneous, the top and bottom-10 average predictions appear to represent well the dispersion of beliefs across the forecaster distribution.

Figure **??** displays the one-two year and the 7-11 year forward term premium implied by the consensus forecaster. We obtain the latter as the difference between observed forward yields and the average short rate expected by the consensus forecast for the same maturity. The consensus forecast, in turn, is given by the average forecast across all forecasters in the survey. We superimpose the difference between the top-10 and bottom-10 average forecasts of the short rate for the same horizon. The charts show that the two measures comove at both horizons. The model presented later in the paper rationalizes this comovement.

The positive correlation between a measure of the disagreement about future short rates and the term premium shown above is not restricted to the term premium implied by the consensus forecast. The upper panel of Figure ?? shows the time series of the two and ten-year Treasury term premium obtained from the ? (ACM) model.[2] This no-arbitrage term structure model uses the first five principal components of Treasury yields as pricing factors and does not include survey forecasts in the estimation. We superimpose the difference between the top and bottom-10 average responses at the two-year horizon and the five and 7-11 year ahead horizons, respectively. The charts show a strong co-movement of the ACM term premium and short rate disagreement at both horizons. This becomes even more apparent when considering scatter plots of the same series in the bottom panel of Figure ??. Both survey-based and statistical term premiums are strongly correlated with measures of longer-term disagreement about the short rate.

3 A GATSM with Shifting Beliefs about the Policy Rate

In this section, we introduce a Gaussian affine term structure model (GATSM) that explicitly allows for shifting beliefs about the future level of rates. We will then use this model to evaluate the differences in the risk-return trade-off of longer-term bonds that investors endowed with the same model but with different beliefs about the longer-run level of yields face.

As is common in GATSM models, we assume that yields are affine functions of three pricing factors which follow a stationary vector autoregression under the pricing measure. The pricing factors are portfolios of yields and can be interpreted as level, slope, and curvature. In the spirit of ?, we extend the standard affine model to feature a level factor with a time-varying long-run mean which we assume to follow a random walk. Importantly, the investors endowed with the same term structure model and observing the same Treasury yields but different beliefs about future rates will obtain different filtered estimates of the model factors and the time-varying long-run mean of the level factor. This assumption is clearly motivated by the time variation in and disagreement about expected longer-run policy rates documented in the previous section. In the following, we introduce the individual elements of this model which will form the basis for our empirical analysis in Section ??.

3.1 The Model

Following a classical setup in the term structure literature, we assume three factors determine the evolution of bond prices, which we denote as 'Level', 'Slope' and 'Curvature'. The pricing factors are given compactly by $\mathbf{X}_t = \bar{\mathbf{X}} + e_j\mu_t + \tilde{\mathbf{X}}_t$, where $\bar{\mathbf{X}}$ is a constant and e_j is a selection vector determining which pricing factors are affected by the long-run drift μ_t. $\tilde{\mathbf{X}}_t$ is stationary and evolves following the process[3]

$$\tilde{\mathbf{X}}_t = \mathbf{\Phi}\tilde{\mathbf{X}}_{t-1} + \Sigma_{\epsilon}^{1/2}\tilde{\epsilon}_t. \tag{1}$$

[2]See https://www.newyorkfed.org/research/data_indicators/term_premia.html.

[3]? introduce a similar model in a representative-agent framework, see the Appendix for details.

Moreover, we assume that there are two investors $i \in \{A, B\}$. Both investors perfectly observe the full set of Treasury yields. They also agree on the the data generating process. However, we assume that the two investors have different beliefs about the drift μ_t which determines the long-run mean of the level factors. Both investors believe μ_t evolves according to a random walk. Their estimates of the drift evolve according to

$$\mu_t^i = \mu_{t-1}^i + \sigma_\eta^i \eta_t^i, \quad \text{for } i \in \{A, B\}. \tag{2}$$

This equation parsimoniously captures the updating of investors' beliefs about slow-moving trends and reflects investors' attempts to disentangle short-run macroeconomic developments from long-term changes in the economy. This difference in beliefs across the two investors could be driven, for example, by idiosyncratic signals or some other form of informational friction.[4] As a result of their different beliefs about the long-run level of rates, agents' estimates of the three pricing factors $\mathbf{X}_t$ and the drift μ_t^i will differ. As a result, their estimated risk-retjurn trade-off for longer-run bonds will also differ. We now turn to zero coupon bond prices. These are perfectly observed by both investors and equal the respective discounted future bond prices of bond with one period less time to maturity:

$$P_t^{(n)} = E_t^i \left(M_{t+1}^i P_{t+1}^{(n-1)} \right), \quad \text{for } \ i \in \{A, B\}.$$

where $P_t^{(n)}$ is the price of an n-period bond in time t and M_t^i denotes the stochastic discount factor of investor i. As is common in the affine term structure literature we assume an exponentially affine functional form:

$$M_{t+1}^i = \exp\left(-r_t - \frac{1}{2} \lambda_t^{i\prime} \Sigma_\epsilon^i \lambda_t^i - \lambda_t^{i\prime} \Sigma_\epsilon^{i,1/2} \epsilon_{t+1}^i \right), \tag{3}$$

where ϵ_t^i are the innovations to the pricing factors and λ_t^i denotes the investor-specific vector of market prices of risk associated with these pricing factors. The short rate is assumed to be linear in the pricing factors:[5]

$$r_t = \delta_0^i + \delta_1^{i\prime} \mathbf{X}_t. \tag{4}$$

Following **?** and many others, we assume that prices of risk are linear in the pricing factors:

$$\lambda_t^i = \lambda_{0,t}^i + \Lambda_1^i \mathbf{X}_t, \tag{5}$$

[4]For example, **?** show that the observed term structures of disagreement can be rationalized in a model with informational frictions where the state variables follow a VAR with slow-moving long-run means which investors filter from the imperfectly observed data.

[5]We will show later ϵ_t involves uncertainty about $\tilde{\mathbf{X}}_t$ as well as μ_t.

where $\lambda^i_{0,t}$ is a 3×1 vector and Λ^i_1 a 3×3 matrix. We assume a time-varying intercept in the market prices of risk to embed the notion that investors' perceived risk-return tradeoff can fluctuate with their assessment of the long-run mean of the risk factors.

Both investors observe bond prices and conjecture that these are exponentially affine in the observed pricing factors:[6]

$$\ln P_t^{(n)} = \mathcal{A}_n + \mathcal{B}_n{}'\mathbf{X}_t^i \quad \text{for } \ i = \{A, B\}.$$

We can then use this in investor i's pricing condition

$$\begin{aligned} P_t^{(n)} &= E_t^i\left(M_{t+1}^i P_{t+1}^{(n-1)}\right) \\ &= E_t^i\left(M_{t+1}^i \exp\left(\mathcal{A}_{n-1} + \mathcal{B}_{n-1}'\mathbf{X}_{t+1}^i\right)\right) \quad \text{for } \ i = \{A, B\}. \end{aligned}$$

This yields the typical Ricatti equations:

$$\mathcal{A}_n = \mathcal{A}_{n-1} + \mathcal{B}_{n-1}'\left(\left(I - \mathbf{\Phi}^i\right)\left(\bar{\mathbf{X}}^i + e_j\mu_t^i\right) - \Sigma_\epsilon^i\lambda_{0,t}^i\right) + \frac{1}{2}\mathcal{B}_{n-1}'\Sigma_\epsilon^i\mathcal{B}_{n-1} - \delta_0^i \tag{6}$$

$$\mathcal{B}_n' = \mathcal{B}_{n-1}'\left(\mathbf{\Phi}^i - \Sigma_\epsilon^i\Lambda_1^i\right) - \delta_1^{i\prime} \quad \text{for } i \in \{A, B\}. \tag{7}$$

This implies that

$$\left(I - \mathbf{\Phi}^i\right)\left(\bar{\mathbf{X}}^i + e_j\mu_t^i\right) - \Sigma_\epsilon^i\lambda_{0,t}^i = \Psi_0^i \quad i \ \in \{A, B\} \quad \forall t \ \in \{1, \ldots, T\} \tag{8}$$

$$\text{and } \ \mathbf{\Phi}^i - \Sigma_\epsilon^i\Lambda_1^i = \Psi_1^i \quad i \ \in \{A, B\} \tag{9}$$

Hence we can express

$$\ln P_t^{(n)} = \mathcal{A}_n\left(\Psi_0^i\right) + \mathcal{B}_n'\left(\Psi_1^i\right)\mathbf{X}_t^i$$

where Ψ_0^i is a 3×1 vector and Ψ_1^i a 3×3 matrix denoting the intercept and autoregressive coefficient of $\mathbf{X}_t^i$ under the pricing measure.

Note that Equation **??** implies a direct link between the individual time-varying drift of the level factor μ_t^i and the market prices of risk $\lambda_{0,t}^i$. Investors observing the same set of yields and relying on the same model will thus perceive different risk-return trade-offs. These will transpire from different return expectations. Expected one-month excess holding period returns across maturities of each investor are given by

$$E_t^i\left[rx_{t+1}\right] = \mathcal{B}_X^i\lambda_t^i - \frac{1}{2}\Sigma^i, \tag{10}$$

[6] In line with the existing affine term structure literature, we only consider a solution with a constant intercept under the pricing measure. While alternative solutions with a time-varying interecept are conceivable, specific parametric assumptions about the intercept would be required to ensure a closed-form system of Ricatti equations as the one we derive below.

where $\mathcal{B}_X^i$ is the vector of corresponding loadings of the log prices of bonds with different maturities on the pricing factors $\mathbf{X}_t^i$ derived above and Σ^i is the model-implied conditional variance-covariance matrix of excess returns given by

$$\Sigma^i = \mathcal{B}_X^i \Sigma_\epsilon^i \mathcal{B}_X^{i\prime}.$$

Inspecting Equation (**??**), a higher perceived μ_t^i results in a higher value of the market price of risk λ_t^i. Since the bond pricing parameters $\mathcal{B}_X$ are negatively related to the yield loadings on the pricing factors, a higher λ_t^i thus implies lower expected returns. In other words, an investor expecting higher future policy rates will expect lower excess returns on longer-term bonds.

