Subjective Risk Premia in Bond and FX Markets

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Abstract

This paper elicits subjective risk premia from professional forecasters' beliefs about sovereign bond yields and exchange rates. Survey-implied risk premia are (*i*) unconditionally negative for bonds, positive for investment currencies and negative for funding currencies, (*ii*) cyclical and correlated with subjective macro expectations and (*iii*) predict future realised returns with positive sign. Deriving a subjective belief decomposition, we estimate a multi-country asset pricing model with three probability measures: the risk-neutral, physical and subjective measures. Through a structural estimation we quantify the size of financial market belief distortions in terms of a positive bias (optimism) in long-run economic growth, with reasonable values of risk aversion and intertemporal elasticity of substitution, and demonstrate that subjective risk premia can be understood in terms of a classical risk-return trade-off.

JEL classification: D9, E3, E4, G12

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Measuring expected returns is a long standing endeavour in financial economics usually inferred by approximating conditional expectations with projections of realizations onto observable factors. However, projection approaches are problematic for a number of well known reasons.[1] This paper explores an alternative approach. We extract subjective expected excess returns on government bonds and exchange rates from a panel of professional forecasters who are local specialists in their respective domestic markets.

The belief panel is available for the G10 countries at a monthly frequency for the sample period January 1995 - December 2020, and allows us to measure (*i*) model free real-time subjective risk premia, (*ii*) country specific subjective growth rate expectations and (*iii*) expectation errors across both sovereign bond and exchange rate markets. The joint properties of subjective risk premia, subjective macro expectations and expectation errors shed light on the validity of existing asset pricing models, and provide guidance for the design of future models which seek to incorporate deviations from full information rationality.

Studying their empirical properties, we first show that subjective bond risk premia (BRPs) are negative on average but can change sign, being persistently positive. An unconditionally negative BRP is consistent with predictions from many leading equilibrium models. Interestingly, the signs of subjective BRP line up with those predicted by the capital asset pricing model (CAPM). To highlight this point, we compute international stock-bond correlations ($corr_{SB}$) and show that the well-studied change from positive to negative $corr_{SB}$ that occurred in the late 1990s in the U.S. carries over to the cross-section of countries in our sample. This is an intriguing observation since observing $corr_{SB} < 0$ suggests bonds should also be hedges according to CAPM logic. Moreover, in the 1990s when $corr_{SB} > 0$ we do, in fact, observe BRPs which are largely positive. Considering country specific $corr_{SB}$ and BRP, we find a strong positive statistical relationship with a pairwise average correlation of 19%; thus, the average sign of subjective BRPs is consistent with a standard risk based view of asset pricing.

Second, we explore the cyclical properties of *subjective* BRP through a series of panel regressions on *subjective* expected real growth measures, also obtained from surveys, estimating a strong and robust negative relationship. Subjective BRP are therefore counter-cyclical even if innovations

[1]Inferring ex-ante beliefs from projections depends crucially on the researcher's choice of covariates; thus, inherits a form of model mis-specification. Proxying for unconditional expected returns from sample mean realized returns assumes that information surprises cancel out during the sample period studied, which in the context of trending interest rate markets is unlikely (Elton, 1999). In statistical terms, typical samples are too short for asymptotic statistical inference to be taken literally; moreover, in-sample inference often paints a different picture than out-of-sample inference (Nagel and Xu, 2023).

to *realized* asset prices rates and macro quantities are largely unspanned. This is an important result since leading asset pricing models featuring priced long run risks (Bansal and Yaron, 2004), habit preferences (Campbell and Cochrane, 1999), and rare disasters (Wachter, 2013) predict that risk premia vary cyclically with the state (realised or expected) of the macro-economy.

Third, subjective BRPs are significantly positively linked to the realized volatility of bond returns and so survey expectations preserve the basic risk-return relation which predicts a tight link between quantities of risk and compensation for risk. This is an additional important take-away since detecting a link between realized returns and measures of volatility is notoriously difficult (see, for example, Eraker, 2018).

Results for subjective exchange rate risk premia (XRP) are consistent with "carry trade" intuition. The average exchange rate risk premium for the standard "funding currencies" within the carry trade, i.e. Switzerland and Japan, is largely negative, while it is largely positive for "investment currencies" such as Norway and Australia. For the remaining countries the average XRP is closer to zero but it is highly time-varying. Exchange rate risk premia display a large degree of co-movement, with all pairwise correlations being positive and significant, equal to 50% on average. The large cross-sectional correlation between individual countries risk premia, which we document in both fixed income and currency spaces, is consistent with highly globalized markets, in which sources of risk and risk compensations are common across countries. Subjective exchange rate risk premia also display strong cyclical properties shedding light on the long withstanding exchange-rate macro-disconnect puzzle, which grapples with the question "why are exchange rates so volatile yet apparently disconnected from fundamentals?".[2] We provide a potential resolution to this puzzle by showing that while volatile realized innovations are largely uncorrelated, subjective expectations of returns and macro aggregates are strongly related.

Next, we estimate a set of return predictability regressions in order to characterise the informational content of subjective risk premia as a signal about future realized returns. We compare forecasts from subjective expectations to benchmark price based predictors, considering interest rate differentials and slopes of the yield curve, for the exchange rate and bond realized returns, respectively. Summarising, we show that ex-ante beliefs about returns significantly forecast realized

[2]This puzzle has been studied in various guises beginning from the seminal works of Meese and Rogoff (1983) in the context of a forecasting puzzle. Rogoff and Stavrakeva (2008) provide a survey of the literature. Related to our paper, Stavrakeva and Tang (2023) provide a complimentary analysis on the disconnect puzzle by exploiting subjective surprises obtained by combining subjective expectations and realized quantities.

returns, in both economic and statistical terms, and with the correct sign according to rational expectations. At the same time, survey expectations are "unspanned" by interest rate spreads which also significantly predict future excess returns. This implies that, while positively correlated with future realized returns, survey implied forecast errors are predictable by date t information, suggesting that beliefs are not fully rational.

Studying the properties of expectation errors more deeply we show that, for all countries in our sample, professional survey forecasters over-predicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante.[3] For exchange rates, expectation errors are mean zero, i.e, there is no systematic bias. Interestingly, forecast errors are highly correlated across countries, and while errors display only mild persistence sampled at annual horizons, this translates into predictability by interest rate spreads.

Finally, we study a subjective multi-country equilibrium model that incorporates the empirical observations that we document. Our approach deviates from full information rationality by introducing a distortion between the subjective beliefs of the representative agent and the physical measure inferred by an econometrician, implemented via a structural estimation, and reveals broad insights on subjective asset pricing.

We consider an equilibrium model with (distorted) long run risks, recursive preferences and constant volatility, and study unconditional moments of risk premia, interest rates and forecast errors. For ease of interpretation we begin by assuming a belief distortion which is constant. Moreover, we assume complete markets so that the risk neutral measure is unique. In this case, it turns out that belief distortions are equivalent to the difference between the market prices of risk under subjective and physical measures.

In estimation we consider a 3-country world economy consisting of Australia (AUD), Switzerland (CHF) and the U.S. and proceed via simulated methods of moments in a sequence of steps. First, we estimate the parameters in the consumption process separately for each country and match the properties of the realized consumption processes with a very persistent long-run risk factor. Second, we estimate the preference and distortion parameters for all countries jointly targeting the term structures, subjective bond risk premia and yield forecast errors. The model can capture well the level of the yield curves in the three countries, the average negative subjective

[3] Buraschi and Whelan (2016) and Hillenbrand (2021) show that Fed Funds futures market was also "surprised" by the secular decline in interest rates by computing shocks to interest rate around FOMC meetings.

bond risk premia and the positive yield forecast errors with a relative risk aversion of 4.0, an elasticity of intertemporal substitution of 1.3, implying a preference for early resolution, and a belief distortion to risk prices that implies subjective optimism. Indeed, mapping the belief distortion in terms of risk prices into a belief distortion about consumption prospects, we find subjective beliefs about long term consumption growth are approximately 1.5% larger than under the physical measure, and this bias is similar across countries.

In a final step, we calibrate the correlations between shocks in the US and in the foreign countries by targeting the subjective exchange rate risk premia, while taking all other parameters as given from the previous steps. Interestingly, the estimated model implies that XRP for AUD is larger than the one for CHF, as in the data, for all values of the correlation between long run risk shocks, even though we do not use any information about the exchange rate dynamics or premia in the estimation, and we can match XRPs exactly by setting correlations between foreign and U.S. latent shocks equal to around 0.3 for CHF and 0.9 for AUD. Similarly, average forecast errors on log exchange rates are consistent with the observed ones, even though we have not used information from exchange rates forecast errors in the estimation. In particular, the forecast errors on exchange rates are close to zero, as they are a function of the difference in belief distortions in the foreign and domestic countries, which tend to be similar.

While the baseline version of the model is able to match unconditional moments, it obviously cannot match the time series properties of the subjective risk premia as model-implied risk premia are constant. Therefore, we also consider an extension with stochastic volatility in order to rationalize the time series dynamics of subjective risk premia and the empirically observed cyclicality and predictability properties. Introducing time-varying economic uncertainty rationalises the cyclical properties of subjective risk premia by allowing for a positive correlation between the latent volatility and expected growth factors. Consistent with the data, in the stochastic volatility version of the model subjective bond risk premia can also positively predict future excess returns as long as the distortion between physical and subjective measure is, importantly, not too large.

I. Relationship to the Literature

Piazzesi, Salomao, and Schneider (2015) are probably the first to construct subjective (bond) risk premia from surveys. These authors point out that while statistical measures of bond risk premia are volatile and countercyclical, subjective premia are far less volatile and not “that” cyclical. The cyclical properties of subjective risk premia takes centre stage in Greenwood and Schleifer (2014) who show that individual investors' stock return expectations display evidence of belief extrapolation and therefore pro-cyclicality rather than counter-cyclicality. More recently, Nagel and Xu (2023) analyse survey-based risk premia across a range of asset classes and forecaster types. These authors argue subjective risk premia are, in fact, a-cyclical which poses a dilemma for rational expectations representative investor asset pricing models, whose basic mechanism generates risk premia via countercyclical variation in perceived risks or effective risk aversion.

While expectations of retail investors are often found to be extrapolative and pro-cyclical, recent studies using survey expectations of professional forecasters and asset managers' capital market assumptions show evidence of counter-cyclicality, see e.g. Wu (2018), Møller, Pedersen, and Steffensen (2020), Renxuan (2021), Dahlquist and Ibert (2024), Couts, S Gonçalves, and Loudis (2023) and Gandhi, Gormsen, and Lazarus (2023). For a recent survey article on the state of this literature see Adam and Nagel (2023). Our paper contributes to this literature by focusing on a cross-section of sovereign bond and currency markets, and demonstrates that subjective risk premia implied by professional forecasters' expectations are indeed cyclical, if one considers measures of *subjective* macro growth rates also extracted from surveys. On the exchange rate side, our findings compliment those of Kremens, Martin, and Varela (2023) who study long horizon survey expectations of exchange rates and show they are strongly correlated with ex-post movements and related to a set of macro-variables that make subjective FX beliefs interpretable.

Deriving a structural subjective belief decomposition, we estimate an equilibrium model with subjective beliefs, risk neutral beliefs, and ex-post objective beliefs that allows us to map forecast errors on asset prices into forecast errors on long run macroeconomic quantities. Thus, our paper relates to a growing literature that seeks to embeds subjective beliefs and belief distortions in equilibrium models (for recent examples, see Maenhout, Vedolin, and Xing, 2023 and Bhandari, Borovička, and Ho, 2022, and Bianchi, Ludvigson, and Ma, 2022). To the best of our knowledge, Chernov and Mueller (2012) is the only other paper to exploit information in risk-neutral, subjective, and physical probability measures in estimation.

II. Data

Survey Data: From Consensus Economics (CE) we collect professional financial market participants monthly forecasts of (*i*) spot exchange rates; and (*ii*) yields on 10 year government bonds for a variety of countries.[4] We focus on the most heavily-traded G10 currencies vis-a-vis the United States (USD): Australia (AUD), Canada (CAD), Switzerland (CHF), Europe (EUR), United Kingdom (GBP), Japan (JPY), New Zealand (NZD), Norway (NOK) and Sweden (SEK). CE reports projections for two horizons, 3 and 12 months, for both exchange rate and interest rate expectations. We focus on the 12-month forecasts which is the horizon where predictable variation in risk premia is most likely to arise. Forecasts begin in (*i*) 1990 for the USD, CAD, EUR, GBP, JPY; (*ii*) in 1995 for AUD, NZD and SEK; and (*iii*) in 1998 for NOK and CHF. All results in the paper, unless otherwise stated, are based on the period from January 1995 to December 2020, for a total of 312 monthly observations for all countries except NOK and CHF. In addition to forecasts of future asset prices CE covers projections for a large set of macroeconomic variables. We focus on 1-year ahead real personal consumption growth and GDP growth, which are available for all G10 countries.

CE has maintained a consistent questioning procedure over time and survey respondents face the same questions for each country. Forecasters receive the questionnaire in the first few days of the month, and survey forecasts are collected the second week of every month on Monday and then released by CE three days after on the Thursday of the same week. We sample all yields, spot rates and exchange rates on the date when the survey goes public, i.e. the release date, that is normally around the middle of the month, in order to avoid any look-ahead bias. Moreover, the survey focuses on experts for each region, with respondents generally located in the country for which they are asked to make predictions. Thus, the dataset is comparable across a large cross-section of countries and is available at monthly frequency for an extended sample period. Section A.1 in the Online Appendix (OA) provides additional details of the CE dataset.

Realized Data: We obtain monthly G10 FX spot and 3-month forwards from Refinitiv Eikon for the sample period January 1995 to December 2020. For the same panel, we obtain zero coupon bond yields, which are generally available for maturities 3, 6, 9 and 12-months, and 3, 5, 10 and

[4] CE panellists provide par yield forecast which we treat as zero coupon forecasts. Section A.1 in the Online Appendix (OA) discusses the approximation error.

15 years from Bloomberg. Below we require country-specific yields for three bond maturities: the 12-month rate (risk-free), the 10-year rate, and the 11-year rate. While Bloomberg provides the former two series, it does not provide us with the latter. To remedy this, we fit a cubic spline to all available maturities and sample the desired yield. Realized second moments of bond and FX returns are measured at daily frequency by computing realized volatility from the sum of squared log returns between subsequent survey release dates, which are approximately $n = 22$ days apart. All volatility estimates are annualised. In addition, in foreign exchange markets, we exploit option-implied risk neutral variances constructed and discussed by Krohn, Mueller, and Whelan (2024).