3.2 The State-Space Model

With these ingredients we now write our model in state-space form. In addition to data on observed bond yields, $\mathbf{y}_t^o$, we use observations of short rate forecasts of investor group i, labeled $\mathbf{y}_t^{E,i}$ to identify the parameters of the model. The observation equation is given by:

$$\begin{bmatrix} \mathbf{y}_t^o \\ \mathbf{y}_t^{E,i} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_X^i \\ \mathbf{H}_0\left(\mathbf{\Phi}^i, \bar{\mathbf{X}}^i\right) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_X^i & 0 \\ \mathbf{H}_X\left(\mathbf{\Phi}^i\right) & \mathbf{H}_\mu\left(\mathbf{\Phi}^i\right) \end{bmatrix} \times \begin{bmatrix} \mathbf{X}_t^i \\ \mu_t^i \end{bmatrix} + \varepsilon_t^i, \tag{11}$$

where the first few N entries of ε_t are yield pricing errors $\varepsilon_t^{(n)}$ where N is the number of yields used in the estimation and ε_t has a diagonal variance-covariance matrix. Moreover, $\mathbf{H}_0\left(\mathbf{\Phi}^i, \bar{\mathbf{X}}^i\right)$, $\mathbf{H}_X\left(\mathbf{\Phi}^i\right)$ and $\mathbf{H}_\mu\left(\mathbf{\Phi}^i\right)$ determine the model-implied loadings of future short rate expectations of investor i on the observed model factors $\mathbf{X}_t$ as well as the updates of the drift (unobserved trend) μ_t^i. These loadings are known, nonlinear transformations of the model parameters.[7]

For identification, the pricing factors in our model are given by $\mathbf{X}_t^i = \bar{\mathbf{X}}^i + e_1\mu_t^i + \tilde{\mathbf{X}}_t^i$, where $\tilde{\mathbf{X}}_t^i$ is stationary following equation (**??**) and $e_1 = (1,0,0)'$. Note that the first element of $\bar{\mathbf{X}}^i$ is set to be zero and, therefore, μ_t^i can be identified and interpreted as the long-run mean of the Level factor. The transition equation is given by

$$\begin{pmatrix} \mathbf{X}_t^i \\ \mu_t^i \end{pmatrix} = \begin{pmatrix} (I-\mathbf{\Phi}^i)\bar{\mathbf{X}}^i \\ 0 \end{pmatrix} + \begin{bmatrix} \mathbf{\Phi}^i & \mathbf{\Gamma}^i \\ 0 & 1 \end{bmatrix} \times \begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mu_{t-1}^i \end{pmatrix} + \begin{bmatrix} \Sigma_{\tilde{\epsilon}}^{i,1/2} & e_1 \\ 0 & \sigma_\eta^i \end{bmatrix} \times \begin{pmatrix} \tilde{\epsilon}_t^i \\ \eta_t^i \end{pmatrix} \tag{12}$$

where $\mathbf{\Gamma^i} = (I-\mathbf{\Phi}^i)e_1$. Let $\mathbf{u}_t^i = \begin{bmatrix} \Sigma_{\tilde{\epsilon}}^{i,1/2} & e_1 \\ 0 & \sigma_\eta^i \end{bmatrix} \times \begin{pmatrix} \tilde{\epsilon}_t^i \\ \eta_t^i \end{pmatrix}$ and $\Sigma_\epsilon^i = \Sigma_{\tilde{\epsilon}}^i + e_1 e_1' {\sigma_\eta^i}^2$, then the covariance

[7] Dropping superscripts i, note that $\mathbf{E}_t^i[\mathbf{X}_{t+k}] = (I-\mathbf{\Phi}^{k-1})(\bar{\mathbf{X}} + e_1\mu_t^i) + \mathbf{\Phi}^k\mathbf{X}_t$, so k-step ahead forecasts of short rates at time t are given by $\mathbf{E}_t^i[r_{t+k}] = \delta_0 + \delta_1'\mathbf{E}_t^i[\mathbf{X}_{t+k}] = \left(\delta_0 + \delta_1'(I-\mathbf{\Phi}^{k-1})\bar{\mathbf{X}}\right) + \delta_1'\mathbf{\Phi}^k\mathbf{X}_t + \delta_1'(I-\mathbf{\Phi}^{k-1})e_1\mu_t^i$.

matrix is given by

$$\Omega^i = E(\mathbf{u}_t^i \mathbf{u}_t{}^{i\prime}) = \begin{bmatrix} \Sigma_{\tilde{\epsilon}}^i + e_1 e_1' {\sigma_\eta^i}^2 & e_1 {\sigma_\eta^i}^2 \\ e_1' {\sigma_\eta^i}^2 & {\sigma_\eta^i}^2 \end{bmatrix} = \begin{bmatrix} \Sigma_\epsilon^i & e_1 {\sigma_\eta^i}^2 \\ e_1' {\sigma_\eta^i}^2 & {\sigma_\eta^i}^2 \end{bmatrix}. \tag{13}$$

Hence, the innovations $\mathbf{u}_t^i$ in the transition equation can be correlated with each other, while the innovations of the pricing factors, ϵ_t^i, are uncorrelated with the innovations of beliefs about the long-run drift, η_t^i. The vector of parameters to be estimated is given by

$$\Theta^i = \Big(\text{vech}(\mathbf{\Phi}^i)' \;\; \bar{\mathbf{X}}^i \;\; \text{vech}(\Sigma_\epsilon^i)' \;\; \sigma_\eta^i \;\; \mathbf{\Psi}_0^{i\prime} \;\; \text{diag}\left(\mathbf{\Psi}_1\right)^{i\prime} \;\; \sigma_y^i \;\; \sigma_{\text{short}}^i \;\; \sigma_{\text{long}}^i \Big)$$

where the last three elements of Θ^i are the variances of the *i.i.d.* measurement error on yields as well as short and long-horizon survey forecasts of the short rate. We estimate the parameters via maximum likelihood and filter the perceived drifts μ_t^i.

4 Bringing the Model to the Data

In this section, we bring our GATSM with disagreement about the short rate to the data. We first describe the data used and the estimation approach. We then discuss the model fit and provide decompositions of Treasury yields into expected short rates and term premiums. In Section **??**, we then use the model to study the perceived risk-return tradeoff of individual investors and their portfolio allocations. Finally, we discuss implications of disagreement about the short rate for the wealth-weighted term premium in the economy.

4.1 Data and Estimation

We jointly estimate our model using zero-coupon Treasury yields as well as survey expectations of short rates for two different groups of investors. We obtain the latter from the *Blue Chip Financial Forecasts* (BCFF) survey. Specifically, we use two- and four-quarter-ahead, one-to-two, four-to-five year-ahead, and long-term forecasts which cover horizons between six and ten or seven and eleven years into the future, depending on when the survey was taken. The short-term forecasts are observed monthly and this is our frequency of observation also for Treasury yields. The medium-term and long-term forecasts are observed biannually. The missing monthly observations in between biannual survey observations can easily be accommodated in our state-space framework. The BCFF provides medium and long-term forecasts for three different cross-sectional averages of the forecaster distribution: the average across all responses (the “consensus” forecast), the average of the top-10 responses and the average of the bottom-10 responses. We employ the top-10 and bottom-10 average responses as proxies representing the two investors at the opposite spectrum of the belief distribution about future short rates. As discussed in Section **??**, the difference between the top-10 and bottom-

10 average responses is closely correlated with common measures of forecaster disagreement, such as the cross-sectional standard deviation or the interquartile range.

We obtain zero coupon Treasury yields from **?** (GSW henceforth).[8] The GSW zero coupon yields are based on fitted Nelson-Siegel-Svensson curves, the parameters of which are published along with the estimated zero coupon curve. We use these parameters to back out the cross-section of zero-coupon yields for maturities up to ten years, using end-of-month values. In our estimation, we use $N = 8$ Treasuries with maturities $n = 3, 6, 12, 24, 36, 60, 84, 120$ months. Our sample period is February 1986 until May 2023 for a total of 448 monthly observations. We employ the normalization scheme in **?** to estimate our model and impose additional zero restrictions in the physical dynamics to ensure a parsimonious model (see Appendix for details).

4.2 Model Fit and Individual Expected Excess Returns

Our model fits both yields and survey forecasts of the short rate very precisely. Figures **??** and **??** compare the time series of observed and model-implied yields for the top-10 and bottom-10 investor, respectively. Although both investors observe the same set of yields and pricing factors, they perceive different time-varying long-run means of the level factor and use this to estimate the risk-return trade-off of Treasury bonds using a separate shifting-endpoint term structure model.

The top two panels of Figure **??** show the time series of observed and model-implied yields for the bottom-10 investor. The associated parameter estimates and standard errors are provided in Table **??**. The bottom two panels of Figure **??** provide the unconditional mean and standard deviation of yields across maturities as observed and fitted by the model. The charts show that the model fits both moments well. The average of yield pricing errors is no more than 5 basis points in absolute value and thus well in line with previous studies. Figure **??** provides the same set of results for the bottom-10 investor. The fit is equally good. The corresponding parameters are provided in Table **??**.

We next turn to the model fit of survey forecasts of the short rate. The top two charts in Figure **??** display the observed and fitted top-10 and bottom-10 average survey forecasts of the federal funds rate, where actual values are plotted by asterisks and model-implied ones by solid lines. These charts document that with only the perceived long-run mean of the level factor being different across investors, our model is able to capture the substantial time variation in investors' disagreement about future short rates precisely. The bottom four panels of Figure **??** provide a plot of unconditional first and second moments of the two groups' survey forecasts as observed and fitted by the model, again documenting that the model fits survey forecasts at all horizons quite precisely. Of note, while the standard deviation of short rate forecasts is similar across the two investors, the average short rate path expected by the bottom-10 investor is substantially flatter than that of the top-10 investor.

[8]See http://www.federalreserve.gov/econresdata/researchdata.htm

We use the two investors' models to study the entire forecast path of individual short rate expectations. Figure **??** displays the evolution of the short rate along with the expected short rate paths ten years into the future. The model generates these paths for every month in the sample, but for ease of exposition we only show them at five year intervals. The following observations are worth highlighting. First, for both investors, the longer-run expected short rate has steadily declined over time. At the same time, the short rate disagreement at the ten year horizon has declined from more than four percentage points in the mid 1980s to around one percentage point at the end of the sample. Disagreement about the long term also results in very different expected paths in the short and medium term. For example, in the mid 1980s and mid 2000s the top-10 investor saw a steepening path of the short rate while the bottom-10 expected a declining path throughout the forecasting horizon. While both investors were too optimistic about policy normalization in the zero lower bound period after the Great Financial Crisis, the bottom-10 investor consistently expected a flatter short rate path. For example, during the period 2011-2013 when the Federal Reserve was heavily relying on forward guidance to steer short-term rate expectations at the zero lower bound, near-term rate expectations of the bottom-10 investor were compressed to zero. In contrast, the top-10 investor saw short rates rise above zero even in the short term. Finally, both investors substantially underestimated the extent and speed of policy tightening in the most recent hiking cycle.

Having discussed the evolution of expected short rates, Figure **??** shows the corresponding perceived term premiums which – as discussed in Section **??** – are equivalent to expected excess one-month returns over the life of the bond. As shown in the upper panel, the top-10 investor expecting high future short rates implicitly expected returns on two-year notes to hover between minus one and one percent over the last 30 years. Similarly, this investor expects ten-year Treasury term premiums to fluctuate around zero in a somewhat wider range. In stark contrast, the bottom-10 investor expecting low future policy rates implicitly saw positive expected excess returns across time and maturity, declining from about three (six) percent at the two-year (ten-year) maturity in the early 1980s to just below one percent for both maturities at the end of the sample. Moreover, the term premiums have become more similar at the end of the sample, consistent with a convergence of longer-term policy rate expectations across the two investors.