We also use realized quarterly observations on real person consumption and consumer price inflation. These time-series are available for all G10 currencies for our sample period. Time-series originate from various sources but can be obtained directly from the "Trading Economics" archive.[5]

III. Framework and Notation

The price of a k-period bond satisfies the first order condition of a representative investor who forms her beliefs under a subjective measure ($\mathbb{S}$). This measure does not necessarily coincide with the objective measure ($\mathbb{S} \neq \mathbb{P}$), in which case the following decomposition holds

$$P_t^{(k)} = E_t^{\mathbb{S}}\left[M_{t+1}P_{t+1}^{(k-1)}\right] \tag{1}$$

$$= \underbrace{\frac{1}{R_t^f}E_t^{\mathbb{P}}[P_{t+1}^{(k-1)}]}_{\text{objective NPV}} + \underbrace{Cov_t^{\mathbb{S}}\left[M_{t+1}, P_{t+1}^{(k-1)}\right]}_{\text{subjective RP}} + \underbrace{\frac{1}{R_t^f}\left[E_t^{\mathbb{S}}[P_{t+1}^{(k-1)}] - E_t^{\mathbb{P}}[P_{t+1}^{(k-1)}]\right]}_{\text{forecast errors}}. \tag{2}$$

where M_{t+1} is the one period stochastic discount factor. The final line shows that subjective risk premia (studied in section IV) and forecast errors (studied in section VI) are flip sides of the same coin. In this section, we introduce notation and formulas used to compute subjective risk premia and later we study forecast errors.[6]

[5] www.tradingeconomics.com.

[6] The literature studying return predictability typically interprets expected excess log returns as risk premia. This is not quite correct since risk premia should be measured as expected excess simple returns. Assuming log-normality, arithmetic average returns differs from the geometric average returns by one half the variance (the Jensen's gap). Computing Jensen's terms from BlueChip Financial Forecasts, Buraschi, Piatti, and Whelan (2022) show that the Jensen's gap is tight using

A. Risk premia in the fixed income market

Let $P_t^{(k)}$ be the time t price of a default risk-free zero-coupon bond of maturity k years. Spot k-year yields are then defined as $i_t^{(k)} = -\frac{\ln P_t^{(k)}}{k}$. Subjective bond risk premia on a k-year bond are defined as 1-year holding period subjectively expected excess returns given by

$$BRP_t^k \equiv E_t^{\mathbb{S}}\left[rx_{t+1}^{(k)}\right] = -(k-1)E_t^{\mathbb{S}}\left[i_{t+1}^{(k-1)}\right] + ki_t^{(k)} - i_t^1, \tag{3}$$

where continuously compounded yields are annualised and k is expressed in years. Note that under the expectation hypothesis (EH), $E_t^{\mathbb{S}}\left[i_{t+1}^{(k-1)}\right] = f_t^{(1,k)}$, where $f_t^{(1,k)} = \frac{ki_t^{(k)} - i_t^1}{k-1}$ is the forward rate for k periods starting 1-year from now, so that the risk premium for investing in long-term bonds is zero, i.e., $BRP_t^k = 0$.

Consensus Economics surveys provide us with the expected yields on a 10-year bonds for a 1-year forecast horizon. In what follows, we compute subjective bond risk premia as the expected change in 11-year log bond prices above the 1-year risk free rate.

B. Risk premia in the foreign exchange market

Denote by x_t the log of the exchange rate, expressed in US Dollars per unit of foreign currency, and Δx_{t+1} the 1-year change in the log exchange rate. A positive Δx_{t+1} corresponds to a depreciation of the US Dollar relative to the foreign currency. Denoting the 1-year interest rate in the foreign country as $i_t^{1,f}$, the annualised log 1-year currency excess return is given by $rx_{t+1}^{FX} = (i_t^{1,f} - i_t^1) + \Delta x_{t+1}$. The subjective exchange rate risk premium is defined as the conditional subjective expectation of this object i.e.,

$$XRP_t \equiv E_t^{\mathbb{S}}\left[rx_{t+1}^{FX}\right] = (i_t^{1,f} - i_t^1) + \left(E_t^{\mathbb{S}}\left[x_{t+1}\right] - x_t\right), \tag{4}$$

According to uncovered interest rate parity (UIP), high interest rate countries are expected to experience an exchange rate depreciation to equalise expected exchange rate adjusted returns on assets. The idea behind UIP is that when the foreign interest rate is higher than the local interest rate, the foreign currency will depreciate by the difference so that in local currency terms the

survey expectations on interest rates. Kyle and Todorov (2023) show that for currencies, the difference is very small and that there are no higher-order risk premiums above the one on variance. In what follows we interpret expected log excess returns as risk premia.

return on investing in the two countries has a zero expected excess return, i.e., $XRP_t = 0$.

IV. Subjective Risk Premia

A. Subjective Bond Risk Premia

Figure 1 shows the time series of survey-based BRP_t and panel (a) of Table I reports summary statistics. Subjective bond risk premia are slightly negative on average, volatile and persistent. For example, the GBP BRP_t ranges between -8% around the year 2000 to +10% in the aftermath of the 2008 financial crisis. The JPY BRP_t is less volatile but still displays significant persistence ranging between -3% pre financial crisis to +7% post financial crisis.

[INSERT FIGURE 1, 2 AND TABLE I HERE]

A negative BRP_t implies that default-free bonds are perceived as hedges, consistent with predictions from many structural models. Re-writing the definition for BRP_t above we have the following decomposition

$$BRP_t = 10 \times \left(i_t^{(11)} - E_t^{\mathbb{S}} \left[i_{t+1}^{(10)} \right] \right) + \underbrace{\left(i_t^{(11)} - i_t^{(1)} \right)}_{\text{Term Spread}}. \tag{5}$$

On average, the second term is positive since the (nominal) term spread is typically positive; thus a negative BRP_t means that the first term in parenthesis is negative. Assume date t interest rate beliefs can be written as a random walk forecast plus an adjustment ϕ_t. The consensus belief can then be written as $E_t^{\mathbb{S}} \left[i_{t+1}^{(10)} \right] = i_t^{(10)} + \phi_t$. Then, since empirically $i_t^{(11)} - i_t^{(10)} \sim 0$ this implies $\phi_t > 0$, i.e. conditional on the current level of the term structure, surveys expected long-term rates to rise. Actually, in our sample there was an unprecedented decline in long term rates and survey forecasters made consistently biased forecasts. This point highlights the link between subjective risk premia and expectational errors, which we study in detail in sections V and VI.

Do forecasters believe bonds are genuinely hedges or is this an artefact of the sample period? Consider again Figure 1. In both first half and second half of the sample (when interest rates were relatively flat) we observe that $BRP_t < 0$ most of the time. Table A.2 in the OA confirms this point more formally by providing subsample statistics. Investigating this question further,

Figure 2 displays stock-bond correlations ($corr_{SB}$) computed as the rolling 200-day correlation between 10-year zero coupon bond returns and the corresponding major equity market indices in each country. The figure demonstrates that the well studied change from positive to negative $corr_{SB}$ that first occurred in the late 1990's in the U.S. is a common feature in the cross-section of G10 countries in our sample. This is an interesting observation since a negative $corr_{SB}$ suggests bonds should also be hedges according to a simple CAPM logic. Moreover, in the 1990s when $corr_{SB} > 0$ we do, in fact, observe BRPs which are largely positive. Computing the average pairwise correlation between $corr_{SB}$ and BRP, we obtain a value of 19%; thus, the average sign and dynamics of subjective BRPs are consistent with a risk based view of asset pricing.

Another notable feature of subjective bond risk premia is their co-movement. Figure 1 displays a clear systematic pattern across countries, which is confirmed by a very high average cross-country correlation equal to 53%. All pairwise correlations are positive ranging from 19% between GBP and NOK, to 76% between EUR and AUD (Table A.4 in the OA). This is an interesting result since it implies that beliefs about future sovereign yields have a strong factor structure. The international finance literature often interprets factor structures as being driven by a combination of global and local shocks - we design and estimate an equilibrium model with this feature in section VII below.

In the time-series expected excess bond returns appear counter-cyclical, with risk premia being high in bad states such as the early 2000s and during the 2008 financial crisis, while in more recent years the average risk premia are lower, mainly negative and less volatile. We study the cyclical properties of subjective bond risk premia formally by estimating pooled OLS regressions of *subjective* BRPs on *subjective* expectations of 1-year growth rates in real consumption and GDP whose dynamics are displayed in Figure 3. Panel (a) of Table II displays findings. A constant is included in the regression but is not reported and we report 95% confidence intervals estimated using a circular block bootstrap with 1000 replications.[7]

[INSERT FIGURE 3 AND TABLE II HERE]

Considering specification (i) we find that lower expected real growth rates are strongly signifi-

[7] The circular block bootstrap takes into account potential heteroskedasticity and autocorrelation. We use a block size of 12 observations, which is in the range of optimal block sizes for each of the countries' bond and exchange rate risk premia. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.6 of the OA. In a panel setting, $N \times T$ autocorrelated and spatial dependent observations contain less information than $N \times T$ independent observations. Driscoll and Kraay (1998) propose a nonparametric covariance matrix estimator that generates consistent standard errors which are robust to heteroskedasticity and general forms of temporal and spatial dependence.

cantly linked to higher subjective bond risk premia with an $R^2 \approx 4\%$, while specification (*ii*) shows a similar relationship with expected GDP growth. Exploring the cyclical properties of subjective risk premia further, we estimate a regression of BPR_t on realized bond return variances ($\widehat{\sigma}_t^2$) and realized excess bond returns measured over the past year, in specifications (*iii*) and (*iv*), respectively. Specification (*iii*) shows a highly significant and positive link between subjective bond risk premia and the realized variance of bond returns, which is a simple proxy for the current quantity of risk in bond markets. Since quantities of risk are high in bad times this also demonstrates a link between survey implied risk premia and the state of the macro economy. Specification (*iv*) also shows a significant relationship to realized excess bond returns, which answers the question of whether subjective beliefs are extrapolative or not. The 95% confidence intervals do not display evidence of extrapolation. In fact, we find the opposite: past bond returns and consensus expectations of future returns are negatively correlated, consistent with predictions from benchmark asset pricing models with rational pricing of risk. Indeed, rational asset pricing models generally predicts that low prices due to discount rate variation generate high expected returns going forward.

B. Subjective Exchange Rate Risk Premia

Figure 4 displays the dynamics of the XRPs and panel (b) of table I reports summary statistics. They are time-varying and volatile relative to their mean, with standard deviations ranging between 2.5% (CAD) and 5.6% (NZD). The average XRP_t is negative for Switzerland and Japan, equal to -1.7% and -2.8%, consistent with the idea that these are "funding currencies" within the carry trade, and positive around 3.5% for Norway and Sweden. For the remaining countries, average exchange rate risk premia are smaller ($< 1.8\%$) but positive. All pairwise correlations between exchange rate risk premia are positive and their average is 50% (Table A.4 in the OA). The XRP_t of Japan is less correlated to the remaining countries' XRP_t, in fact excluding Japan from the sample of countries the average pairwise correlation increases to around 63%. Interestingly, XPRs flip sign in a systematic fashion throughout the sample, being largely positive between 1995 – 2004 and oscillate between positive and negative values thereafter.

[INSERT FIGURE 4 HERE]

We investigate the cyclicality of subjective exchange risk premia via pooled OLS regressions of

XRP_t on *subjective* macro expectations. From the perspective of a U.S. investor who sells USD and buys foreign currencies, from a risk-sharing perspective, one should expect exchange rate risk premia to be correlated with both foreign growth and foreign growth relative to domestic growth.

Panel (b) of Table II shows the results of a panel regression of XRPs on subjective foreign country macro expectations. Specifications (i) and (ii) show that positive shocks to foreign country consumption and GDP growth rate expectations are associated with an increase in XRP_t, thus demonstrating that subjective expectations of business cycle dynamics are clearly linked to subjective exchange rate risk premia. Table A.7 in the OA shows that differences in subjective macro expectations defined as the foreign expected growth rate minus the U.S. expected growth rate are also, intuitively, associated with an increase in XRP_t. In bad times, the US Dollar is expected to appreciate due to its perceived safe-haven role, leading to a decrease in the expected exchange rate (expressed as US Dollars per foreign currency) and therefore a lower XRP.

Panel (b) of Table II also shows a positive and significant link between XRP_t and risk neutral return variance implied by FX options.[8] Finally, XRPs display evidence of mean-reversion around past realized returns, which contrasts with extrapolative beliefs that has been proposed as an explanation for return predictability by the behavioural finance literature.

C. Comparison with statistical risk premia

We compare the dynamics of our survey-based bond risk premia (BRP) and exchange rate risk premia (XRP) with standard statistical models. Projection-based bond risk premia are obtained by regressing realized excess returns at time $t + 1$ (one year holding period) on the slope of each yield curve at time t, defined as the spread between the 10-year and the 1-year yield. For exchange rate risk premia, we use the forecasts implied by 1-year interest rate differentials.[9]

Figure 5 displays time series of equally-weighted averages for each country's subjective risk premia against the same time series obtained with statistical models. The dynamics of the statistical model based premia are clearly different from survey-based premia. In fact, their correlation is slightly negative. Bond risk premia using statistical models are uniformly positive and do not

[8] We also find a positive and significant link to realized exchange rate variance although the statistical significance is smaller.

[9] Statistical forecasts are computed in sample so are subject to a look-ahead bias, contrary to the survey-based forecasts which are real-time. However, we do not attempt to compute projection-based estimates in real time, i.e. out-of-sample, as our goal here is just to compare the time series dynamics of the survey and statistical-based foreign bond risk premia, not their predictive power.

switch sign, contrary to subjective BRP, while the ranking of the country average exchange rate risk premia are clearly different than survey-based XRP. In particular, the negative exchange rate risk premia of SEK and NOK appear inconsistent with the standard intuition behind carry trade strategies.[10]

[INSERT FIGURE 5 HERE]

V. Predictability Regressions

We estimate predictability regressions in order to characterise the informational content of survey-based risk premia as signals about future realized returns, and contrast these to statistical expected return signals. The statistical benchmark signals we consider are the slope of the yield curve (for bonds) and the interest rate differential (for FX) which we denote

$$IRD_t \equiv i_t^{(1)} - i_t^{(1),f} \quad , \quad Slope_t \equiv i_t^{(10)} - i_t^{(1)}. \tag{6}$$

A. Bond Risk Premia

Panel (a) of Table III reports bond market expectations hypothesis predictability regressions where point estimates are pooled and confidence intervals are computed using circular block bootstrap.[11]

$$rx_{t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t + \epsilon_{t+1}. \tag{7}$$

[INSERT TABLE III HERE]

The slope of the yield curve significantly positively predicts future realized returns, as expected. More importantly, survey-based expected excess bond returns are also positively and highly significantly linked to future realized excess bond returns. A common null hypothesis in rational expectations tests would include $H_0 : b_2 = 1$. These estimates deliver a 95% upper confidence interval of 0.77 which rejects but is not "miles away" from the rational expectations beta loading null, economically speaking. In any case there are many well known consensus aggregation reasons for which this is not an appropriate null hypothesis for testing rationality. More importantly, we

[10] Table A.3 in the OA reports summary statistics for risk premia based on statistical projections.

[11] For robustness, we report an equivalent table using the estimator of Driscoll and Kraay (1998) in the OA, see Table A.8.

infer that $b_2 > 0$ with a high degree of statistical confidence. Interestingly, the predictive power of our survey-based measure does not disappear when adding the slope as an additional predictor, even if the estimated factor loading b_2 drops from 0.52 to 0.30. This result suggests that survey forecasts contain valuable information to predict future bond returns, which is not completely spanned by the information in current interest rate term structures. This 'lack of spanning' also suggests that forecast errors are predictable by date t information since they do not subsume the predictive ability of the *Slope*. This suggests that bond market beliefs are not fully rational. Section VI immediately below elaborates on this point.

B. Exchange Rate Risk Premia

Beginning with Fama (1984) a vast literature has tested the UIP condition via a predictability regression of log spot exchange rate changes on lagged interest rate differentials (or forward-spot spreads). The zero currency expected excess return prediction is commonly tested by the time-series predictive regression

$$\Delta x_{t+1} = \alpha + \beta IRD_t + \epsilon_{t+1}. \tag{8}$$

The UIP condition predicts that $\alpha = 0$ and $\beta = 1$ so that earning positive carry from the perspective of a U.S. investor ($IRD_t < 0$) is offset by a capital loss (a foreign currency depreciation, $\Delta x_{t+1} < 0$) when repatriating the initial investment. Subtracting $i_t^{(1)} - i_t^{(1),f}$ from both sides of the regression,

$$\Delta x_{t+1} - IRD_t = \alpha + \underbrace{(\beta - 1)}_{b_1} IRD_t + \epsilon_{t+1}, \tag{9}$$

we see that considering a Fama regression with excess returns as a dependent variable, UIP implies a regression coefficient $b_1 = 0$. Testing the predictability of excess exchange rate returns, we estimate the following pooled OLS regressions:

$$rx_{t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t+1}. \tag{10}$$

Panel (b) of Table III shows that the coefficient for the usual UIP predictor, the interest rate differential, is significantly negative, consistent with the literature. The estimated b_1 is -1.56, meaning that not only do U.S. investors in high interest rate currencies earn positive carry but they also earn a capital gain when closing out their positions.

As above, a natural null hypothesis for the XRP_t coefficient is $H_0 : a = 0$ and $b_2 = 1$, i.e. if surveys were fully rational (in a full information sense) we should expect $rx^{FX}_{t+1} = XRP_t + \epsilon_{t+1}$. Testing this null, we do not reject at conventional levels based on bootstrap standard errors (nor based on Driscoll and Kraay (1998) standard errors in Table A.8). However, moving to specification (*iii*) we find that both the interest rate differential and XRP_t are statistically significant. Moreover, the point estimate of coefficient b_2 does not change much and remains statistically significant. In summary, survey forecast of exchange rate returns not only significantly positively predict future realized excess returns but they do not *only* use information completely spanned by current interest rate spreads. As with the bond predictability regression above, while positively correlated with future realized returns their errors are predictable by the current observables (IRD_t in this case) again suggesting that beliefs are not fully rational.

VI. Expectation Errors

From survey forecasts about a target variable y_{t+1} we compute expectation errors as follows

$$y_{t+1} - y_t = E^{\mathbb{S}}_t \Big[y_{t+1} - y_t \Big] + FE_{t,t+1} \tag{11}$$

Table IV provides insights into the properties of forecast errors, by showing summary statistics of 10-year yield and exchange rate expectation errors for all countries in the sample. For interest rates, the mean expectational error is negative for all countries, meaning that all forecasts overpredicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante. For exchange rates, the mean expectational errors are close to zero, meaning that there is no systematic bias. To the best of our knowledge, the finding of a over prediction in yields and a zero bias in exchange rate predictions from the same set of forecasters is new to the literature. A plausible explanation for this finding is a central feature of the structural estimation which follows below.

Studying dynamics in errors, Figure 6 displays the time series of 10-year yield forecast errors (left panel) and exchange rate errors (right panel). Interestingly, forecast errors across countries are highly correlated suggesting there is a common factor in belief formation. However, we note that errors are volatile around zero and display only mild persistence, as evidenced by the AR(1) coefficients sampled at annual horizons (final row of each panel in Table IV).

[INSERT FIGURE 6 AND TABLE IV HERE]

Next, we ask whether the mild persistence observed in the autocorrelation of $t+1$ errors results in predictability by factors observable at date t. Such a question is important since forecast error predictability suggests that consensus beliefs (measured from surveys) are not incorporating all available information. Table V shows pooled regressions of (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on the level and slope of the yield curve and the 1-year interest rate differential, respectively.

$$FE^i_{t,t+1} = a^i + b^i X^i_t + \eta^i_{t+1}, \tag{12}$$

where for interest rate forecast errors X_t includes the $Level_t = i^{(1)}_t$ and $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ and for exchange rates $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$.

Considering first panel (a) which displays interest rate forecast error predictability, we observe that the $Level_t$ of the yield curve is significant at 5%, while the $Slope_t$ of the yield curve is not significantly linked to future forecast errors. The point estimates on the $Level_t$ is positive, meaning that a negative shock to the date t interest rate causes agents to under predict future interest rates, consistent with the idea that agents generally perceive rate shocks to be *more* persistent than they turns out to be ex-post.

Consider now panel (b) which analyzes the predictability of exchange rate forecast errors. Exchange rate errors are also predictable by IRD_t with a point estimate that is negative and statistically significant at 5%. The negative sign has the opposite interpretation than for interest rate errors: a negative shock to IRD_t causes agents to over predict future exchange rates, consistent with the idea that agents tend to perceive rate shocks to be *less* persistent than they turns out to be ex-post.

[INSERT TABLE V HERE]

These findings suggest agents do not exploit all available information when forming their beliefs, which is inconsistent with the basic idea of rational expectations. However, to the best of our knowledge it remains an open question whether statistical predictability can be exploited to correct errors in real-time. In the OA (Section A.2) we provide a partial answer to this question by designing an experiment which constructs fictitious expectations that correct for predictability in errors using date t observables. Summarising, we find that 'uncorrected' beliefs dominate their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements.

VII. An Equilibrium Model with Subjective Beliefs

In this section, we adapt an off-the-shelf asset pricing model to incorporate a wedge between objective beliefs and subjective beliefs and use the model to shed light on the joint empirical properties of (*i*) subjective risk premia; (*ii*) expectation errors; (*iii*) physical and risk neutral FX and interest rate dynamics. We consider a 3-country world consisting of Australia, Switzerland and the U.S. and estimate the model via simulated methods of moments by exploiting beliefs from surveys and asset prices jointly. A full derivation and estimation details are reported in Sections A.3 and A.4 of the OA, respectively.

A. Stylised Facts

The main empirical characteristics of subjective risk premia that we have outlined in Section IV can be summarised as follows:

- **Risk Premia:** Subjective bond risk premia are negative on average but can also take positive values. This is consistent with the idea that bonds are usually hedges but can sometimes be risky bets. From the perspective of a U.S. investors, foreign currency risk premia display a clear carry trade pattern: positive expected returns are earned on investment currencies and negative expected returns are paid on funding currencies.

- **Cyclicality:** Both subjective bond risk premia and exchange rate risk premia vary countercyclically with subjective macro-expectations of real growth rates and quantities of risk.

- **Belief Distortions:** Ex-ante beliefs about returns significantly forecast realized returns and with the correct sign according to rational expectations. At the same time, survey expectations are not completely "spanned" by interest rate spreads, which also significantly predict future excess returns. This implies, and indeed we show, that while positively correlated with future realized returns, survey implied forecast errors are predictable by date t information; thus, there exists a distortion between subjective and physical measures.

Our model estimation is designed to understand these facts in a structural setting. We begin with a standard equilibrium model in which risk premia are constant and we target unconditional moments of risk premia, as well as interest rates and forecast errors, in order to learn about the magnitude of the distortion between subjective and physical measures. Then, we extend the model introducing a stochastic volatility factor in order to rationalize the time series dynamics of risk premia in the data and in particular the observed cyclicality and predictability properties.

B. Fundamentals and Preferences

Consider an economy where the state-variables are adapted to the filtered probability space $(\Omega, \mathcal{F}_{t\in T}, \mathbb{P})$ and $\mathbb{P}$ denotes the physical probability measure of the econometrician. The level of the aggregate consumption flow in this economy, C_t ($c_t = \ln C_t$), evolves according to the following dynamics, and its conditional mean is driven by a "long run risk" factor g_t:

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sigma_c dW_{c,t}, \tag{13}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g dW_{g,t}, \tag{14}$$

We assume a constant conditional mean Geometric Brownian Motion for the price level

$$\frac{dQ}{Q} = \pi dt + \sigma_Q dW_{Q,t} \tag{15}$$

and that all Brownian motions are independent.

A continuum of identical agents hold the Duffie and Epstein (1992a) parameterisation of the continuous time Epstein and Zin (1989) and Weil (1989) recursive preferences, where β is a time discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of inter-temporal substitution (EIS). As discussed in Bansal and Yaron (2004), recursive preferences allow for separation of time-and state-preferences for consumption. When $\gamma \neq \psi$ agents have a preference for early (late) resolution of uncertainty when risk aversion is larger (smaller) than the inverse of EIS. A preference for early resolution combined with $\psi > 1$ implies that $\theta \equiv \frac{1-\gamma}{1-1/\psi} < 1$.

C. Bond Pricing

An approximate solution for asset prices in this economy follows from a standard log-linearisation of the consumption-wealth ratio. Denoting $h_1 = E\left[\frac{C_t}{W_t}\right]$ the diffusion for the nominal SDF is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{16}$$

$$r_t = r_0 + \frac{1}{\psi} g_t \tag{17}$$

$$\Theta_c = \gamma\sigma_c \quad , \quad \Theta_g = \left(\frac{1}{h_1 + \kappa_g}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \quad , \quad \Theta_Q = \sigma_Q \tag{18}$$

Bond prices are exponentially affine and given by

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] = E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right]$$

Under the $\mathbb{Q}$-measure the risk neutral diffusion for g_t is given by

$$dg_t = \kappa_g(\mu_g^{\mathbb{Q}} - g_t)dt + \sigma_g dW_{g,t}^{\mathbb{Q}}, \tag{19}$$

so that the unconditional mean $\mu_g^{\mathbb{Q}} = \mu_g - \Theta_g \frac{\sigma_g}{\kappa_g}$ is different under the physical and the risk-neutral measure. The bond price factor loading is obtained from standard risk-neutral pricing techniques and is given by $B_\tau = -\frac{1}{\psi}\frac{1}{\kappa_g}\left(1 - e^{-\kappa_g \tau}\right) < 0$ so that higher real growth rates lower bond prices and increase interest rates. A_τ is also known in closed form and reported in the OA. The term structure of interest rates is then given by $y_t^\tau = a_\tau + b_\tau g_t$ where the yield factor loadings are equal to the bond pricing factor loadings multiplied by $-\tau^{-1}$.

D. Subjective Measure and Belief Distortion

The state variables in the previous subsections are all adapted to the filtered probability space $(\Omega, \mathcal{F}_{t\in T}, \mathbb{P})$, where $\mathbb{P}$ represents the physical probability distribution. When investors only have a short sample of data available, learn sub-optimally, or suffer from some behavioural biases, their subjective measure $\mathbb{S}$ will not necessarily coincide with the physical (objective) measure $\mathbb{P}$ of an econometrician.

Formally, the relationship between the physical measure and the subjective measure is given by $E_t^{\mathbb{P}}\left[\frac{d\mathbb{S}}{d\mathbb{P}} z_s\right] = E_t^{\mathbb{S}}\left[z_s\right]$, where from Girsanov[12]

$$\frac{d\mathbb{S}}{d\mathbb{P}} = \exp\left(\int_0^t \phi_{z,u} dW_u^{\mathbb{P},z} + \frac{1}{2}\int_0^t \phi_{z,u}^2 du\right), \tag{20}$$

$$dW_t^{\mathbb{S},z} = dW_t^{\mathbb{P},z} - \phi_z dt. \tag{21}$$

We call ϕ_z a BELIEF DISTORTION. For simplicity and ease of interpretation we assume that the distortion is constant and that only the latent long run risk process g_t has different shocks under the subjective and physical measures, so that $\phi_g \neq 0$, while all other Brownian motions are the same under $\mathbb{S}$ and $\mathbb{P}$. Potentially, a similar change of measure could hold for the other shocks and in different settings.

Abstracting from a specific mechanism for subjective belief formation, we have two changes of measure which satisfy[13]

$$\begin{aligned} dW_{g,t}^{\mathbb{Q}} &= dW_{g,t}^{\mathbb{P}} + \Theta_g^{\mathbb{P}} dt \quad , \quad dW_{g,t}^{\mathbb{Q}} = dW_{g,t}^{\mathbb{S}} + \Theta_g^{\mathbb{S}} dt, \\ dW_{g,t}^{\mathbb{S}} &= dW_{g,t}^{\mathbb{P}} - \phi_g dt \quad \rightarrow \quad \phi^g = \Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{P}} \end{aligned} \tag{22}$$

So, the SDF dynamics under the subjective measure is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c^{\mathbb{S}} dW_{c,t}^{\mathbb{S}} - \Theta_g^{\mathbb{S}} dW_{g,t}^{\mathbb{S}} - \Theta_Q^{\mathbb{S}} dW_{Q,t}^{\mathbb{S}} \tag{23}$$

$$\Theta_c^{\mathbb{S}} = \Theta_c^{\mathbb{P}} \quad , \quad \Theta_g^{\mathbb{S}} = \Theta_g^{\mathbb{P}} + \phi_g \quad , \quad \Theta_Q^{\mathbb{S}} = \Theta_Q^{\mathbb{P}} \tag{24}$$

This implies that only the market price of risk for the latent long-run risk factor g is different under the two measures and in particular it increases by the distortion ϕ_g. Under the subjective measure $\mathbb{S}$, the diffusion for g_t is given by

$$dg_t = \kappa_g(\mu_g^{\mathbb{S}} - g_t)dt + \sigma_g dW_{g,t}^{\mathbb{S}},$$

where $\mu_g^{\mathbb{S}} = \mu_g + \frac{\sigma_g}{\kappa_g}\phi_g$. Note that bond prices can equivalently be solved for under the subjective

[12] We assume $\mathbb{S}$ is absolutely continuous with respect to $\mathbb{P}$.