The bottom-left panel of Figure **??** provides the time series average of the implied term premiums across maturities for the two investors. While the bottom-10 investor has an upward sloping term structure of term premiums ranging between 50 basis points at the one-year maturity and 200 basis points at the ten-year maturity, the top-10 investor essentially sees term premiums on average slightly negative across all except the very long maturities. In other words, to this investor, Treasuries provide insurance for which she is willing to pay a premium. The bottom-right chart presents the differences between the two term premium estimates across time and maturities, showing that these differences have been somewhat less pronounced since around the year 2000.

5 Portfolio Allocation and Aggregate Term Premium

We have documented the evolution of individual short rate expectations and their implications for subjective term premiums. As discussed in **?** and **?**, in economies with hterogeneous beliefs, disagreement about relevant state variables will give rise to speculative trading and as such can generate risk premium variation.

In the spirit of these models, in this section we use the estimates from the previous section to measure an aggregate term premium of an economy with heterogeneous beliefs about the long-run level of yields. To do so, we have to evaluate how important these two investors have been for marginal bond prices over time. We proceed in two steps. First, we track the portfolio performance (and therefore their hypothetical wealth) of the two investors over time. Second, we construct an aggregate term premium by appropriately weighting the individual term premiums using the two investors' relative wealth.

5.1 Portfolio Allocation and Investment Performance

What do the different short rate forecasts imply for investors' bond portfolio allocation? Intuitively, the investor predicting higher future short rates would go long in short-term maturities while the investor predicting lower future short rates should tilt her portfolio towards longer maturities. Given the secular decline of interest rates over the sample period, one would thus expect the investor with a larger portfolio weight on longer maturities to accumulate larger wealth. While our reduced-form approach does not allow to derive portfolio allocations from explicit micro foundations, we can use the model-implied expected excess returns to compute hypothetical mean-variance portfolio weights.

Specifically, we assume that each investor constructs a portfolio by choosing the weights that minimize a quadratic utility function subject to the constraint that they add up to one. We follow **?** and obtain the optimal weights by solving in every period

$$w_{i,t}^{*} = \arg\min_{w} \left\{ w' E_t^i \left[rx_{t+1} \right] - \frac{\gamma}{2} w^{i\prime} \Sigma^i w \right\}, \tag{14}$$

where γ denotes the coefficient of relative risk aversion. Consistent with much of the prior literature, we set $\gamma = 3$ in our baseline analysis. In our analysis of the quantitive importance of short rate disagreement for term premium variation, we report results based on a grid of values for γ.

Recall that the expected excess return of investor i is

$$E_t^i \left[rx_{t+1} \right] = \mathcal{B}_X' \lambda_t^i - \frac{1}{2} \Sigma^i,$$

As shown in **?**, the solution to this problem is

$$w_{i,t}^{*} \;\; = \;\; \phi + \Phi E_t^i \left[rx_{t+1} \right],$$

$$\text{where } \phi \;=\; \frac{\Sigma^{-1}\iota}{\iota'\Sigma^{-1}\iota} \text{ and } \Phi = \frac{1}{\gamma}\left(\Sigma^{-1} - \frac{\Sigma^{-1}\iota\iota'\Sigma^{-1}}{\iota'\Sigma^{-1}\iota}\right) \tag{15}$$

are forecaster-specific parameters for which we have dropped the superscripts for brevity. Based on these optimal portfolio weights, we then obtain realized excess returns for investor i according to

$$\begin{aligned} \boldsymbol{rx}^i_{t+1} &= \hat{w}^{*'}_{i,t} \times \left(rx^i_{t+1} + r_t\right) \\ &= (\frac{1}{\gamma} - \frac{1}{2\gamma^2})\lambda^{i'}_t(\Sigma^i_\epsilon)^{-1}\lambda^i_t + \frac{1}{\gamma}\lambda^{i'}_t(\Sigma^i_\epsilon)^{-1/2}\epsilon^i_{t+1} \end{aligned} \tag{16}$$

The top panel of Figure **??** shows the expected excess one-month return for a ten-year Treasury note. The top-10 investor's expected return (solid blue line) has been slightly negative throughout the sample. In contrast, the bottom-10 investor (dashed green line) has been expecting positive but declining returns over time. This pattern of return expectations is the mirror image of the short rate expectations of the two investors. While the difference in expected returns was sizable at the beginning of the sample in the mid 1980s, it has been shrinking over the later part of the sample. Of note, even the bottom-10 investor who has predicted lower future short rates implicitly expected negative returns from the mid 2000s until the recent hiking cycles of policy rates.

The bottom panel of Figure **??** displays the cumulative realized returns from both investors' portfolios according to Equation (**??**). Not surprisingly, the bottom-10 investor strongly outperforms the top-10 investor over the sample period. The reason is that the bottom-10 investor is tilting her portfolio towards longer maturities, in line with the higher expected returns. Given the observed secular decline of rates over the sample, the price of longer-term bonds appreciated substantially, delivering higher returns compared to short-term maturities. This can be seen by the additional two lines in the figure which display the cumulative returns from rolling over a three-month Tbill (solid gray) and a ten-year Treasury (dashed gray), respectively. Given his portfolio allocation tilted towards shorter maturities, the top-10 investor's cumulative return is comparable to that of the Tbill until the early 2000s and then significantly falls short, only to recover in the recent hiking cycle. Conversely, the bottom-10 investor even outperformed the ten-year Treasury until the mid 2010s as she combines the holdings of longer maturities with a short position in short-term maturities.

5.2 Implications for the Term Premium

In structural asset pricing models with heterogeneous agents, the evolution of relative wealth is an important determinant of asset prices. Commonly, in these models the investor with a higher share of the total wealth has a greater impact on the marginal valuation of risky assets. Specifically, a central result in heterogeneous asset pricing models with general preferences is that the discount factor and beliefs of the representative investor can be represented as weighted averages of the discount factors and beliefs of the individual agents (see, e.g., **?** and **?**). Applied to a bond-pricing model ? show that the bond prices in a representative economy equal a wealth-weighted average of individual investors' bond prices. Interestingly, while in their model individual term premiums are constant due to specific assumptions about preferences, the aggregate term premium fluctuates over time because of changes in relative wealth and disagreement.

Inspired by this result, we provide a measure of the aggregate term premium that would arise if the Treasury market was only populated by the top-10 and bottom-10 investors. We begin by approaching this problem in a agnostic way by utilizing a nonparametric approach to estimating the weights. The nonparametric approach relies on two assumptions. The first, as discussed above, is that we can write the expected returns for the marginal investor in the economy as a weighted average of the top 10 and bottom-10 investor. Second, we assume that these weights vary smoothly over time in a way that is made precise in the Appendix. With these two assumptions, we can then use realized returns over our sample to estimate the corresponding weights on each of our investors.

Figure **??** plots the estimated wealth weights of the bottom-10 investor using the two different approaches. The red line corresponds to the point estimate of the nonparametric approach, with 68% confidence intervals represented by the black lines. The estimated wealth weight of the bottom-10 investor at the beginning of the sample is about 20%. Over the course of the sample, this weight rises monotonically, ending the sample a bit above 80%. The blue line in Figure **??** represents the wealth share derived from the cumulative returns for each investor obtained from the mean-square portfolio problem shown in Figure **??**. As initial condition, we use the 20% initial weight obtained from the nonparametric approach. Both the nonparametric estimated weights and the wealth share for the bottom-10 investor follow broadly similar dynamics, rising steadily across our sample. We proceed by estimating the economy-wide term premium as the wealth-weighted term premium of each of our individual investors according to the portfolio allocation in Equation (**??**).

The bottom panel of Figure **??** shows the resulting wealth-weighted term premium as the blue line. This term premium measure hovers between zero and two percent for most of the sample period and is quite stationary. For comparison, we superimpose two alternative term premium measures. The first is from **?** (ACM) who estimate a standard affine term structure model with five factors. A key difference with respect to the wealth-weighted term premium is that this "statistical term premium" is substantially higher than the wealth-weighted term premium until the mid 1990s and thus features more of a downward trend. This could be driven by at least two different effects. First, the ACM term premium is based on a model with constant parameters. As such, it features a constant instead of a time-varying long-run mean of the level factor. Since this long-run mean is estimated to be lower than the values of the level factor at the beginning of the sample, the model initially implies a reversion to a lower mean than the one obtained from the model with a time-varying intercept. As a result, the term premium is initially larger. The second difference is likely related to disagreement about future short rates which is high before the late 1990s and thus magnifies the effect of the differential wealth weights on the term premium. Conversely, later in the sample the bottom-10 investor increases her relative wealth share and contemporaneously the expected paths of future short rates converge. As a result, the wealth-weighted term premium appears more stationary.

As a second benchmark, we superimpose the term premium for the consensus forecaster, obtained by estimating the term structure model using the average forecasts from the BCFF survey at all horizons, shown as the yellow dash-dotted line in Figure **??**. While also somewhat larger than the wealth-weighted term premium at the beginning of the sample, the consensus term premium is also considerably more stationary and range-bound than the ACM term premium. This likely owes to the fact that the consensus term premium is based on a model with a time-varying long-run mean of the level factor and as such implies reversion of the short rate to a lower mean at the beginning of the sample. Yet, despite this shared feature with the wealth-weighted term premium, there are notable differences. The consensus term premium is larger than its wealth-weighted counterpart at the beginning and lower at the end of the sample. This is in line with the fact that the top-10 investor initially carries a larger wealth weight but perceives a higher term premium due to higher expected future short rates. At the end of the sample, in turn, the bottom-10 investor features a larger wealth-weight but a higher subjective term premium and thus the wealth-weighted term premium exceeds the consensus.

In summary, our results thus show that the distribution of short-rate forecasts is informative about the pricing of risk in economies with heterogeneous beliefs about the short rate. In the next and final section, we quantify the role of short rate disagreement for time variation in the term premium.

5.3 A Simple Variance Decomposition

In this section, we come back to our initial question: how much of the time variation of term premiums is accounted for by disagreement about future policy rates? We answer this question using a simple variance decomposition. Specifically, we decompose the consensus term premium into a component that is due to variations of the wealth-weighted term premium and a component that is due to deviations of the wealth-weights from the equal weighted consensus term premium. The latter component thus captures the component of the term premium that is directly attributable to time-variation of the wealth weights.