[13] Micro-foundations for belief distortions arise when investors are subjective to behavioural biases or information frictions (see Coibion and Gorodnichenko (2015) for an overview) or in settings where agents have preferences for statistical robustness (for recent examples, see Maenhout, Vedolin, and Xing (2023) and Bhandari, Borovička, and Ho (2022)).

or physical measure via the changes of measure defined above:

$$P_t^\tau = E_t^{\mathbb{P}}\left[\frac{\Lambda_T}{\Lambda_t}\right] = E_t^{\mathbb{P}}\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] = E_t^{\mathbb{P}}\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\frac{d\mathbb{S}}{d\mathbb{P}}\right] = E_t^{\mathbb{S}}\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\right] = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right]$$

whose solution is unique and reported above.

E. *Subjective Bond Risk Premia and Yield Forecast Errors*

The subjective price of risk outlined above also drives subjective expected excess bond returns. Namely, the subjective instantaneous bond risk premium on a T period bond is given by the negative of the covariance between SDF changes and bond returns:

$$BRP_t = -E_t^{\mathbb{S}}\left[\frac{d\Lambda_t^{\mathbb{S}}}{\Lambda_t^{\mathbb{S}}}\frac{dP_t^\tau}{P_t^\tau}\right] = \Theta_g^{\mathbb{S}} B_\tau \sigma_g \tag{25}$$

In this economy, the bond risk premium is constant and negative, consistent with the unconditional mean of the subjective bond risk premia observed in the data, as for realistic parameter choices, we have $\Theta_g^{\mathbb{S}} > 0$ and $B_\tau < 0$. Forecast errors on yields are given by $FE_{t+1}^\tau = y_{t+1}^\tau - E_t^{\mathbb{S}}\left[y_{t+1}^\tau\right]$ which can be written as

$$FE_{t+1}^\tau = \frac{-B_\tau}{\tau}\left[g_{t+1} - E_t^{\mathbb{S}}(g_{t+1})\right], \tag{26}$$

where $E_t^{\mathbb{S}}(g_{t+1}) = g_t e^{-k_g} + \mu_g^{\mathbb{S}}(1 - e^{-k_g})$. The physical expectation of the survey forecast errors therefore depends on the difference between the unconditional mean of the g process under $\mathbb{P}$ and $\mathbb{S}$, which is proportional to the belief distortion ϕ_g:

$$\begin{aligned} E_t^{\mathbb{P}}\left(FE_{t+1}^\tau\right) &= \frac{-B_\tau}{\tau}\left[E_t^{\mathbb{P}}(g_{t+1}) - g_t e^{-k_g} - \mu_g^{\mathbb{S}}(1 - e^{-k_g})\right] \\ &= \frac{B_\tau}{\tau}\left[\left(1 - e^{-k_g}\right)\phi_g \frac{\sigma_g}{k_g}\right] \end{aligned} \tag{27}$$

In order to match the negative yield forecast errors in the data, since B_τ is negative, we need a positive belief distortion ϕ_g, so that the unconditional mean of the long-run risk factor under the subjective measure is larger than under the physical measure.

F. *Foreign Countries*

In order to solve for the exchange rate risk premium we need to make an assumption about the foreign countries. Let us assume there are $N+1$ consumption goods and $N+1$ countries: "home" country (the U.S.) and N "foreign" countries. As in Colacito and Croce (2011), we assume that each country behaves as in autarky, in both consumption and financial assets (total home bias) so that a representative investor within each country only consumes the good which they are endowed.

We assume foreign country dynamics have the same structure as in (13) but with different shocks and potentially different parameters. We model cross-country correlation by assuming that shocks in the foreign country, i.e. $dW_{c,t}^{\mathbb{S},f}$, $dW_{g,t}^{\mathbb{S},f}$ and $dW_{Q,t}^{\mathbb{S},f}$, are correlated with the corresponding shocks in the home country, with correlations ρ_c, ρ_g and ρ_q, respectively. Cross-country correlation among the state variables potentially generates global and local factor pricing of risk as in Lustig, Roussanov, and Verdelhan (2011). We also assume that the correlation between shocks in the foreign countries is completely driven by their exposure to the home country shocks.

Investors can trade in both domestic and foreign bond markets. Denote X_t the real exchange rate in the U.S good per the foreign good. When X_t goes up, the US dollar depreciates in real terms. If markets trade without frictions then exchange rates are pinned down by the ratio of the SDFs, i.e. $X_t = \Lambda_t^f \Lambda_t^{-1}$, where the dynamics of the foreign SDF Λ_t^f follows exactly the same structure as the domestic SDF in Equation (16) but with the country-specific parameters and shocks. The U.S. dollar price of the foreign bond ($P_t^{\tau,f}$) is given by

$$\Lambda_t X_t P_t^{\tau,f} = E_t^{\mathbb{S}}\Big[\Lambda_{t+1} X_{t+1} P_{t+1}^{\tau-1,f}\Big] \tag{28}$$

which implies that the exchange rate risk premium is obtained as the negative of the covariance between the SDF and exchange rate changes, which is equal to the drift in the exchange rate dynamics plus the interest rate differential (equivalent to Equation (4)):

$$XRP_t = -\frac{1}{dt} Cov_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}, \frac{dX_t}{X_t}\right) = \frac{1}{dt} Var_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}\right) - \frac{1}{dt} Cov_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}, \frac{d\Lambda_t^f}{\Lambda_t^f}\right) \tag{29}$$

$$= \Theta_c^{\mathbb{S}}\left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f}\rho_c\right) + \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right) + \Theta_Q^{\mathbb{S}}\left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f}\rho_Q\right) \tag{30}$$

As for the bond risk premium, in this simple setting the exchange rate risk premium is constant, but it can take both positive and negative values, depending on the correlation across countries and their respective market prices of risk. Forecast errors on log exchange rates are given by $FE^{FX}_{t+1} = x_{t+1} - E^{\mathbb{S}}_t\Big[x_{t+1}\Big]$. The physical expectation of the survey forecast errors therefore depends on the difference between the $\mathbb{P}$ and $\mathbb{S}$ expectations of log exchange rates:

$$E^{\mathbb{P}}_t\left(FE^{FX}_{t+1}\right) = E^{\mathbb{P}}_t\Big[x_{t+1}\Big] - E^{\mathbb{S}}_t\Big[x_{t+1}\Big] \tag{31}$$

Using Equation (4) we can rewrite this as the difference between the exchange rate risk premia under the physical and subjective measures:

$$\begin{aligned} E^{\mathbb{P}}_t\left(FE^{FX}_{t+1}\right) &= E^{\mathbb{P}}_t\Big[\Delta x_{t+1}\Big] - E^{\mathbb{S}}_t\Big[\Delta x_{t+1}\Big] = XRP^{\mathbb{P}}_t - XRP^{\mathbb{S}}_t \\ &= \Theta^{\mathbb{P}}_g\left(\Theta^{\mathbb{P}}_g - \Theta^{\mathbb{P},f}_g \rho_g\right) - \Theta^{\mathbb{S}}_g\left(\Theta^{\mathbb{S}}_g - \Theta^{\mathbb{S},f}_g \rho_g\right), \end{aligned} \tag{32}$$

which depends on the relative magnitudes of the belief distortions in the foreign and domestic country.

G. Estimation Results

We estimate the model in several steps. First, we estimate the parameters in the consumption process (Equations (13)-(14)) for each country separately using the simulated method of moments and targeting mean, standard deviation, skewness, kurtosis and autocorrelations (with 1, 5 and 10 lags) of the realized consumption processes. Tables A.15, A.16 and A.17 in the OA report estimation results for the US, Switzerland and Australia, respectively. Panel (a) shows parameter estimates alongside 95% confidence intervals and panel (b) reports the empirical moments as well as the model-implied moments and their confidence intervals.

Consistent with the literature on long-run risk models we match the properties of realized consumption processes in the three countries very well, with a very persistent long-run risk process g. The estimated mean reversion coefficient κ is around 0.05 for the US, 0.20 for Switzerland and 0.06. The unconditional mean of the log-run growth process is 1.3%, 1.6% and 3.2%, for the US, Switzerland and Australia, respectively.

Inflation parameters π and σ_Q (see Equation (15)) are set equal to the mean and volatility of inflation in each country.[14]

With these parameters at hand we turn to the estimation of the preference parameters, i.e. the time preference parameter β, the coefficient of risk aversion γ, and the elasticity of inter-temporal substitution ψ. These preference parameters affect the term structures in each country, as well as the risk premia. However, the subjective bond risk premia depend on both the preferences and the distortion between physical and subjective beliefs. Therefore, we estimate the preference parameters and the distortions jointly using term structure moments and the average bond risk premia, as well as the average 10-year bond yield forecast errors. The preference parameters are assumed the same across countries, while the belief distortion ϕ_g could potentially be country-specific. Overall, we have 6 parameters to estimate (β, γ, ψ, ϕ_g^{US}, ϕ_g^{CHF}, ϕ_g^{AUD}) and 18 moment conditions (mean of the 3-month yield, standard deviation of 3-month yield changes, mean of 5-year yield, mean of 10-year yield, mean bond risk premium and average 10-year yield forecast error), for a total of 6 moments for each of the 3 countries.[15] Preference and distortion parameter estimates are summarised in Table VI, as well as the value of targeted moments in the model and in the data.

[INSERT TABLE VI HERE]

The model estimation fits the average subjective bond risk premia and yield forecast errors quite well. In particular, the estimated bond risk premia are all negative and around -1%, as in the data. The model also captures the standard deviation of yields and the level of the term structure. The estimated values of the risk aversion and intertemporal elasticity of substitution parameters are 4.0 and 1.3, respectively, implying a preference for early resolution of uncertainty. The main shortcoming of the model is the fitting of the shape of the yield curves, with the model implying slightly downward sloping term structures while the observed term structures are all upward sloping.

Interestingly, the estimated belief distortions for the latent factor shocks (ϕ_g) are all relatively small, with values of 0.22, 1.31 and 0.14, for the US, Switzerland and Australia, respectively. In order to better understand the economic magnitude of these distortions, we can compare the

[14] In our sample, $\pi^{US} = 2.29\%$, $\pi^{CHF} = 0.53\%$ and $\pi^{AUD} = 2.15\%$, while the unconditional volatilities of inflation are $\sigma_Q^{US} = 0.81\%$, $\sigma_Q^{CHF} = 1.12\%$ and $\sigma_Q^{AUD} = 0.67\%$.

[15] In the estimation, instead of the instantaneous bond risk premium we compute the annualised expected excess bond return with a 1-year holding horizon, as in Equation (3), to be consistent with the empirical counterpart. We find that the model-implied values of the instantaneous and 1-year ahead bond risk premia are quite close.

unconditional mean of the latent risk factor g under the physical and the subjective measure. For the US, the physical mean is $\mu_g = 1.31\%$ while the subjective mean is $\mu_g^{\mathbb{S}} = 2.64\%$, which implies a 1.30 percentage points difference between the two. For Switzerland this difference is a little higher, at 1.72%, since we find $\mu_g = 1.55\%$ and $\mu_g^{\mathbb{S}} = 3.27\%$. For Australia instead the difference is a little smaller at 1.13%, with $\mu_g = 3.22\%$ and $\mu_g^{\mathbb{S}} = 4.45\%$. An alternative way to interpret the magnitude of the belief distortion is to look at the difference in the market prices of risk for the factor g under the two measures, i.e. $\Theta_g^{\mathbb{P}}$ and $\Theta_g^{\mathbb{S}}$, as well as the model-implied subjective and physical instantaneous bond risk premia. Table VII summarises these differences and confirms that, while the belief distortion helps matching the negative and relatively sizeable bond risk premia implied by survey forecasts, its magnitude in economic terms is not large.

[INSERT TABLE VII HERE]

Finally, we calibrate the correlations between shocks in the US and in the foreign countries by targeting the subjective exchange rate risk premia, while taking all other parameters as given from the previous steps. The correlations between consumption growth and inflation shocks, ρ_c and ρ_q, are observable, so we set them equal to the correlation between US and foreign consumption growth and inflation in the data (see Tables A.12 and A.14 in the OA). For Switzerland, we have $\rho_c = 0.97$ and $\rho_q = 0.89$, while for Australia we have $\rho_c = 0.76$ and $\rho_q = 0.95$. Panel (a) of Figure 7 shows the model-implied exchange rate risk premia for the two foreign countries as a function of the correlation between the unobservable long-risk risk factors, ρ_g, fixing all other parameters to their estimated values. The estimated model implies that XRP for AUD is always larger than the one for CHF, as in the data, even if we have still not used any information about the exchange rate dynamics or premia in the estimation. Note also that we can set the ρ_g parameters in order to exactly match the subjective exchange rate risk premia in the data, which are $XRP^{CHF} = -1.69\%$ and $XRP^{AUD} = 1.74\%$. The corresponding values of ρ_g are 0.32 for Switzerland and 0.90 for Australia.

[INSERT FIGURE 7 HERE]

The main driver of the sign of the XRP in the two countries is the relative magnitude of the market prices of risk for the long-run risk factor in the US versus the foreign country, weighted by the correlation ρ_g as the market prices of risk for the consumption and price level shocks are relatively smaller. Under the subjective measure, $\Theta_g^{\mathbb{S}}$ is similar in Australia and in the US, leading to positive values of the exchange rate risk premia, while for Switzerland the market price of risk is much larger, so that XRP can become negative when the correlation ρ_g is large enough (larger than 0.27). It is interesting to note that the ranking of the market prices of risk is very different under the physical measure, i.e. $\Theta_g^{\mathbb{P},CHF} < \Theta_g^{\mathbb{P},US} < \Theta_g^{\mathbb{P},AUD}$, which would imply a positive XRP for Switzerland and a possibly negative XRP for Australia.

Average forecast errors on log exchange rates are consistent with the observed ones, even though we have not used information from exchange rates forecast errors in the estimation. Panel (b) of Figure 7 shows the model-implied average log exchange rate forecast errors for the two foreign countries as a function of the correlation between the unobservable long-risk risk factors, ρ_g, fixing all other parameters to their estimated values. The values in the data are 1.01% for CHF and -0.34% for AUD, denoted by the horizontal dashed lines in the figure. The model-implied values are generally positive for CHF and negative for AUD, as in the data, and we can match the observed value for CHF with a ρ_g not far from the one required to fit the exchange rate risk premium ($\rho_g = 0.23$ vs $\rho_g = 0.32$). Our three step estimation approach has a harder time fitting the FX forecast error for AUD but the value of ρ_g required to fit the FX error and FX risk premium jointly is indeed close to 1.

H. Introducing Stochastic Volatility

The model presented so far is able to match the unconditional mean of the subjective risk premia, in both fixed income and exchange rate markets, and allows us to clearly quantify and interpret the belief distortion between $\mathbb{P}$ and $\mathbb{S}$. However, it obviously cannot match the time series properties of the subjective risk premia, such as their volatility and cyclicality, as model-implied risk premia are constant. Therefore, in this subsection we look at the implications of introducing stochastic volatility in the model.