Let TP_t^{cons} denote the consensus term premium and TP_t^A and TP_t^B the subjective term premiums perceived by the top-10 and bottom-10 investor, respectively, Then, we can write:[9]

$$\begin{aligned} TP_t^{cons} &\approx 0.5TP_t^A + 0.5TP_t^B \\ &= (0.5 - w_t^A + w_t^A)TP_t^A + (0.5 - w_t^B + w_t^B)TP_t^B \end{aligned}$$

[9]The first equality only holds approximately for the following reason: While the consensus term premium is a weighted average of all forecasters in the survey and thus not strictly equal to the equal-weighted average of the bottom-10 and top-10 average forecaster that we use here as our measure of consensus, we show in unreported results that the two are almost perfectly correlated in our sample.

$$= \underbrace{(0.5 - w_t^A)TP_t^A + (0.5 - w_t^B)TP_t^B}_{\text{Deviations from Consensus due to Disagreement}} + \underbrace{w_t^A TP_t^A + w_t^B TP_t^B}_{\text{wealth-weighted TP}}$$
$$= TP_t^{\text{Disag}} + TP_t^{\text{ww}}.$$

Hence, we can perform the following simple variance decomposition

$$1 = \frac{Cov(TP_t^{cons}, TP_t^{cons})}{Var(TP_t^{cons})} = \frac{Cov(TP_t^{cons}, TP_t^{\text{Disag}})}{Var(TP_t^{cons})} + \frac{Cov(TP_t^{cons}, TP_t^{\text{ww}})}{Var(TP_t^{cons})}. \tag{17}$$

Table **??** provides the results. The row and first column correspond to our baseline specification with an initial wealth weight of 20% for the bottom-10 investor and a coefficient of relative risk aversion $\gamma = 3$. The remaining columns and rows provide the corresponding variance shares for different initial wealth ratios and different values for γ. We report these as well as these choices arguably affect time variation in wealth weights that our model and portfolio allocation exercise give rise to.

Several remarks appear in order. First, in our baseline specification, the variance share of the consensus term premium that is explained by short rate disagreement is estimated to equal 27 percent. While this represents meaningful variation, it also shows that more than two thirds of the variation in term premiums are not driven by short rate disagreement. This is somewhat at odds with the implications of the model by **?** in which *all* variation in the real term premium is due to time-variation in disagreement about the real short rate. Since their structural model of the bond market with heterogeneous investors does not feature time-varying market prices of risk, we interpret our result as suggesting that a quantitatively more plausible model of the term structure features both, time-varying prices of risk as well as time-varying disagreement about short rates. Second, varying the coefficient of relative risk aversion while keeping the initial 20% wealth weight of the bottom-10 investor constant, does not meaningfully affect this result. The corresponding variance shares explained by the short rate disagreement component vary between 13% and 20%. Third and finally, however, when we increase the initial wealth ratios of the bottom-10 investor to 50% or even 80%, the quantitative role of short rate disagreement substantially declines and even attains negative values for some specifications. These arise as the covariance of the consensus term premium with the wealth-weighted term premium in these cases exceeds the variance of the consensus term premium itself. The reason is that the larger the initial wealth weight of the bottom-10 investor, the lower the overall time variation in that wealth ratio as it is increasing over time due to falling short rates and superior realized returns of the bottom-10 investor's portfolio allocation. Overall, since the nonparametric portfolio weights indicate an initial wealth ratio of the bottom-10 investor of around 20% and since short rates had been relatively high for an extended period before the beginning of our sample, we consider these alternative initial wealth ratios as less plausible.

6 Conclusion

Bond investors disagree about the future path of policy rates, and particularly about their long-run level. Accordingly, they disagree about the risk-return tradeoff of longer-term bonds and engage in speculative trading. This induces shifts in their relative wealth which, in turn, affects the marginal pricing of risk in the economy. Hence, the term premium of an econometrician observing only yields partly reflects disagreement-driven changes in the marginal pricing of risk.

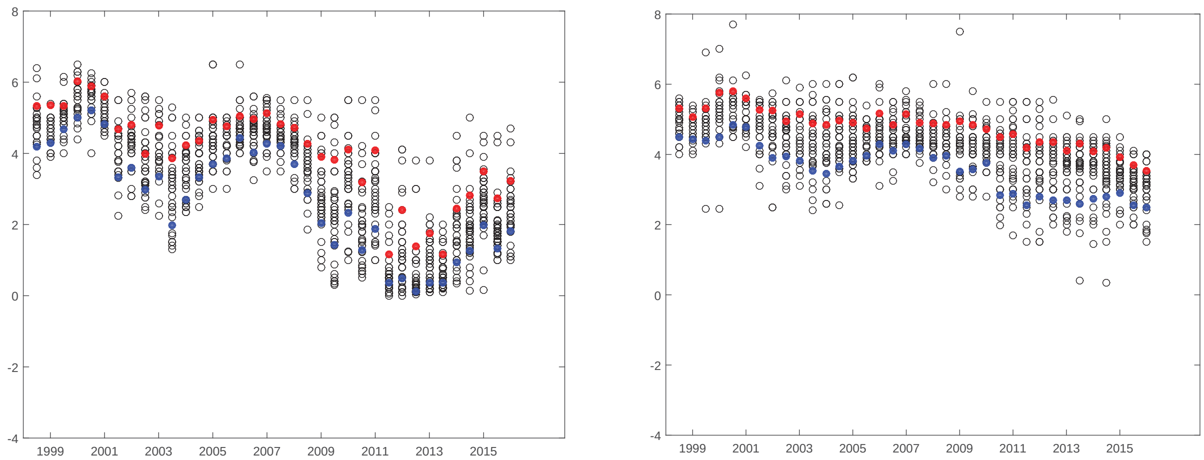
In this paper, we have formalized this intuition endowing investors with different beliefs about the long-run level of yields with an affine term structure with shifting endpoints. Specifically, in the model, investors perfectly observe yields but have different beliefs about the long-run level of rates. We estimate the models for two investors with polar beliefs about future short rates using short rate expectations from the Blue Chip Financial Forecasts (BCFF) survey. We show that the model fits both yields and survey-implied short rate forecasts very precisely across two different investors representing the tails of the distribution of short rate forecasts.

Both investors use the model to predict excess bond returns and form mean-variance optimal bond portfolios. The investor who projected a faster decline in the short rate expected positive and sizable excess returns across maturities, while the one who anticipated a more modest decline expected near zero excess bond returns. With realized yields falling since the early 1980s, this implies that the first investor would have consistently accumulated more wealth. As a result, we show that the wealth-weighted term premium displays essentially no trend, in sharp contrast to term premiums from statistical arbitrage-free affine models of the Treasury market.

We also find that the term premium perceived by the consensus forecaster from the BCFF survey features much less of a trend than the statistical term premium. We attribute this to the fact that the term structure model that we endow investors with features a time-varying long-run mean of the level factor and as such implies reversion of future short rates to a higher mean early in the sample and to a lower mean later in the sample as compared to the affine model with constant parameters. Yet, there are notable differences between the consensus term premium and the wealth-weighted term premium, in line with differences in short rate expectations across the two investors. In a final step of our analysis, we provide a variance decomposition to quantify the importance of short rate disagreement for the consensus term premium. In our baseline specification, policy rate disagreement is estimated to explain somewhat less than a third of the variance of the consensus term premium. This suggests that while disagreement is a quantitatively meaningful driver of term premium variation, a larger share of variation is explained by time-varying market prices of risk.

Figures and Tables

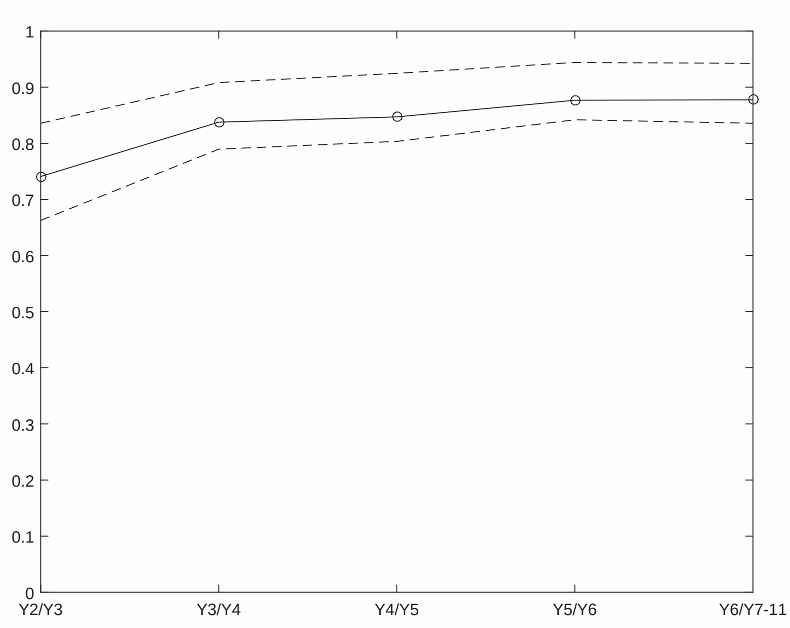
Figure 1: Disagreement about Short Rates at Medium and Long Horizons



Notes:

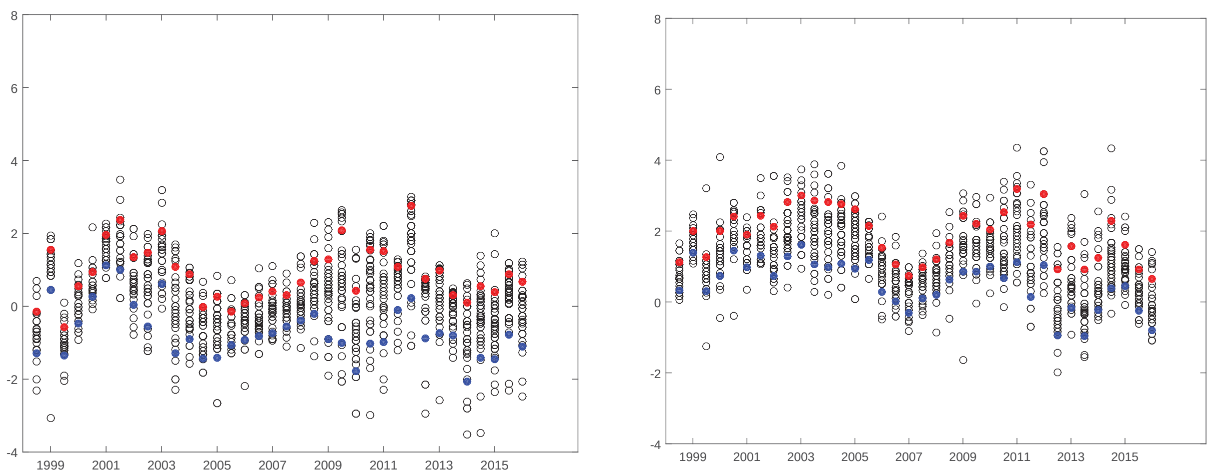
This figure plots individual forecasts from the Blue Chip Economic Indicators survey for the three-month Treasury Bill at forecast horizons of two (left chart) and 7-11 years (right chart) into the future. The red and blue dots represent the top and bottom-10 average responses, respectively. The sample is from 1999-2016.