The only difference compared to the model above is that consumption growth and the long-run risk factor have stochastic volatility, so that Equations (13) and (14) are substituted by the following:

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sqrt{v_t}dW_{c,t}, \tag{33}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g\sqrt{v_t}dW_{g,t}, \tag{34}$$

$$dv_t = \kappa_v(\mu_v - v_t)dt + \sigma_v\sqrt{v_t}dW_{v,t}, \tag{35}$$

where v_t is the stochastic volatility factor, which follows a CIR process, and all Brownian motions are independent.

We assume that the preference structure is unchanged so that the nominal SDF satisfies the following process:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} - \Theta_v dW_{v,t}, \tag{36}$$

where the market prices of risk for consumption and the latent g and v processes are proportional to $\sqrt{v_t}$. Bond prices are still exponentially affine but they also depend on the stochastic volatility factor:

$$\frac{dP_t^\tau}{P_t^\tau} = \mu_P(\tau)dt + B_g(\tau)\sigma_g\sqrt{v_t}dW_{g,t} + B_v(\tau)\sigma_v\sqrt{v_t}dW_{v,t}, \tag{37}$$

where $B_g(\tau) < 0$ and $B_v(\tau) > 0$ are the bond factor loadings.

As above, we assume that only the latent long run risk process g_t has different shocks under the subjective and physical measures, but instead of a constant distortion we assume that $\phi_g = \phi\sqrt{v_t}$, so that the market prices of risk for the long-run risk factor, Θ_g, have the same form under $\mathbb{P}$ and $\mathbb{S}$, i.e. they are both proportional to $\sqrt{v_t}$, just with a higher proportionality coefficient under $\mathbb{S}$. Note that under the subjective measure $\mathbb{S}$, the diffusion for g_t is now given by

$$dg_t = [\kappa_g(\mu_g^{\mathbb{P}} - g_t) + \sigma_g\phi v_t]dt + \sigma_g dW_{g,t}^{\mathbb{S}}, \tag{38}$$

so that the distortion affects not only the unconditional mean of the long-run risk factor, but also its persistence. The model-implied instantaneous subjective bond risk premium in this case is given by:

$$BRP_t = \Theta_g^{\mathbb{S}} B_g(\tau)\sigma_g\sqrt{v_t} + \Theta_v^{\mathbb{S}} B_v(\tau)\sigma_v\sqrt{v_t}. \tag{39}$$

Since both market prices of risk are proportional to $\sqrt{v_t}$, the subjective bond risk premium is proportional to v_t. So, in this economy, the bond risk premium is time varying with economic uncertainty. We have $\Theta_g^{\mathbb{S}} > 0$ and $B_g(\tau) < 0$, as well as $\Theta_v^{\mathbb{S}} < 0$ and $B_v(\tau) > 0$, so BRP is still negative and larger in absolute than in the base model, all else equal. This stochastic volatility version of the model still does not allow bond risk premia to switch sign, but it is straightforward to think about further extensions of the model that could help matching this feature of the data, such as priced inflation risk, credit risk, or allowing for (possibly stochastic) correlations across Brownian motions in the model.

The subjective exchange rate risk premium implied by the stochastic volatility model is given by:

$$XRP_t = \Theta_c^{\mathbb{S}} \left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f} \rho_c \right) + \Theta_g^{\mathbb{S}} \left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f} \rho_g \right) + \Theta_Q^{\mathbb{S}} \left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f} \rho_Q \right) + \Theta_v^{\mathbb{S}} \left(\Theta_v^{\mathbb{S}} - \Theta_v^{\mathbb{S},f} \rho_v \right), \quad (40)$$

so it has exactly the same form as in Equation (30) with an additional component that depends on the market prices of risk for the domestic and foreign volatility factors. Assuming that the stochastic volatility factor is global, i.e. common across all countries, the last term disappears (as we would have $\rho_v = 1$ and $\Theta_v^{\mathbb{S},f} = \Theta_v^{\mathbb{S}}$) and XRP_t is affine in v_t. The subjective exchange rate risk premium can be both positive or negative, as in the base model, but it is also time-varying and could potentially switch sign depending on the relative sign and magnitude of the constant term in the expression (the $\Theta_Q^{\mathbb{S}}$ component) and the terms proportional to v_t.

In order to match the cyclicality of subjective bond risk premia we would need a negative covariance (under the subjective measure) between subjective bond risk premia and subjective expectations of consumption growth, i.e. $Cov^{\mathbb{S}}(BRP_t, g_t) < 0$. Since BRP_t is a negative linear function of v_t, this can be obtained with a positive correlation between v_t and g_t, i.e. between the latent volatility and expected growth factors. This same positive correlation is also consistent with the observed positive covariance between XRP and expected growth, at least for the countries in which the model-implied exchange rate risk premium is positive, that is the majority of countries in the data.

The predictability properties of subjective risk premia require a positive covariance between subjective risk premia and future realized excess returns, i.e. $Cov(BRP_t, rx_{t+1}) > 0$ for bonds. By construction, under the probability measure of the econometrician we have $rx_{t+1} = BRP_t^{\mathbb{P}} + \epsilon_{t+1}$, where ϵ_{t+1} is a mean-zero error uncorrelated with excess returns. Therefore,

$$\begin{aligned} rx_{t+1} &= BRP_t^{\mathbb{S}} + (BRP_t^{\mathbb{P}} - BRP_t^{\mathbb{S}}) + \epsilon_{t+1} \\ &= BRP_t^{\mathbb{S}} - \phi B_g(\tau)\sigma_g v_t + \epsilon_{t+1}, \end{aligned} \tag{41}$$

from which

$$Cov\left(BRP_t^{\mathbb{S}}, rx_{t+1}\right) = Var\left(BRP_t^{\mathbb{S}}\right) - \phi B_g(\tau)\sigma_g Cov\left(v_t, BRP_t^{\mathbb{S}}\right) \tag{42}$$

where both $B_g(\tau)$ and $Cov\left(v_t, BRP_t^{\mathbb{S}}\right)$ are negative, so that the second term is positive and reduces the covariance between BRP and realized returns. This means that subjective bond risk premia can positively predict future excess returns if the distortion between physical and subjective measure, ϕ, is not too large, as we have found in the baseline version of the model.

VIII. Conclusion

We show that subjective risk premia on sovereign bonds and exchange rates are countercyclical: they are significantly linked to subjective macro expectations in a manner consistent with canonical asset pricing models. Moreover, subjective risk premia are significantly positively linked to measures of quantity of risk, such as realised and implied volatility on bonds and exchange rates, consistent with the basic idea of a risk-return tradeoff.

Subjective risk premia significantly positively predict future realized excess returns and the predictive power goes beyond that of standard predictors like the interest rate differential and the slope of the term structure. This finding also implies (and we show) that forecast errors based on survey expectations are predictable by yield spreads, even if they are only mildly autocorrelated at annual horizons.

A significant link between subjective risk premia, subjective macro expectations and the quantity of risk supports asset pricing models that generate return predictability through cyclical variation in risk aversion, uncertainty, the likelihood of disasters or rational learning, but require the addition of a belief distortion between the probability measures of the econometrician ($\mathbb{P}$) and of a subjective measure ($\mathbb{S}$) under which the representative agent optimises.

We conclude by deriving an equilibrium asset pricing model, with complete markets, recursive preferences, long run risks and a long run-risk belief distortion. We estimate the model exploiting information from the yield curves, subjective risk premia and forecast errors, to jointly estimate $\mathbb{Q}$, $\mathbb{P}$ and $\mathbb{S}$. The model implied unconditional moments line up closely with the empirical survey based unconditional moments, with a belief distortion implying a positive bias in long-run growth of approximately 1.5%, with reasonable values of risk aversion and intertemporal elasticity of substitution.

Collectively, the findings of this paper demonstrate that subjective risk premia are well explained by standard asset pricing models, irrespective of whether belief formation is rational or not, so long as one allows for a distortion between subjective and objective beliefs that is not too large.

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IX. Figures

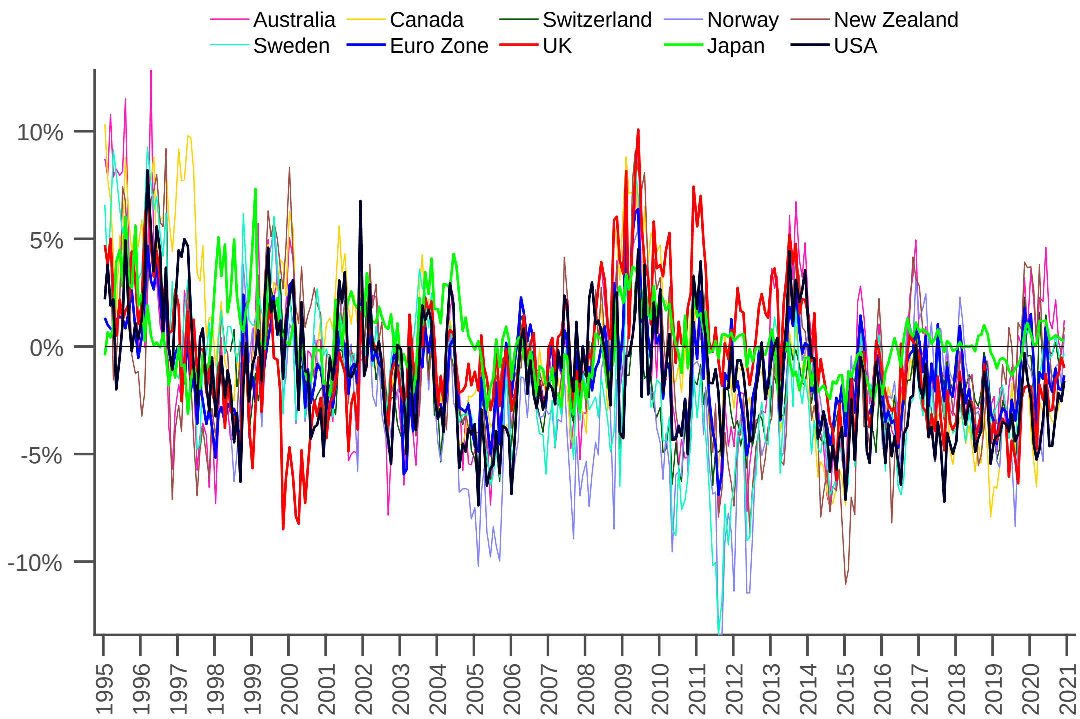


Figure 1. Subjective Bond Risk Premia

Figure displays real-time bond risk premia, defined as:

$$BRP_t^{11} = E_t^{\mathbb{S}}\left[rx_{t+1}^{(11)}\right] = E_t^{\mathbb{S}}\left[p_{t+1}^{(10)}\right] - p_t^{(11)} - i_t^{(1)},$$

where p_t^n are the log zero coupon bond prices for maturity n and i_t^1 is the continuously compounded one-year interest rate. The sample period is 1995.1 to 2020.12.

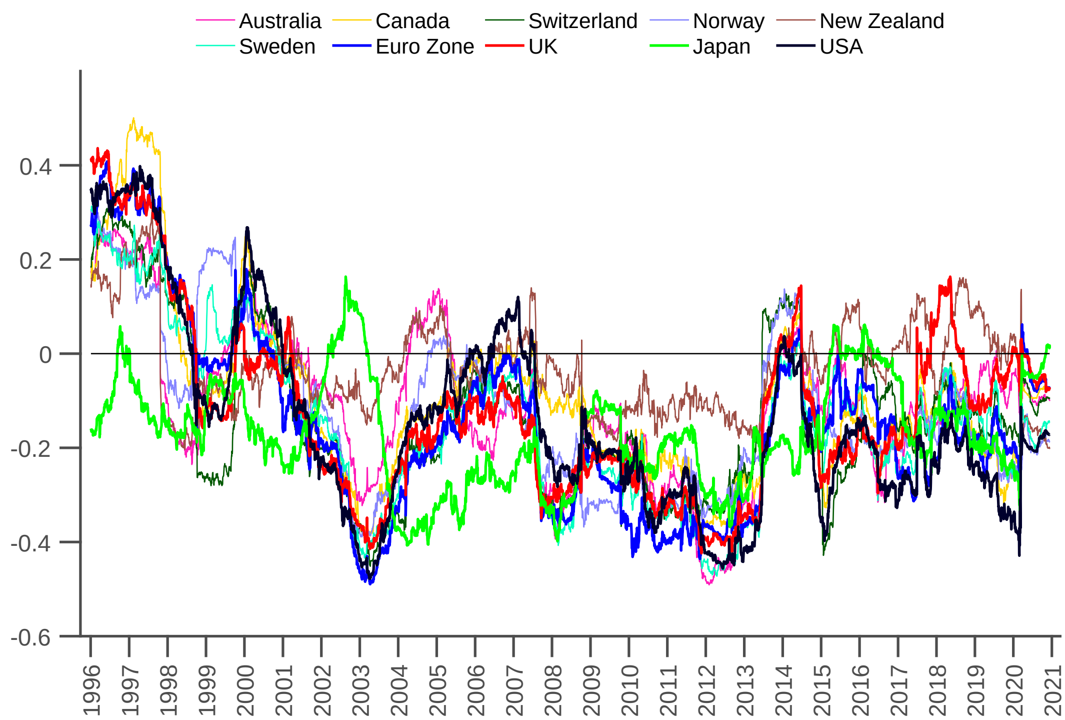
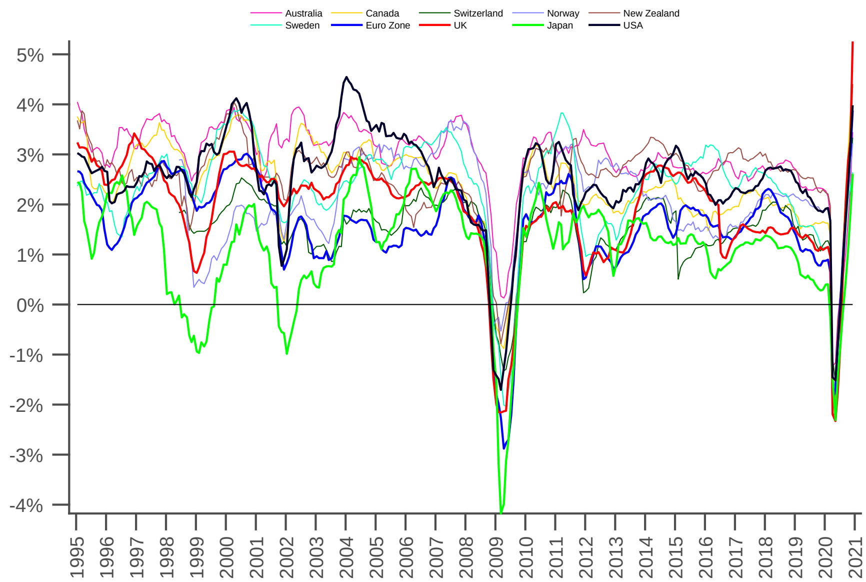
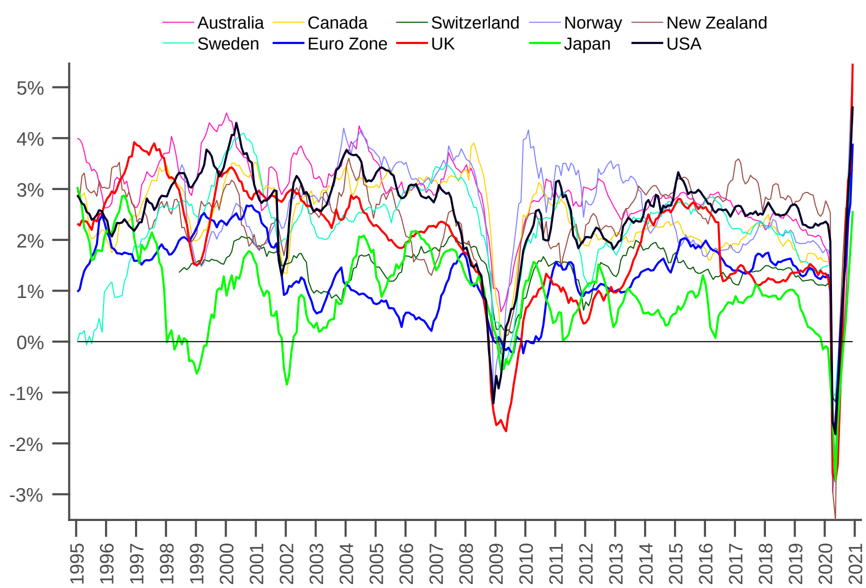


Figure 2. Stock-Bond Correlation

Figure displays the time series of rolling window correlations between ten-year log bond returns and log equity returns for a given country. We use the respective country-specific benchmark equity indices. The rolling window length is 222 daily observations. The sample period is 1995.1 to 2020.12.