Figure 2: Consistency of Individual Beliefs about the Short Rate Across Forecast Horizons



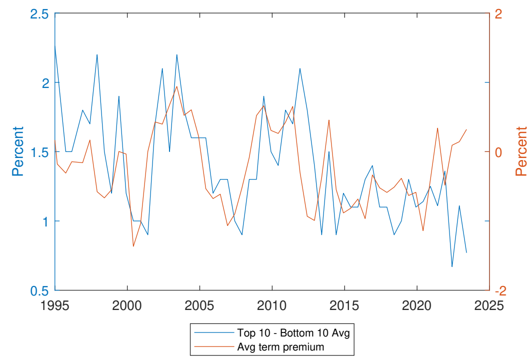
Notes:
This figure plots the rank correlation among individual forecasts from the Blue Chip Economic Indicators (BCEI) survey for the three-month Treasury Bill at adjacent forecast horizons between two and 7-11 years into the future. The dashed lines provide the 5th and 95th pointwise confidence intervals. The sample is from 1999-2016.

Figure 3: Disagreement About Term Premiums at Medium and Long Horizons



Notes:
This figure plots forward term premiums implied by individual forecasts of the three-month Treasury Bill from the Blue Chip Economic Indicators (BCEI) survey at forecast horizons 1-2 and 7-11 years into the future. The red and blue dots represent the top and bottom-10 average responses, respectively. The sample is from 1999-2016.

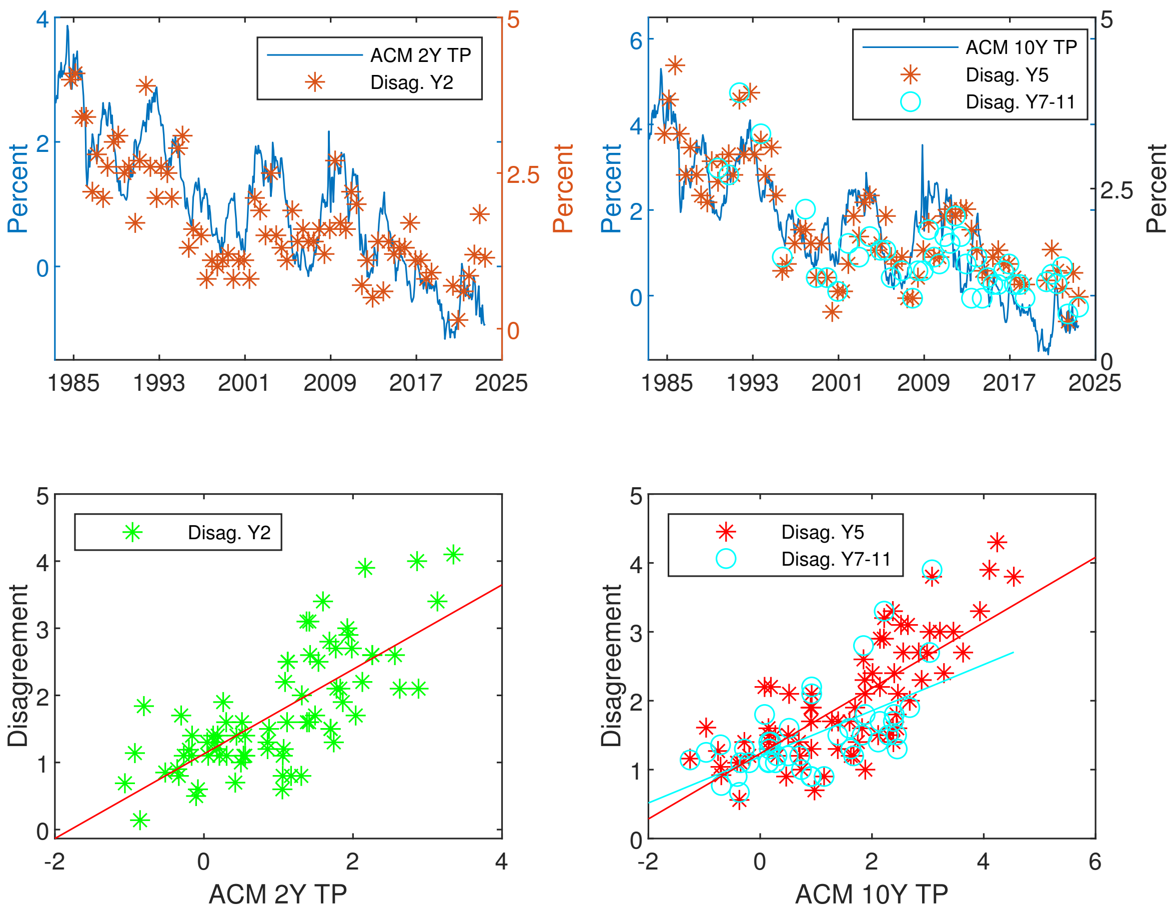
Figure 4: Consensus Term Premium and Disagreement About Short Rates



Notes:

This figure plots the forward term premium implied by the consensus forecaster along with the difference between the top and bottom-10 average forecasts for the three-month Treasury Bill from the Blue Chip Economic Indicators (BCEI) survey at the 7-11 years forecast horizon. The sample is from 1994-2022.

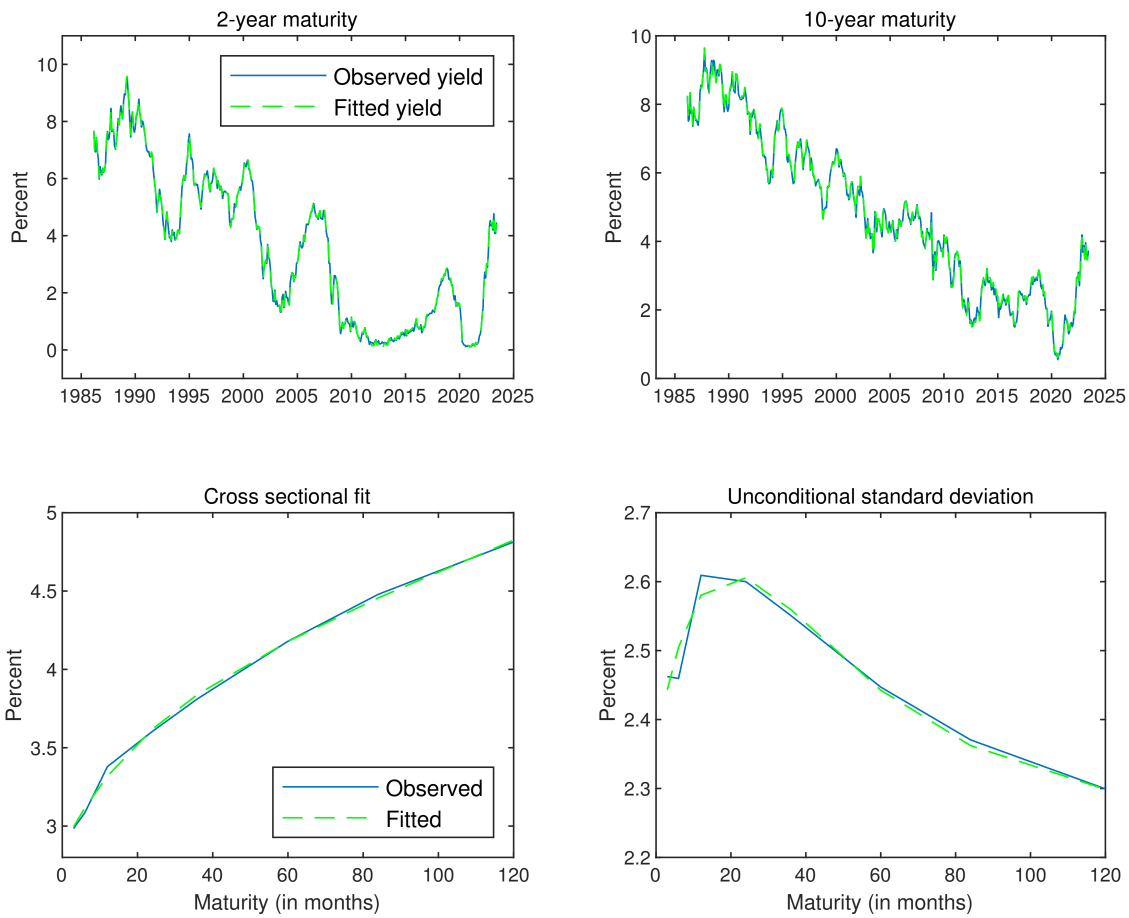
Figure 5: Disagreement About Short Rates and Term Premiums



Notes:

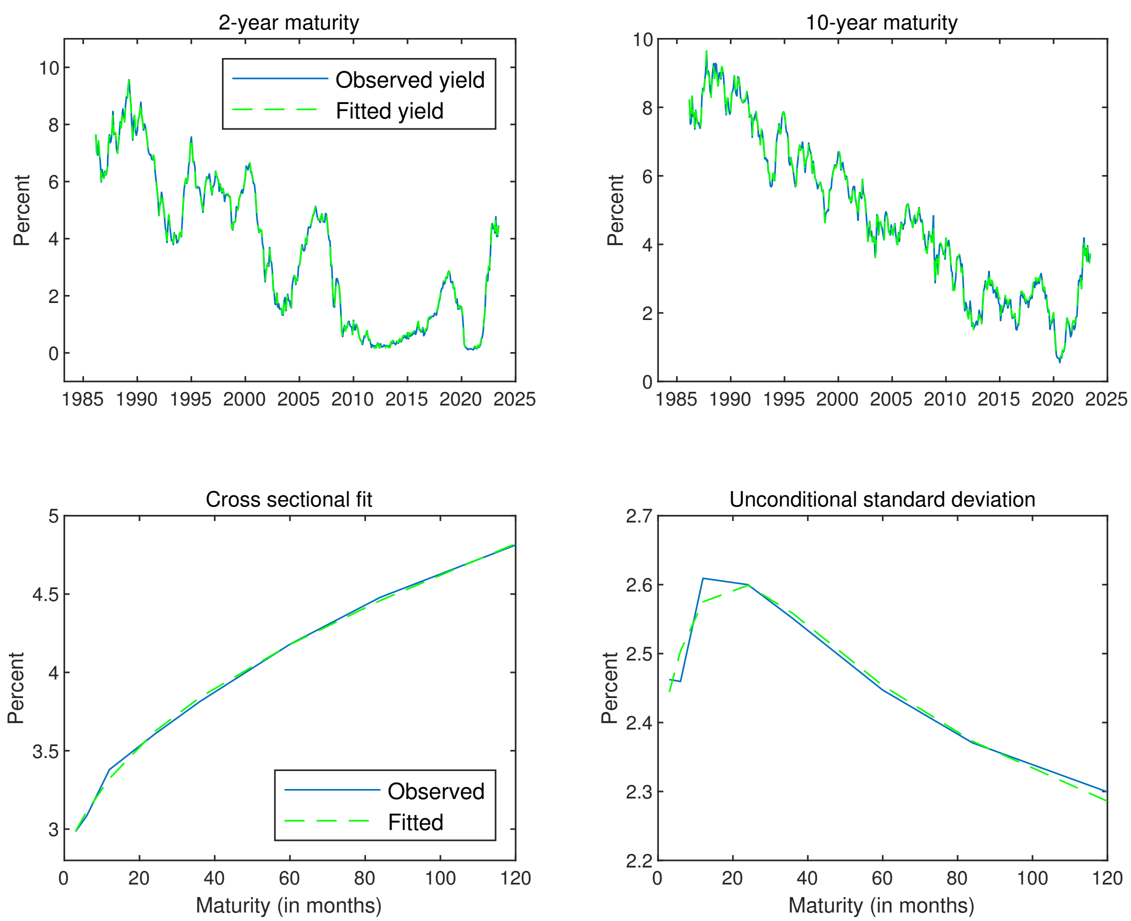
This figure plots disagreement measures calculated using survey forecasts and ACM term premiums obtained from the model described in **?**. The upper two charts display the difference between top-10 and bottom-10 average forecasts of the federal funds rate obtained from the Blue Chip Financial Forecasts (BCFF) survey. One- to four-quarter ahead, five-year ahead and long-horizon (7-11 years) survey forecasts are used. The lower two charts compare two-year and ten-year ACM term premiums with disagreement over similar horizons. Asterisks and circles in the charts are long-horizon forecasts from surveys conducted biannually.