(a) GDP



(b) Consumption

Figure 3. Subjective Macro Expectations

Figure displays subjective expectations of 12-month ahead GDP growth (%) and real private consumption growth (%), for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The sample period is 1995.1 to 2020.12.

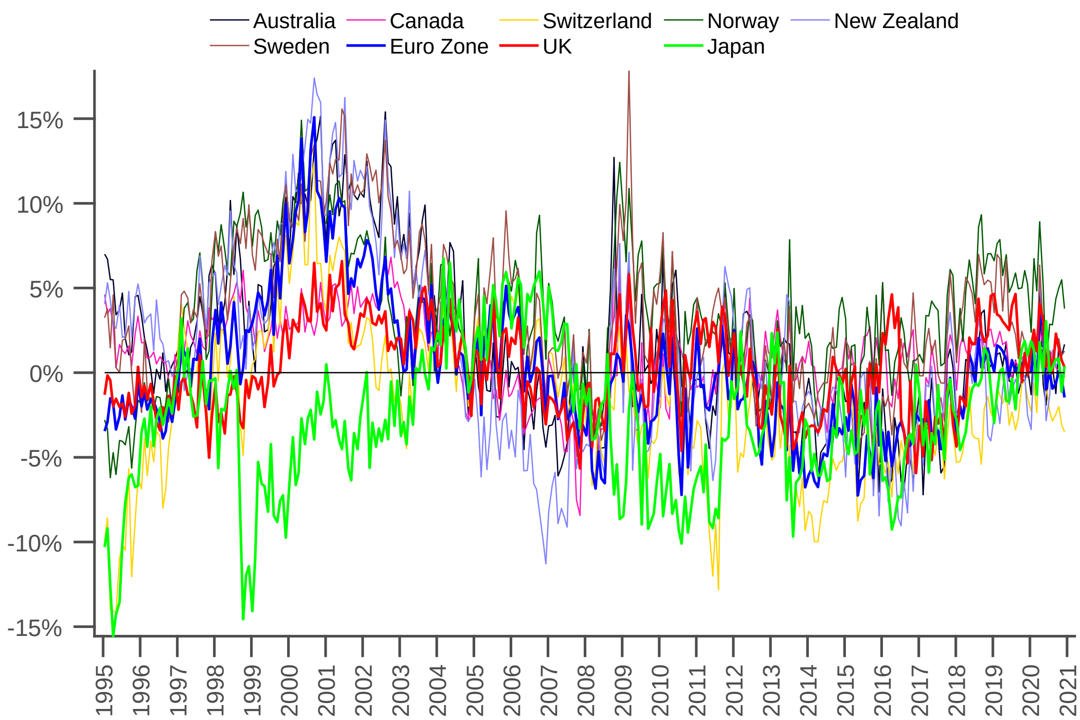
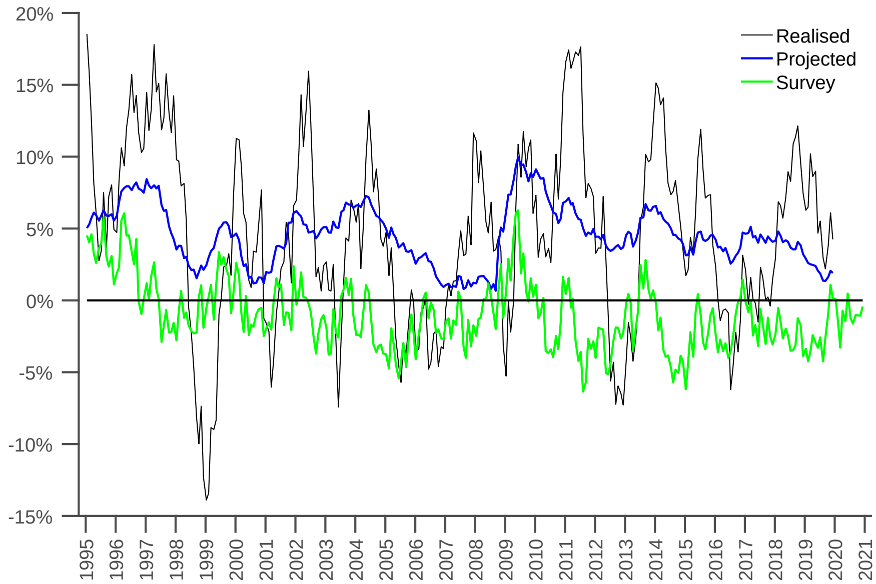


Figure 4. Subjective Exchange Rate Risk Premia

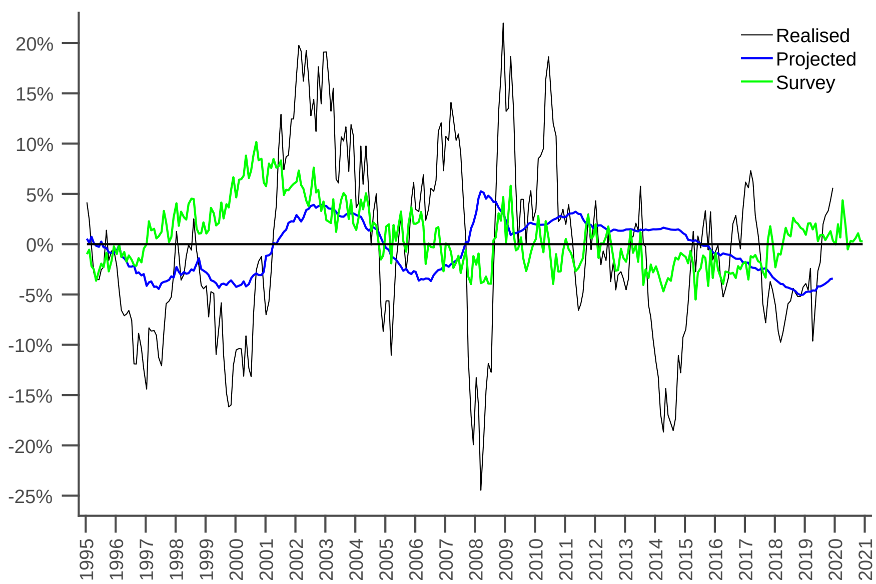
Figure displays real-time exchange rate risk premia, defined as:

$$XRP_t = E_t^{\mathbb{S}}\left[rx_{t+1}^{FX}\right] = (i_t^{(1),f} - i_t^{(1)}) + E_t^{\mathbb{S}}\left[\Delta x_{t+1}\right],$$

where the 1-year change in the log exchange rate is denoted Δx_{t+1}, and $i_t^{(1)}$ is the continuously compounded one-year interest rate in the foreign (f) versus domestic (U.S.) markets. The sample period is 1995.1 to 2020.12.



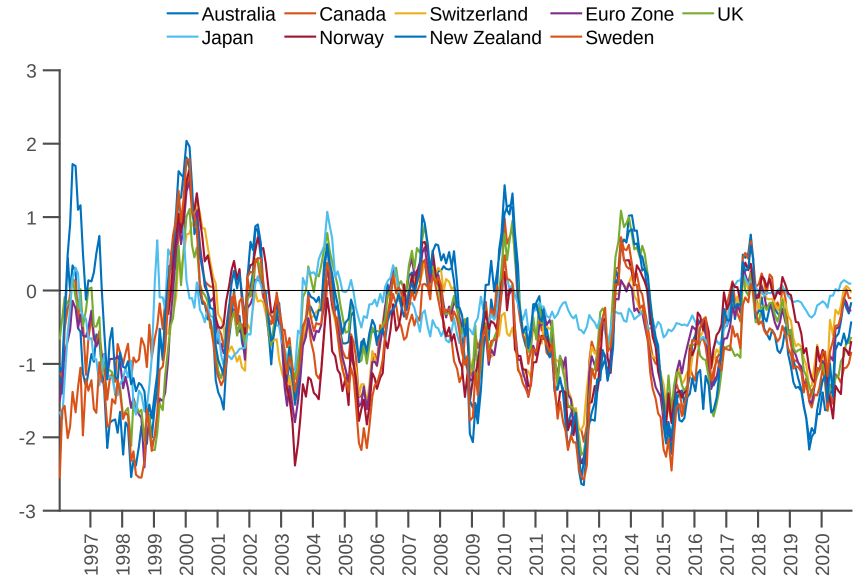
(a) Bond Risk Premia



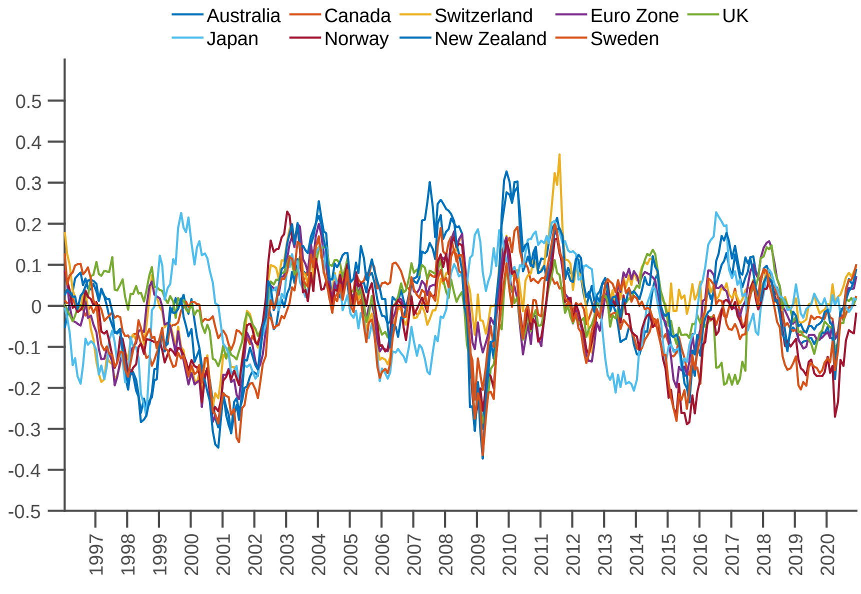
(b) Exchange Rate Risk Premia

Figure 5. Survey vs Projected Risk Premia

The figure displays an equally-weighted average of subjective risk premia across countries. Blue lines represent the average projected risk premia, while green lines represent average survey-implied risk premia. In panel (a), projections for bond risk premia are obtained by regressing realized excess returns on the slope of the yield curve. In panel (b), projections for exchange rate risk premia are obtained by regressing realized excess returns on the interest rate differential between the foreign country and the home country. The sample period is 1995.1 to 2020.12 for survey forecasts and 1995.1 to 2019.12 for projection-based forecasts.



(a) Interest Rates



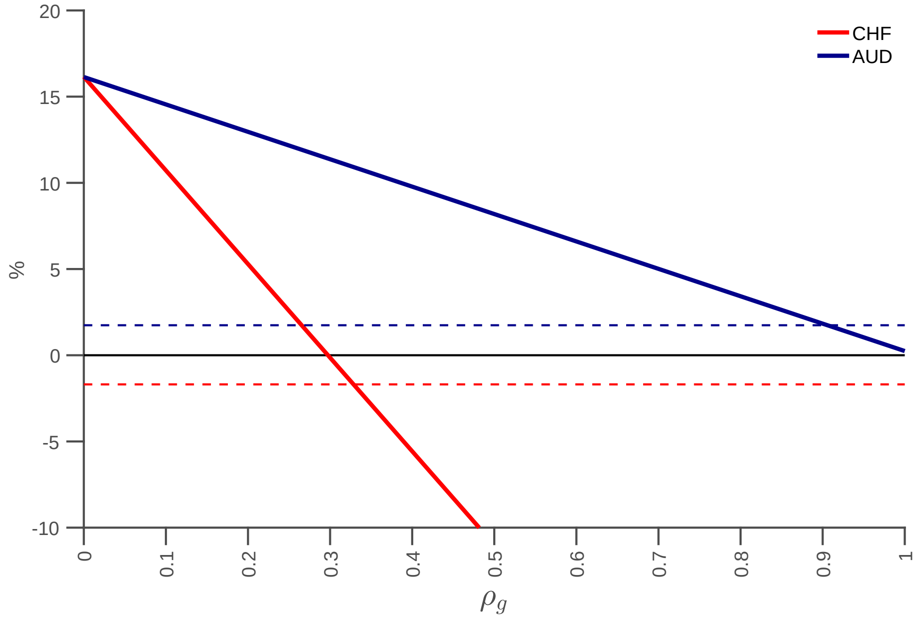
(b) Exchange Rates

Figure 6. Forecast Errors

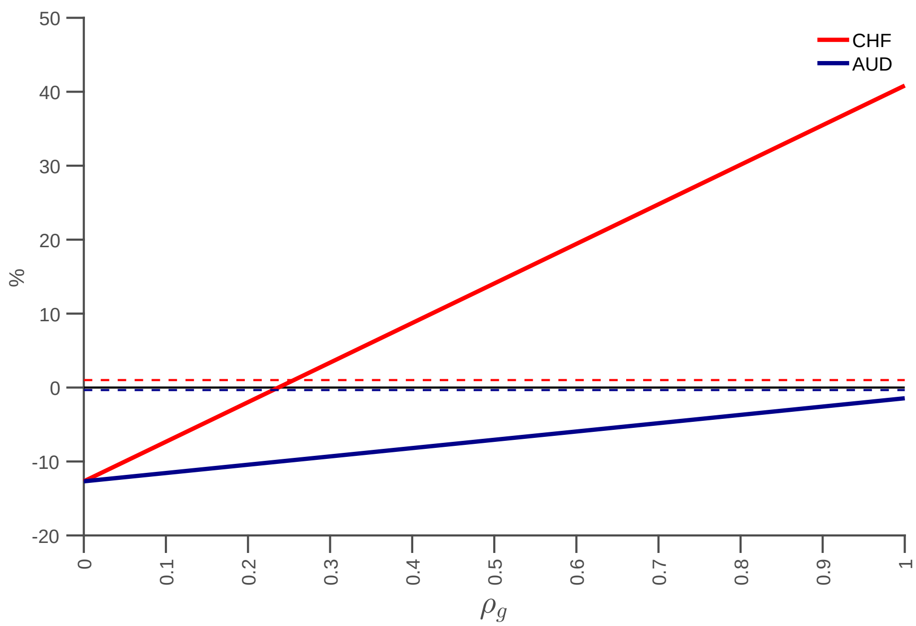
Forecast errors are calculated as

$$y_{t+1} - y_t = E_t^{\mathbb{S}}\left[y_{t+1} - y_t\right] + \epsilon_{t+1}$$

and plotted for 10-year interest rates and log exchange rates for a 1-year forecast horizon. Forecast errors are realized over the sample period 2001.1 to 2020.12.