Figure 6: Time-series and Cross-sectional Fit of Yields (Top-10 Model)



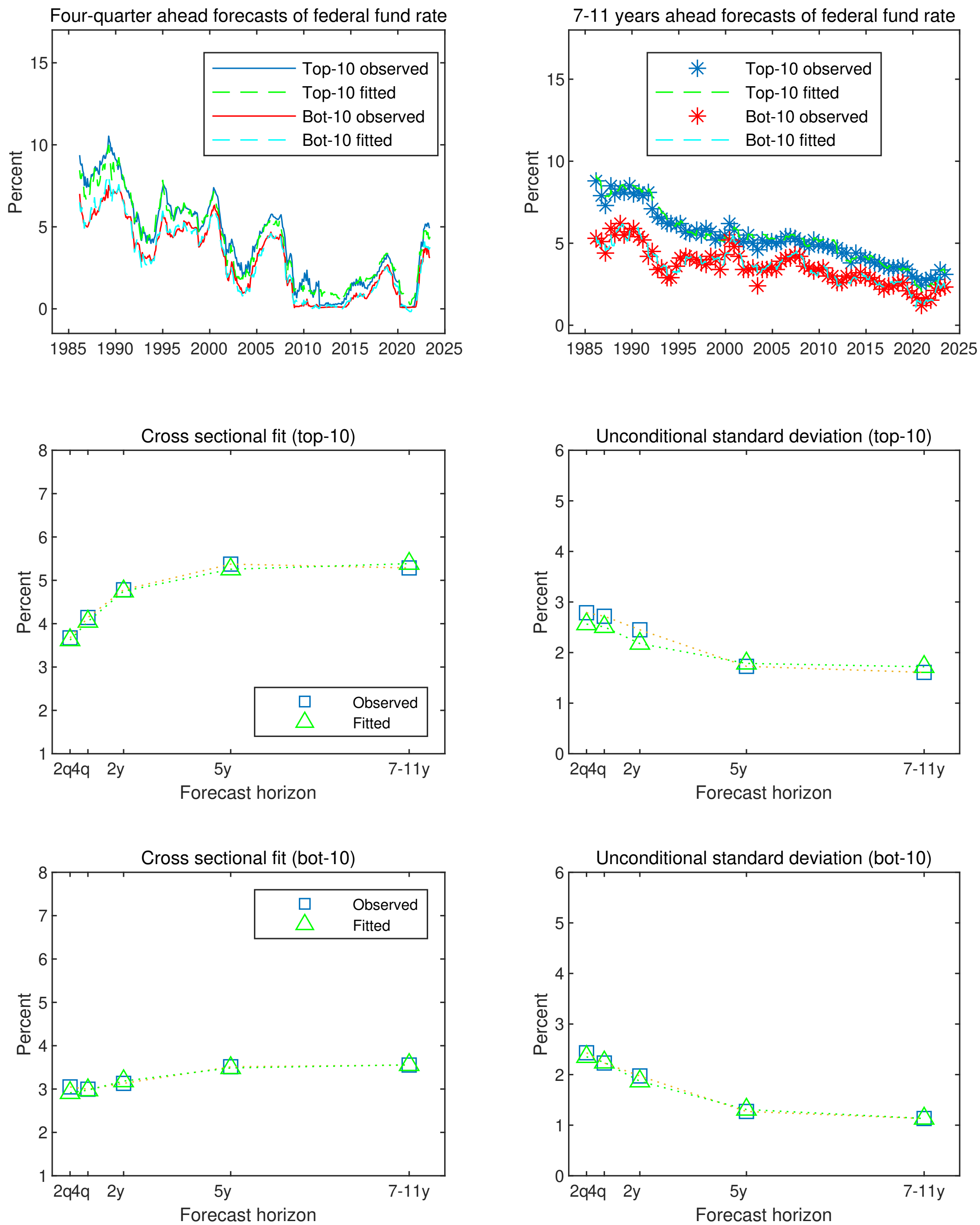
Notes:
This figure provides plots of observed and model-implied yields for the top-10 investor. The upper two panels show time series of observed and model-implied two- and ten-year yields. Observed yields are displayed by solid lines, dashed lines correspond to model-implied yields. The bottom panels plot unconditional averages and standard deviations of observed yields against those implied by the model.

Figure 7: Time-series and Sross-sectional Fit of Yields (Bottom-10 Model)



Notes:
This figure provides plots of observed and model-implied yields for the bottom-10 investor. The upper two panels show time series of observed and model-implied two- and ten-year yields. Observed yields are displayed by solid lines, dashed lines correspond to model-implied yields. The bottom panels plot unconditional averages and standard deviations of observed yields against those implied by the model.

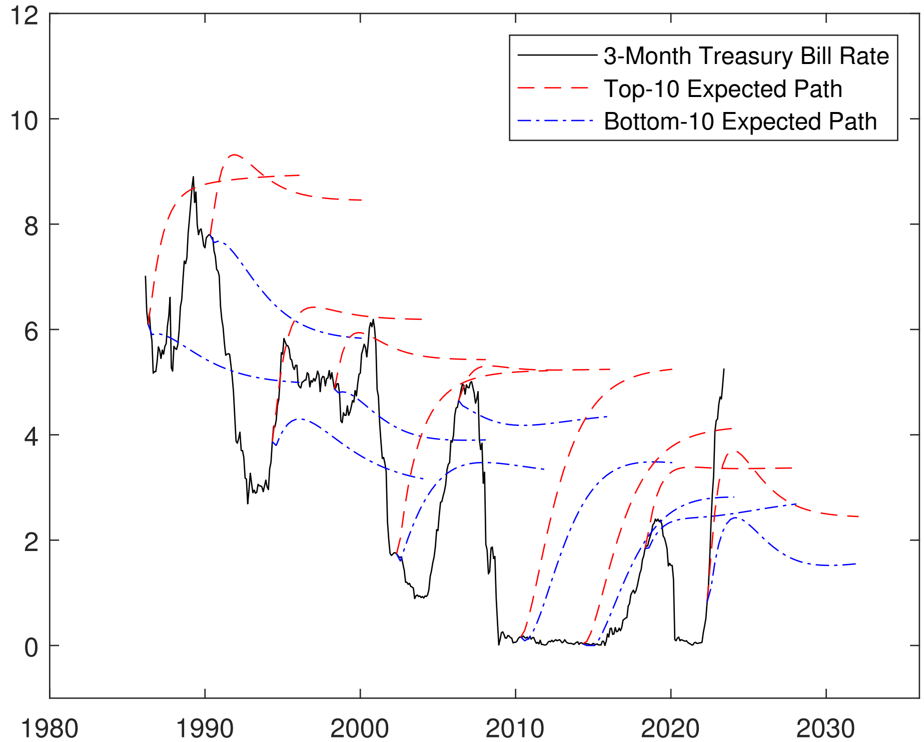
Figure 8: Time-series and Cross-sectional Fit of Survey Forecasts



Notes:

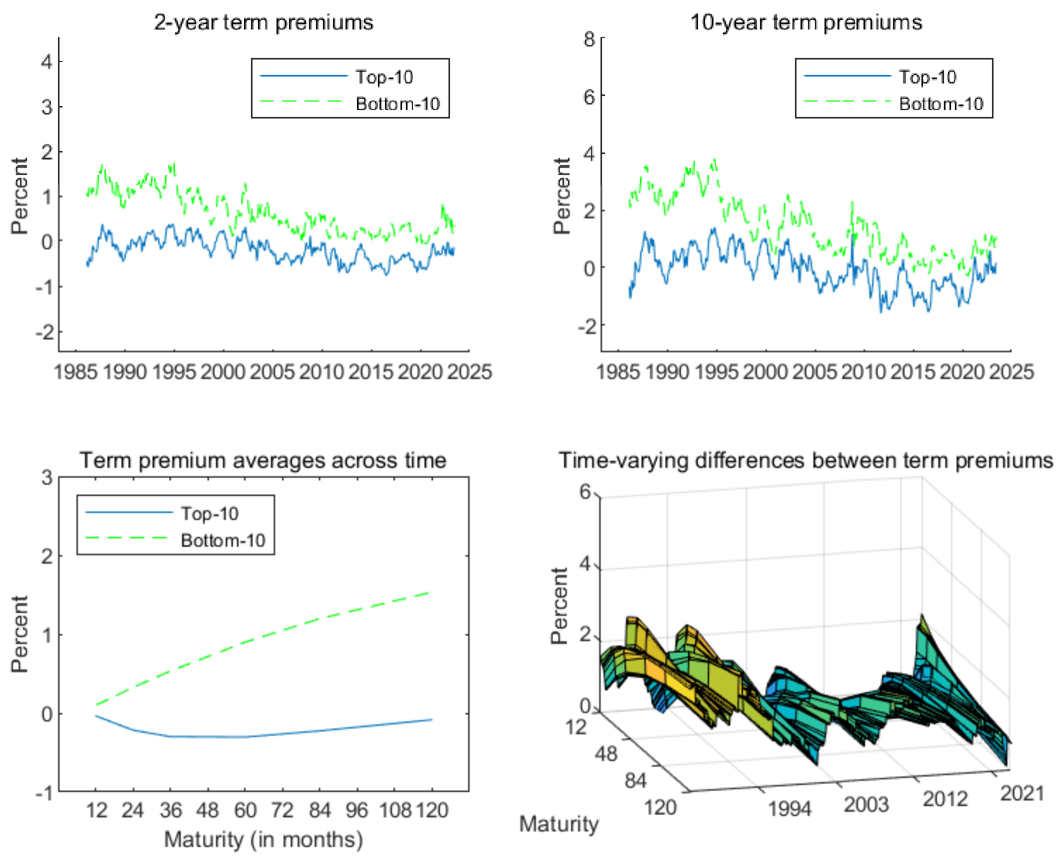
This figure provides plots of observed and model-implied survey forecasts of the fed funds rate. Observed survey forecasts are displayed as solid lines, dashed lines correspond to model-implied survey forecasts. The top two charts show the Blue Chip Financial Forecasts (BCFF) four-quarter ahead and 7-11 years ahead top-10 and bottom-10 average forecasts of the federal funds rate. Asterisks in the top right chart are long-term forecasts which are observed biannually. The bottom four panels plot unconditional means and standard deviations of survey forecasts of the top-10 and bottom-10 average responses (shown as blue and green triangles and squares) against those implied by the model (shown as solid lines).