(a) XRP



(b) FX FE

Figure 7. Model Implied Exchange Rate Risk Premia

This figure shows the model-implied exchange rate risk premia (Panel (a)) and average log exchange rate forecast errors (Panel (b)) for CHF and AUD as a function of the correlation between foreign and domestic long run risk factor, ρ_g, taking all other parameters as given from the previous steps of the model estimation. The horizontal dashed lines in both panels denote the corresponding values in the data.

X. Tables

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
					Panel (a): BRP					
Mean	−0.83	−0.43	−1.91	−1.18	−0.55	0.29	−2.72	−0.94	−1.56	−1.41
Std	3.45	3.78	1.00	2.14	3.03	1.74	3.34	3.58	3.58	2.85
Skew	0.85	0.72	0.11	0.47	0.43	0.73	−0.30	0.32	0.31	0.46
AR(1)	0.06	0.45	−0.18	0.00	0.49	0.30	0.03	−0.02	0.38	0.20
					Panel (b): XRP					
Mean	1.74	1.03	−1.69	0.17	0.20	−2.76	3.55	1.14	3.48	
Std	4.91	2.48	4.48	4.04	2.64	4.12	3.00	5.58	4.32	
Skew	0.63	−0.33	0.11	0.84	0.06	−0.24	0.11	0.58	0.52	
AR(1)	0.68	0.34	0.56	0.66	0.26	0.25	0.20	0.71	0.62	

Table I. Descriptive Statistics for Subjective Risk Premia

This table presents the means, standard deviations, skewness, and AR(1) coefficients for subjective bond risk premia (BRP) and subjective exchange rate risk premia (XRP) as defined in Equations (3) and (4), respectively. The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

	(i)	(ii)	(iii)	(iv)
		Panel (a): BRP		
E[*con*]	−0.57			
	[−0.92 −0.24]			
E[*gdp*]		−0.55		
		[−0.90 −0.12]		
$BondVar$			2.16	
			[0.96 3.14]	
$rx^{(11)}_{t-12,t}$				−0.09
				[−0.13 −0.05]
R^2 (%)	3.78	3.57	5.85	5.11
		Panel (b): XRP		
E[*con*]	1.08			
	[0.54 1.76]			
E[*gdp*]		0.95		
		[0.39 1.81]		
$FXVar$			0.78	
			[0.35 1.55]	
$rx^{FX}_{t-12,t}$				−0.09
				[−0.18 −0.02]
R^2 (%)	6.36	4.87	3.58	4.81

Table II. Cyclicality Regressions: Subjective Bond & FX Risk Premia
This table reports estimates from pooled OLS regressions of the forms

$$BRP_t = a + b^\top X_t + \epsilon_t, \text{ and}$$
$$XRP_t = a + b^\top X_t + \epsilon_t,$$

BRP_t is the survey-implied bond risk premium, XRP_t is the survey-implied currency risk premium, and X_t is a vector of explanatory variables. For the specifications in panel (a), X_t contains the 1-year *expected* consumption (*con*) growth and *gdp* growth, the realized bond variance ($BondVar$), and the excess return on an 11-year bond, realized between $t-1$ year and t. For the specifications in panel (b), X_t contains the 1-year *expected* consumption (*con*) growth and *gdp* growth in the foreign country, the FX option-implied risk neutral variance constructed and discussed by Krohn, Mueller, and Whelan (2024) ($FXVar$), and the excess currency return realized between $t-1$ year and t. A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.6. The sample period is 1995.1 to 2020.12 except for specification (iii) in panel (b) which only contains data from 1999.1 to 2019.4.

	(i)	(ii)	(iii)
	Panel (a): Bond Return		
Slope	2.81		2.49
	[1.96 3.69]		[1.53 3.46]
BRP		0.52	0.30
		[0.27 0.77]	[0.03 0.58]
R^2 (%)	13.85	4.96	14.79
	Panel (b): FX Return		
IRD	−1.56		−1.41
	[−2.38 −0.45]		[−2.19 −0.36]
XRP		0.55	0.45
		[0.08 1.01]	[0.03 0.90]
R^2 (%)	8.89	5.43	12.54

Table III. Bond & FX Return Panel Predictability Regressions

This table reports estimates from pooled OLS regressions of the forms

$$rx^{(11)}_{t,t+1} = a + b_1 Slope_t + b_2 BRP_t \epsilon_{t,t+1},$$
$$rx^{FX}_{t,t+1} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1},$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively. $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ is the slope of the domestic yield curve, $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$ is the one-year interest rate differential, BRP_t is the survey-implied bond risk premium, and XRP_t is the survey-implied currency risk premium. We also report univariate regressions with the same predictors in columns (i) and (ii). A constant is included but not reported. Confidence intervals in [·] are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.8. The sample period is 1995.1 to 2020.12.

Panel (a): IR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Mean	-0.57	-0.55	-0.52	-0.64	-0.52	-0.33	-0.55	-0.45	-0.78	-0.60
Std	0.89	0.65	0.58	0.67	0.71	0.47	0.78	0.87	0.81	0.77
Min	-2.65	-2.06	-1.90	-2.41	-2.24	-1.75	-2.38	-2.58	-2.58	-2.37
Max	2.04	1.55	1.24	1.48	1.11	1.07	1.66	1.81	1.79	1.80
AR(1)	-0.18	-0.03	-0.08	-0.18	-0.10	-0.05	0.07	-0.10	-0.12	-0.23
Panel (b): XR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	
Mean	0.00	-0.01	0.01	-0.01	0.00	0.00	-0.04	0.01	-0.05	
Std	0.13	0.08	0.10	0.10	0.08	0.11	0.11	0.13	0.11	
Min	-0.31	-0.23	-0.25	-0.28	-0.29	-0.27	-0.29	-0.37	-0.36	
Max	0.33	0.19	0.37	0.20	0.15	0.23	0.23	0.30	0.20	
AR(1)	0.10	0.12	0.23	0.09	-0.07	0.19	0.10	0.08	0.01	

Table IV. Summary Statistics: Expectation Errors

This table shows the means, standard deviations, minima and maxima of the forecast errors of survey forecasts. It also shows, at an annualised horizon, the 12 month autocorrelation. For each country in our sample, statistics are reported for long-term (10Y) interest rates, and for log spot exchange rates w.r.t the US in panels (a) and (b), respectively. The reported AR(1) coefficients relate to the yearly autocorrelation. Realized forecast errors are computed for the sample period 2000.1 to 2020.12.

	Yields	FX
$Level_t$	0.08	
	[0.03 0.15]	
$Slope_t$	0.08	
	[−0.03 0.20]	
IRD_t		−0.95
		[−1.41 −0.45]
R^2 (%)	5.71	5.73

Table V. Forecast Error Predictability

This table shows pooled OLS regressions of (a) 10-year yields and (b) foreign exchange forecast errors, both at an annual horizon, on either the level and slope of the yield curve or the 1-year interest rate differential:

$$FE_{t,t+1} = a + bX_t + \eta_{t+1},$$

where for interest rate forecast errors X_t includes $Level_t = i_t^{(1)}$ and $Slope_t = (i_t^{(10)} - i_t^{(1)})$ and for exchange rate forecast errors it includes $IRD_t = (i_t^{(1)} - i_t^{(1),f})$. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.9. The sample period is 1995.1 to 2020.12

Panel A: Distortions and Preferences

Parameter	Estimate	Lower	Upper
$\phi_{g,d}$	0.22	0.07	0.37
ϕ_{g,f_1}	1.31	0.78	1.84
ϕ_{g,f_2}	0.14	0.09	0.20
$\beta(\%)$	0.58	-0.25	1.41
γ	4.01	0.27	7.75
ψ	1.30	1.02	1.58

Panel B: Term Structure

Moment	Data	Model	Lower	Upper
		USD		
$E[y_t^{0.25}]$	2.33	3.76	2.89	4.63
std $\Delta y_t^{0.25}$	0.21	0.24	0.19	0.29
$E[y_t^5]$	3.37	3.66	2.80	4.52
$E[y_t^{10}]$	3.98	3.56	2.69	4.43
$BRP^{\mathbb{S}}$	-1.41	-0.81	-1.27	-0.35
FE^{10}	-0.60	-0.81	-1.27	-0.35
		CHF		
$E[y_t^{0.25}]$	0.82	2.27	1.41	3.13
std $\Delta y_t^{0.25}$	0.19	0.20	0.16	0.24
$E[y_t^5]$	1.42	2.26	1.40	3.11
$E[y_t^{10}]$	1.96	2.25	1.39	3.10
$BRP^{\mathbb{S}}$	-1.91	-1.22	-1.76	-0.69
FE^{10}	-0.52	-0.57	-0.81	-0.32
		AUD		
$E[y_t^{0.25}]$	4.13	4.97	3.97	5.97
std $\Delta y_t^{0.25}$	0.25	0.35	0.27	0.43
$E[y_t^5]$	4.65	4.77	3.83	5.71
$E[y_t^{10}]$	4.96	4.59	3.65	5.52
$BRP^{\mathbb{S}}$	-0.83	-1.13	-2.07	-0.20
FE^{10}	-0.57	-0.65	-0.92	-0.39

Table VI. Term Structure Estimation

This table presents the parameter estimates (Panel A) and moments (Panel B) from the SMM estimation of the distortion and preference parameters of the model presented in Section VII of the paper. Panel A reports point estimates alongside 95% confidence intervals. Panel B reports moments in the data and model implied moments alongside 95% confidence intervals.

	USD	CHF	AUD
Panel A: Consumption growth			
$\mu_g^{\mathbb{S}}$	2.64	3.27	4.35
$\mu_g^{\mathbb{P}}$	1.31	1.55	3.22
$\mu_g^{\mathbb{S}} - \mu_g^{\mathbb{P}}$	1.34	1.72	1.13
Panel B: Risk pricing			
$\Theta_g^{\mathbb{S}}$	40.18	135.16	39.56
$\Theta_g^{\mathbb{P}}$	18.60	4.22	25.06
$BRP^{\mathbb{S}}$	-0.81	-1.22	-1.13
$BRP^{\mathbb{P}}$	-0.38	-0.04	-0.72

Table VII. Belief Distortion

Panel A reports the estimated unconditional mean of the long-run growth process g_t, i.e. μ_g, under the subjective and physical measures, for the three countries. Panel B reports the estimated market prices of risk for the g factor, Θ_g, and the model-implied bond risk premia, BRP, under the two measures for the three countries.

Subjective Risk Premia in Bond and FX Markets
Online Appendix

This online appendix is not intended for publication. Section A.1 contains additional details on the construction of our survey dataset. Section A.2 reports additional results related to section VI. Section A.3 reports the derivations of the subjective beliefs equilibrium model discussed in Section VII. Section A.4 provides details of the simulated method of moments estimation approach. Sections A.5 and A.6 contains supplementary tables and figures to the main body of paper.

A.1. Data Appendix

Integral part of this paper is the measurement of subjective beliefs. We use the Consensus Economics dataset to get these beliefs for a set of professional forecasters such as banks, funds, and economic advisors. In particular, we use the Consensus Forecasts - G7 & Western Europe and Asia Pacific Consensus Forecasts datasets. These two datasets contain a wide cross-section of countries. While a large cross-section of countries is available, the number of forecasters participating for less developing economies is significantly smaller than the number of forecasters for developed economies. Furthermore, FX data is not available at all for some currencies and realized data is sparse for the smaller economies. Therefore, we restrict our attention to the universe of countries outlined in Section II of the main body of the paper to maintain a satisfactory number of observations for each date.

Included in the two datasets are expectations about several macroeconomic variables, interest rates, and exchange rates. While we know the identity of forecasters predicting macroeconomic variables and interest rates, Consensus Economics only reports a consensus estimate for historical exchange rates. Figure A.1 displays the number of forecasters that submitted estimates of future 10-year yields for each of the ten countries selected.

[INSERT FIGURE A.1 HERE]

A. Interest Rate Expectations

Consensus Economics asks its panellists to provide estimates of "yields on 10 year government bonds", without specifying what type of yield. However, it is generally understood that they are providing estimates of the on-the-run bond yield to maturity, which is effectively a par yield forecast. Since we only have two maturities available, we cannot bootstrap zero coupon bond yield estimates from the par yields provided. Therefore, in the main body of the text, we treat par yield forecasts as zero coupon forecasts. Moreover, the compounding frequency of the yields provided is also not explicitly stated, so we assume they are continuously compounded, i.e. log yields. This appendix shows the robustness of our results with respect to these assumptions, by comparing empirically par yields and zero-coupon bond yields for 10-year government bonds, as well as yields and bond returns based on different compounding frequency assumptions.

Panel A of Table A.1 displays the mean and standard deviation for (1) US 10-year par yields obtained from the Fed (H15), (2) US 10-year zero yields obtained from Bloomberg (BB), and (3) their differences. Figure A.2 shows the time series of the same par (H15) and zero (BB) yields, as well as their difference. We can see that the two series are extremely close and their difference is close to zero and insignificant. A similar picture arises when looking at the bond returns implied by par and zero yields.

Panel B of Table A.1 contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Again, both mean and standard deviation of the yields are extremely similar (see also Figure A.3).