Figure 9: Expected Short Rate Paths



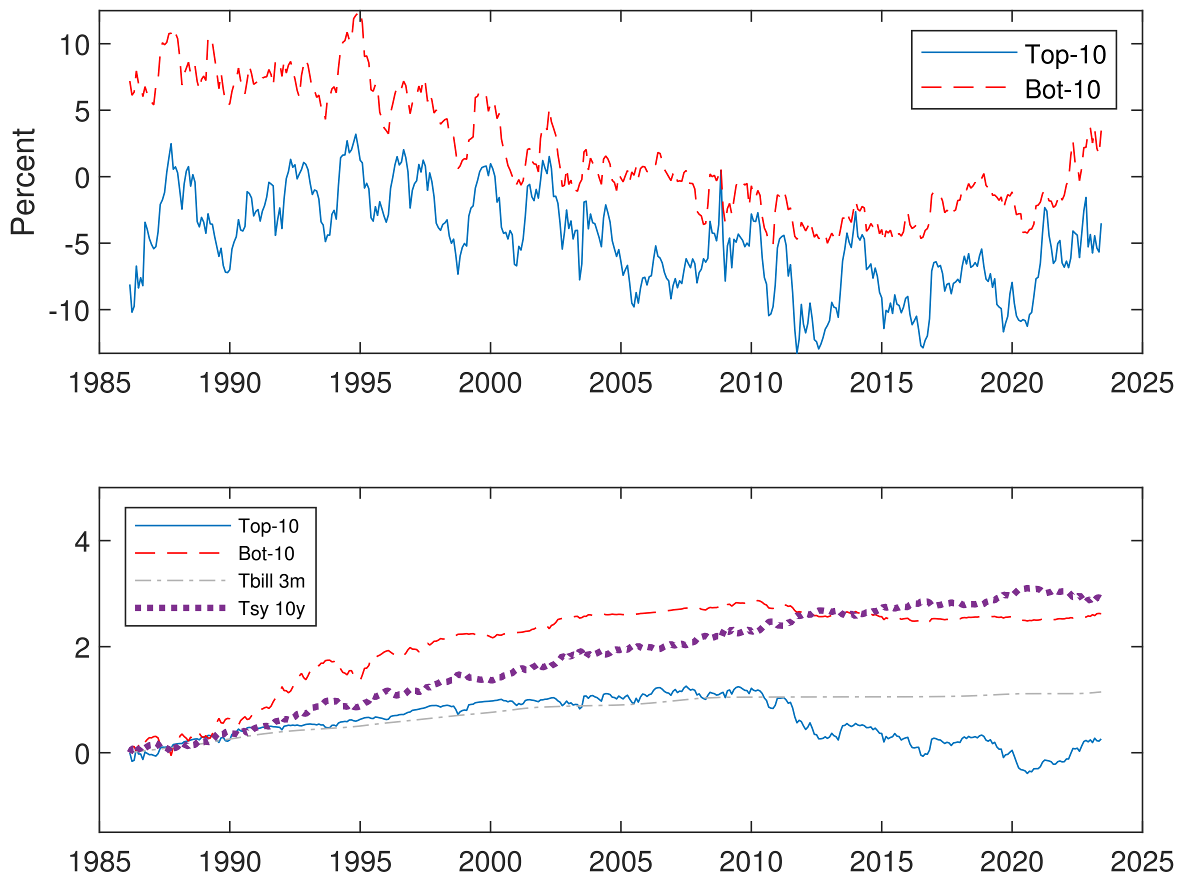
Notes:
This figure plots the realized three-month Treasury Bill along with the expected Treasury Bill paths for the top-10 and bottom-10 investors.

Figure 10: Short Rate Expectations and Term premiums



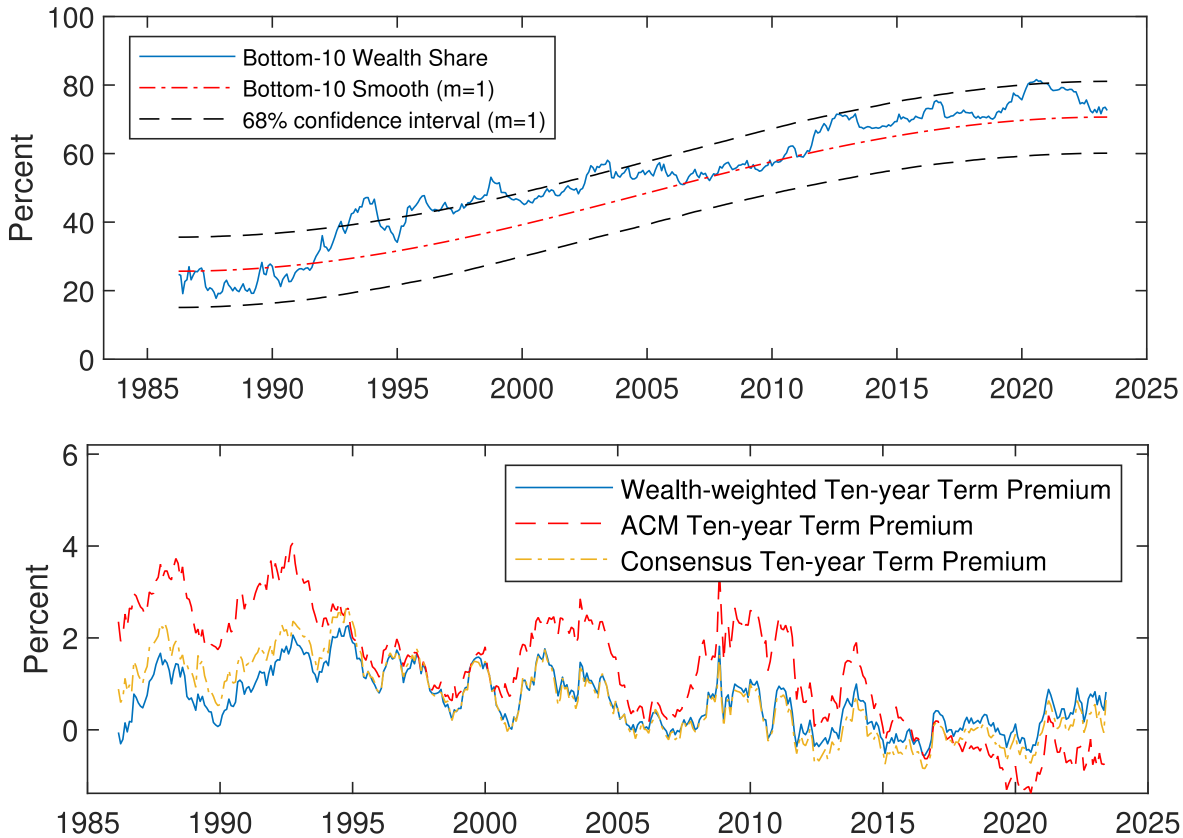
Notes:
This figure plots the term premiums implied by the top-10 and bottom-10 investors' beliefs about future short rates. The upper panels plot the the term premium estimates for two- and ten-year treasury notes. The lower left panel plots the sample averages of term premium estimates in the investors' beliefs for different maturities. The lower right panel displays time-varying differences across maturities between the two investors' beliefs.

Figure 11: Expected Returns and Investment Performance



Notes:
The upper panel of this figure plots the top-10 and bottom-10 investor's expected excess return on the ten-year Treasury. The bottom panel shows the cumulative returns on the mean-variance portfolios chosen by the two investors, along with the cumulative return of rolling over the three-month TBill and that of rolling over the ten-year Treasury.

Figure 12: Wealth-Weighted Term Premium



Notes:
The upper panel of this figure plots the estimated weight of the bottom-10 investor in the economy-wide term premium (red line) with associated 68% confidence intervals (black lines) along with the wealth share (blue line) derived from the cumulative returns for each investor. The bottom panel shows the wealth-weighted ten-year term premium based on a 20% initial wealth share (blue line) along with the term premium of **?** (red line) and the consensus term premium (yellow line) obtained from estimating the model for the consensus forecaster from the BCFF survey.

Table 1: Model Parameter Estimates (Bottom-10)

$k_\infty^{\mathbb{Q}}$	3.53E-04	(1.09E-4)				
$\lambda^{\mathbb{Q}}$	-0.002	(5.34E-4)	-0.037	(2.54E-2)	-0.094	(7.20E-2)
$\text{chol}(\Sigma_\epsilon)$	3.73E-06	(3.64E-6)	-3.29E-03	(2.26E-3)	-3.01E-04	(1.58E-4)
			1.75E-05	(7.87E-6)	-6.85E-03	(1.62E-3)
					5.56E-05	(2.92E-5)
$\mathbf{\Phi}$	0.973	(1.85E-2)	-0.010	(7.35E-3)	-0.005	(2.43E-3)
	0.004	(4.36E-3)	0.962	(1.83E-2)	-0.013	(5.91E-3)
	-0.006	(2.81E-3)	0.012	(7.83E-3)	0.946	(5.09E-2)
$\bar{\mathbf{X}}$			0.016	(2.55E-2)		
σ_η	3.47E-06	(2.47E-6)				
	short		*long*		*y*	
σ_j	4.73E-03	(1.91E-3)	1.18E-03	(6.10E-4)	7.79E-04	(2.02E-4)

Notes:
This table reports parameter estimates for the affine term structure model of bottom-10 investors. The sample period is 1986:02-2023:05, and standard errors are reported in parentheses. σ_y is the standard deviation of bond yield observational errors, while σ_{short} and σ_{long} denote observational error standard deviations of short-horizon forecasts (less than one year) and long-horizon forecasts, respectively.

Table 2: Model Parameter Estimates (Top-10)

$k_\infty^{\mathbb{Q}}$	2.41E-04	(3.69E-5)				
$\lambda^{\mathbb{Q}}$	0.000	(3.06E-6)	-0.042	(1.81E-2)	-0.078	(1.96E-2)
$\text{chol}(\Sigma_\epsilon)$	3.91E-06	(1.26E-6)	-2.08E-03	(7.51E-4)	-4.13E-04	(1.61E-4)
			1.18E-05	(5.34E-6)	-1.00E-02	(3.32E-3)
					1.14E-04	(1.16E-4)
$\mathbf{\Phi}$	0.964	(5.81E-3)	-0.019	(7.49E-3)	0.070	(1.26E-2)
	-0.020	(8.72E-3)	0.954	(3.28E-2)	0.094	(1.64E-2)
	0.089	(7.98E-2)	-0.093	(3.11E-2)	0.915	(3.87E-2)
$\bar{\mathbf{X}}$			0.004	(1.08E-2)		
σ_η	1.06E-06	(3.84E-6)				
	short		*long*		*y*	
σ_j	7.76E-03	(3.79E-3)	6.06E-03	(2.52E-3)	7.54E-04	(5.19E-5)

Notes:
This table reports parameter estimates for the affine term structure model of top-10 investors. The sample period is 1986:02-2023:05, and standard errors are reported in parentheses. σ_y is the standard deviation of bond yield observational errors, while σ_{short} and σ_{long} denote observational error standard deviations of short-horizon forecasts (less than one year) and long-horizon forecasts, respectively.

Table 3: Variance Shares of Policy Rate Disagreement Components in Consensus Term Premium

Variance shares explained by Disagreement

	Initial wealth ratios		
	20-80	50-50	80-20
$\gamma = 3$	**27.53%**	7.27%	-9.67%
$\gamma = 1$	13.86%	-3.21%	-13.59%
$\gamma = 6$	23.05%	7.72%	-8.74%
$\gamma = 10$	20.89%	6.75%	-8.81%
$\gamma = 30$	19.15%	5.20%	-9.36%

Notes:
This table reports variance shares explained by the component of the ten-year consensus term premium driven by disagreement about future policy rates according to Equation (**??**). The columns report these shares for three different initial wealth ratios: 20% initial wealth of the bottom-10 investor and 80% initial wealth by the top-10 investor (labeled "20-80"), 50% initial wealth shares for both investors ("50-50"); and 80% initial wealth of the bottom-10 investor and 20% initial wealth by the top-10 investor ("80-20"). The rows provide estimates based on different levels of the coefficient of relative risk aversion γ. The baseline specification is based on $\gamma = 3$ and a 20% initial wealth share of the bottom-10 investor. The sample period is 1986:02-2023:05.

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Appendix

A Nonparametric Estimation of Time-Varying Weights

In our model we have that

$$rx_{t+1}^{(n-1)} = \mathcal{B}_{n-1}\left(w_t^A \lambda_t^A + w_t^B \lambda_t^B\right) + \mathcal{B}_{n-1} v_{t+1},$$

where we have used that $\lambda_t^R = w_t^A \lambda_t^A + w_t^B \lambda_t^B$ for some weights $w_t^A + w_t^B = 1$. Importantly we have that $\mathbb{E}_t\left[v_{t+1}\right] = 0$. Suppose that we assume that w_t^A is a smooth process over time. Then it can be well approximated by a linear combination of an appropriate set of basis functions:

$$w_t^A \approx \sum_{j=0}^{m} \zeta_j \cdot P_{j,t},$$

where

$$\begin{aligned} P_{0,t} &= 1 \\ P_{i,t} &= \sqrt{2}\cos\left(i\pi\left(t - .5\right)/T\right), \end{aligned}$$

and m is a user-chosen parameter which governs how much smoothness to impose (see, e.g., **?**). For example if $m = 0$ then we would set w_t^A to be constant over time. Next note that,

$$\begin{aligned} & rx_{t+1}^{(n-1)} \\ = \; & \mathcal{B}_{n-1}\lambda_t^B + \mathcal{B}_{n-1} w_t^A \left(\lambda_t^A - \lambda_t^B\right) + \mathcal{B}_{n-1} v_{t+1} \\ = \; & \mathcal{B}_{n-1}\lambda_t^B + \zeta_0 \cdot \mathcal{B}_{n-1}\left(\lambda_t^A - \lambda_t^B\right) P_{0,t} + \cdots + \zeta_m \cdot \mathcal{B}_{n-1}\left(\lambda_t^A - \lambda_t^B\right) P_{m,t} + \mathcal{B}_{n-1} v_{t+1}. \end{aligned}$$

We observe one-period excess returns across time and maturity. From our model outputs we have $\left(\mathcal{B}_{n-1}, \lambda_t^A, \lambda_t^B\right)$. However, we do not observe $\mathcal{B}_{n-1} v_{t+1}$ because this is the realized return error relative to the representative investor. Thus, we can estimate the coefficients on each basis function directly by linear regression. To choose m in practice, we use the BIC,

$$BIC = nT\log\left(\sum\nolimits_{n,t}\left(\xi_t^{(n)}\right)^2\right) + m\log\left(nT\right),$$

where

$$\begin{aligned} & \xi_t^{(n-1)} \\ = \; & rx_{t+1}^{(n-1)} - \hat{c} - \mathcal{B}_{n-1}\lambda_t^B - \\ & \hat{\zeta}_0 \cdot \mathcal{B}_{n-1}\left(\lambda_t^A - \lambda_t^B\right) P_{0,t} - \cdots - \hat{\zeta}_m \cdot \mathcal{B}_{n-1}\left(\lambda_t^A - \lambda_t^B\right) P_{m,t} \end{aligned}$$

and $\hat{c}$ is the estimated constant. The red line in Figure **??** shows the nonparametrically estimated weights using the optimal choice based on BIC ($m = 1$). These model-free estimated weights are well aligned with the mean-variance optimal weights we use in our baseline analysis.

B Normalization Scheme and Parameter Estimates

To estimate the model, we employ the normalization scheme proposed by **?** (henceforth, JSZ). Under the JSZ normalization scheme, we have a risk-neutral parameter set $\Theta^{\mathbb{Q}} \equiv (\Sigma_\epsilon, \lambda^{\mathbb{Q}}, k^{\mathbb{Q}}_\infty)$. Let $\mathbf{X}_t$ denote a set of risk factors with

$$
\begin{aligned}
&r_t = \mathbf{1}'\mathbf{X}_t,\\
&\mathbf{X}_{t+1} = C(k^{\mathbb{Q}}_\infty) + J(\lambda^{\mathbb{Q}})\mathbf{X}_t + \Sigma_\epsilon^{1/2}\epsilon^{\mathbb{Q}}_{t+1}.
\end{aligned}
$$

JSZ show that there exists a unique rotation of $\mathbf{X}_t$ so that the factors are portfolios (or principal components) of bond yields:

$$
\mathcal{P}_t = v(\Theta^{\mathbb{Q}}, W) + L(\lambda^{\mathbb{Q}}, W)\mathbf{X}_t,
$$

where W denoteS weights used to construct factor-mimicking portfolios such that the latent states are portfolios of yields.[10] That is, $\mathcal{P}_t = \mathbf{y}^o_t \cdot W'$. It can be shown that the parameters controlling the risk neutral dynamics (Ψ_0, Ψ_1, δ_0, δ_1) are all functions of the elements in $\Theta^{\mathbb{Q}}$.

The physical dynamics can be written in a similar normalized form

$$
\mathbf{X}_{t+1} = C(\mu_t) + \mathbf{\Phi}\mathbf{X}_t + \Sigma_\epsilon^{1/2}\epsilon_{t+1}.
$$

We identify μ_t as the long-run mean of the short rate, i.e., $r^\infty_t = \mu_t$, consistent with **?**. Since $\mathbf{X}_t = \bar{\mathbf{X}} + e_1\mu_t + \tilde{\mathbf{X}}_t$, we have $r^\infty_t = \mathbf{1}'\mathbf{X}^\infty_t = \mathbf{1}'(\bar{\mathbf{X}} + e_1\mu_t) = \mu_t$, which directly gives $\mathbf{1}'\bar{\mathbf{X}} = 0$.

The JSZ normalization also imposes restrictions on short rate parameters about factor-mimicking portfolios. We define that $\mathcal{P}_t = \bar{\mathcal{P}} + \gamma^{\mathcal{P}}\mu_t + \tilde{\mathcal{P}}_t$, where $\bar{\mathcal{P}}$ is constant and $\tilde{\mathcal{P}}_t$ is the stationary component which equals to zero in the long run. The mapping from $\mathbf{X}_t$ to $\mathcal{P}_t$ gives

$$
\begin{aligned}
\mathcal{P}_t &= \bar{\mathcal{P}} + \gamma^{\mathcal{P}}\mu_t + \tilde{\mathcal{P}}_t\\
&= \big((\Theta^{\mathbb{Q}}, W) + L(\lambda^{\mathbb{Q}}, W)\bar{\mathbf{X}}\big) + L(\lambda^{\mathbb{Q}}, W)e_1\mu_t + L(\lambda^{\mathbb{Q}}, W)\tilde{\mathbf{X}}_t,
\end{aligned}
$$

where $\bar{\mathcal{P}} = v(\Theta^{\mathbb{Q}}, W) + L(\lambda^{\mathbb{Q}}, W)\bar{\mathbf{X}}$, $\gamma^{\mathcal{P}} = L(\lambda^{\mathbb{Q}}, W)e_1$ and $\tilde{\mathcal{P}}_t = L(\lambda^{\mathbb{Q}}, W)\tilde{\mathbf{X}}_t$. Note that short rates are affine in factor-mimicking portfolios, i.e., $r_t = \delta^{\mathcal{P}}_0 + \delta^{\mathcal{P}\prime}_1\mathcal{P}_t$, so we have the expected long-run short rate $r^\infty_t = \delta^{\mathcal{P}}_0 + \delta^{\mathcal{P}\prime}_1\bar{\mathcal{P}} + \delta^{\mathcal{P}\prime}_1\gamma^{\mathcal{P}}\mu_t$, and the identification scheme implies that $\delta^{\mathcal{P}}_0 + \delta^{\mathcal{P}\prime}_1\bar{\mathcal{P}} = 0$ and $\delta^{\mathcal{P}\prime}_1\gamma^{\mathcal{P}} = 1$. These restrictions are the same as those imposed by **?**, which is shown in the following section.

[10] We choose the portfolio weights following **?**. The portfolios can be interpreted as empirical Level, Slope and Curvature.