[INSERT TABLE A.1 AND FIGURES A.2 AND A.3 HERE]

Summarising, we show that par yields and zero-coupon bond yields for 10-year government bonds are empirically very close and that the compounding frequency has little impact on the bond yields, so our results would be practically unchanged if we assumed that yields are annually or semi-annually compounded instead of continuously compounded, and they are robust to our assumption that survey forecasters provide zero yields.

B. Macroeconomic Expectations

In addition to interest rate and foreign exchange forecasts CE covers a large set of macroeconomic variables. We focus on real GDP growth and personal consumption growth, and the rate of unemployment. A complication with the survey projections is that respondents are asked to report expectations over the current and the next calendar year (except for interest rates, which are constant maturity forecasts); thus, the dataset represents a set of variable maturity events. For example, in July 2003 each contributor to the survey made a forecast for the percentage change in GDP for the remaining two quarters of 2003 (6 months ahead), and an average percentage change for 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induces a seasonal pattern in the survey. We compute an implied constant maturity forecast for each individual forecaster as in Buraschi and Whelan (2022) and Fendel, Lis, and Rülke (2011). Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2, \ldots, 12$. A constant maturity expectation is formed taking as weight $(1 - \frac{j}{12})$, for the short term projection (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term projection (the forecast for the following year). Figure A.4 illustrates the weighting procedure visually.

[INSERT FIGURE A.4 HERE]

A.2. Economic Significance of Deviation from Rational Expectations

To study the economic significance of behavioural components in agents expectations, we design an experiment in which we construct fictitious expectations by correcting the predictable errors using information available in date t observables. In this real-time experiment, we initialise a rolling regression with a window of w-years of data and recursively estimate a projection of realized errors on yields or interest rate differentials. The loadings available in the forecast error regression at date t can only be learned from errors realized one year earlier. These loadings are then applied to date t observables in order to build a 'corrected' beliefs from the following system

$$\widehat{FE}^{n}_{t-1,t} = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_{t-1} \tag{A.1}$$

$$\xi_t = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_t \tag{A.2}$$

$$\widehat{Y}_t = E_t^{\mathbb{S}}\Big[Y_{t+1}\Big] + \xi_t \tag{A.3}$$

The subscript t in the parameters $\hat{\alpha}_t$ and $\hat{\beta}_t^{\top}$ indicate that the correction is restricted to use only real-time information which is available at time t. The predictable component of forecast error is estimated using a rolling window to replicate real-life conditions of a trader. In unreported results we also consider expanding windows after a 5-year initial burn in period; the main message that follows is quantitively similar.

Panel (a) of figure A.9 displays the change in the $RMSE$'s (y-axis) from the original forecasts to the corrected forecasts for various rolling window lengths (x-axis). We find that, although the initial regressions indicate the existence of predictability in the forecast errors, the $RMSE$ of the corrected beliefs are unambiguously *higher* than the uncorrected ones. For instance, using a rolling window of 5 years in the estimation of the correction parameters, the RMSE increase by around 96% for the 10-year bond. This shows that the expectations extracted from surveys cannot be easily improved using market based state-dependent information. In panel (b) of figure A.9 we show that if one were to correct interest rate expectations for a constant bias, obtained from the (ex-post) mean of the forecast errors, the RMSE of the forecast would decrease by about 11%. However, we note that this is due to the bias in forecasts coming from over-prediction in sample rather than agents omitting useful information from the term structure. The result of replicating this experiment for exchange rate expectations yields similar results and is visualised in figure A.10.

Summarising, the findings of this section show that 'uncorrected' beliefs dominate their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements. This provides a possible explanation for why subjective expectation can be persistently different from the null implied by rational expectations.

[INSERT FIGURE A.9 AND A.10 HERE]

A. Note on the Driscoll and Kraay (1998) estimator

In Section IV, we discuss tables that show evidence about the explainability and predictability of survey-implied risk premia. For better readability, we choose to report point estimates as well as 95% confidence intervals. These intervals are estimated using a circular block bootstrap that uses a block length of 12 and 1000 bootstrap samples. As a robustness check to our predicability and cyclicality results, we also present confidence intervals using the estimator of Driscoll and Kraay (1998).

As shown in Tables A.4 and **??**, our panel dataset is characterised by significant cross-sectional correlation. We choose the Driscoll and Kraay (1998) estimator to account for this cross-sectional dependence in the estimation. Hoechle (2007) provides a discussion of the inner workings of the estimator and includes a STATA program called *xtscc* to run the estimation. In short, the estimator of Driscoll and Kraay (1998) applies a correction similar to Newey and West (1987) but adds robustness to cross-sectionally clustered standard errors. By combining these two properties, it yields a method to obtain standard errors that are not only robust to autocorrelation and heteroskedasticity but also consistent in settings of cross-sectional dependence. Another appealing property of the estimator is that it can, with a minor adjustment, handle unbalanced samples by applying the Newey and West (1987) correction at every time t to the moment conditions of individuals $N(t)$.

In terms of hypothesis testing, Tables A.6 and A.8 yield similar results as their counterparts (Tables II and III) in the main body of the paper.

[INSERT TABLES A.6 AND A.8 HERE]

A.3. Asset Pricing Derivations

A. Economic Dynamics

Consider the following model. Aggregate consumption flow is given by C_t ($c_t = \ln C_t$) whose conditional mean is driven by a "long run risk' factor g_t

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sigma_c dW_{c,t}, \tag{A.4}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g dW_{g,t}, \tag{A.5}$$

$$= (a_{g0} + a_{g1}g_t)dt + \sigma_g dW_{g,t}, \tag{A.6}$$

where $\langle dW_{c,t}, dW_{g,t}\rangle = \rho_{cg}$ is potentially non-zero. However, in estimation we assume short run and long run risks are independent. We assume a constant conditional mean geometric Brown motion for the price level

$$\frac{dQ}{Q} = \pi dt + \sigma_Q dW_{Q,t} \tag{A.7}$$

with $dW_{Q,t}$ independent of all other Brownian motions.

B. Preferences

We employ the Duffie and Epstein (1992a) parameterisation of the continuous time Epstein and Zin (1989) and Weil (1989) recursive preferences

$$J_t = E_t\left(\int_t^\infty f(C_s, J_s)ds\right) \tag{A.8}$$

where $f(C,J)$ is the normalised Kreps and Porteus (1978) aggregator

$$f(C,J) = (1-\gamma)J\left(\frac{\beta}{1-\frac{1}{\psi}}\right)\left(\frac{C^{1-1/\psi} - [(1-\gamma)J]^{1/\theta}}{[(1-\gamma)J]^{1/\theta}}\right) \tag{A.9}$$

$$f(C,J) = \beta\theta J\left[\frac{C^{1-1/\psi}}{[(1-\gamma)J]^{1/\theta}} - 1\right] \tag{A.10}$$

$$= \beta\theta J\left[Z(C,J) - 1\right], \tag{A.11}$$

where we have defined

$$Z(C,J) \equiv [C^{1-1/\psi}]\,[(1-\gamma)J]^{-1/\theta} \tag{A.12}$$

$$\theta \equiv \frac{1-\gamma}{1-1/\psi}. \tag{A.13}$$

β is a time discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of intertemporal substitution (EIS). As discussed in Bansal and Yaron (2004), EZ preferences allow for separation of time-and state-preferences for consumption, a property unavailable in time-separable specifications where consumption profiles are governed by a single parameter, $\gamma = 1/\psi$. When $\gamma \neq \psi$ agents have a preference for early (late) resolution of uncertainty when risk aversion is larger (smaller) than the inverse of EIS. A preference for early resolution combined with $\psi > 1$ implies that $\theta < 1$. Note that f_J and f_C are given by

$$f_J = \beta Z(C,J)(\theta - 1) - \beta\theta \tag{A.14}$$

$$f_C = \beta(1-\gamma)JC^{-1}Z(C,J). \tag{A.15}$$

C. Bond Pricing

Duffie and Epstein (1992a) also show that

$$\Lambda_t^* = \beta e^{\int_0^t f_J(C_s,J_s)ds} f_c(C_t,J_t) \tag{A.16}$$

which defines the wealth process defined as the present value of future consumption

$$\Lambda_t^* W_t = E_t\left[\int_t^\infty \Lambda_s^* W_s ds\right] \tag{A.17}$$

The real risk-free interest rate is given by

$$r_t^* = -\frac{1}{dt}E_t\left(\frac{d\Lambda_t^*}{\Lambda_t^*}\right) \tag{A.18}$$

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = \frac{df_c}{f_c} + f_J dt \tag{A.19}$$

and prices of risk come from the diffusion terms in $\frac{df_c}{f_c}$. Solving for real bond prices, note that in what follows the logarithm of Λ_t^* follows the SDE

$$d\log\Lambda_t^* = \left(-r_t^* - \frac{1}{2}(\Theta_c)^2 - \frac{1}{2}(\Theta_g)^2\right)dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} \tag{A.20}$$

$$r_t^* = r_0 + r_g g_t. \tag{A.21}$$

This implies for $\tau = T - t$

$$P_\tau^* = E_t\left[\frac{\Lambda_T^*}{\Lambda_t^*}\right] = E_t\left[e^{\log\Lambda_T^* - \log\Lambda_t^*}\right] \tag{A.22}$$

$$= E_t\left[e^{-\int_t^T\left[r_u^* du + \frac{1}{2}\Theta_c^2 + \frac{1}{2}\Theta_g^2\right]du - \int_t^T \Theta_c dW_{c,u} - \int_t^T \Theta_g dW_{g,u}}\right] \tag{A.23}$$

Notice that

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_t^T\left[\Theta_c^2 + \frac{1}{2}\Theta_g^2\right]du - \int_t^T \Theta_c dW_{c,u} - \int_t^T \Theta_g dW_{g,u}\right) \tag{A.24}$$

is the Radon-Nikodym derivative between physical probability measure $\mathbb{P}$ and another measure risk-adjusted measure $\mathbb{Q}$. With the Radon-Nikodym derivative at hand we can re-write $P^\tau = E_t^{\mathbb{P}}[\dots]$ as

$$P_\tau^* = \frac{E_t\left[\frac{d\mathbb{Q}}{d\mathbb{P}} e^{-\int_t^T r_u^* du}\right]}{E_t^{\mathbb{P}}\left[\frac{d\mathbb{Q}}{d\mathbb{P}}\right]} = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^* ds}\right] \tag{A.25}$$

and from Girsanov theorem we know that this change of measure satisfies

$$W_{i,t}^{\mathbb{Q}} = W_{i,t} + \int_0^t \Theta_i^{\mathbb{P}} du. \tag{A.26}$$

Notice the SDF can be equivalently expressed as

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^* dt - \Theta_c\sqrt{1-\rho_{cg}^2}\,dW_{c,t}^{\top} - \left(\rho_{cg}\Theta_c + \Theta_g\right)dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.27}$$

where $dW_{c,t}^{\top}$ is an orthogonal Brownian. Under the $\mathbb{Q}$-measure the risk neutral diffusion for g_t is given by

$$dg_t = [a_{g0}^{\mathbb{Q}} + a_{g1}^{\mathbb{Q}} g_t]dt + \sigma_g dW_{g,t}^{\mathbb{Q}}, \tag{A.28}$$

$$a_{g0}^{\mathbb{Q}} = a_{g0} - (\rho_{cg}\Theta_c + \Theta_g)\sigma_g \tag{A.29}$$

$$a_{g1}^{\mathbb{Q}} = a_{g1} \tag{A.30}$$

Computing bond prices follows from the standard risk neutral pricing approach requiring a solution to

$$\frac{\partial P^*_\tau}{\partial \tau} = \Big[a^{\mathbb{Q}}_{g0} + a^{\mathbb{Q}}_{g1} g_t\Big]\frac{\partial P^*_\tau}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 P^*_\tau}{\partial g_t^2} - r_t P^*_\tau \tag{A.31}$$

Conjecture the solution

$$P^*_\tau = \exp\Big(A^*(\tau) + B(\tau) g_t\Big). \tag{A.32}$$

Taking partials, substituting in the PDE above yields the system of equations

$$\frac{\partial A^*(\tau)}{\partial \tau} = B(\tau) a^{\mathbb{Q}}_{g0} + \frac{1}{2}B(\tau)^2\sigma_g^2 - r_0 \tag{A.33}$$

$$\frac{\partial B(\tau)}{\partial \tau} = B(\tau) a_{g1} - r_g \tag{A.34}$$

The boundary conditional for the ODEs will have to be $A(0) = B(0) = 0$ which implies $P^*_0 = 0$. $B(\tau)$ is a first order non-homogeneous linear DE, which can be solved using an integrating factor where

$$B(\tau) = \frac{r_g}{a_{g1}}\big(1 - e^{a_{g1}\tau}\big) \tag{A.35}$$

Note that $B(\tau) < 0$ because higher real growth rates increases short rates and lowers prices. Then $A(\tau)$ is an ugly expression that follows by direct integration. The specific preferences specifications that we consider below will generate differences in bond pricing solutions through the risk adjustments $\{\Theta_c, \Theta_g\}$ and through the short rate loadings $\{r_0, r_g\}$. The nominal SDF is given by $\Lambda_t = d\Lambda^*_t Q_t^{-1}$

$$d\Lambda_t = d\Lambda^*_t Q_t^{-1} + \Lambda^*_t dQ_t^{-1} + d\Lambda^*_t dQ_t^{-1} \tag{A.36}$$

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{d\Lambda^*_t}{\Lambda^*_t} - \frac{dQ}{Q} - \sigma_q^2 dt - \frac{d\Lambda^*_t}{\Lambda^*_t}\frac{dQ_t}{Q_t} \tag{A.37}$$

Thus,

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.38}$$

$$r_t = r^*_t + \pi - \sigma_Q^2 \tag{A.39}$$

$$\Theta_Q = \sigma_Q \tag{A.40}$$

and the nominal bond pricing function is given by

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\frac{Q_t}{Q_T}\right] = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] E_t\left[\frac{Q_t}{Q_T}\right] = \exp(A^*(\tau) + B(\tau)g_t)\exp\left([\pi - \frac{1}{2}\sigma_Q^2]\tau\right) \tag{A.41}$$

$$= \exp(A(\tau) + B(\tau)g_t) \quad \text{where } A(\tau) = A^*(\tau) + [\pi - \frac{1}{2}\sigma_Q^2]\tau \tag{A.42}$$

The diffusion for bond returns follows as an application of itô's lemma

$$\frac{dP_\tau}{P_\tau} = \mu_\tau dt + \sigma_{\tau,g} dW_{g,t} \tag{A.43}$$

where

$$\mu_\tau = \frac{1}{\mu_{t,\tau}}\left[\frac{\partial P}{\partial g}\mu_g + \frac{1}{2}\frac{\partial^2 P}{\partial g^2}\sigma_g^2 + \frac{\partial P}{\partial \tau}\right] \tag{A.44}$$

$$= \left[B(\tau)(a_{g0} + a_{g1}g_t) + \frac{1}{2}B(\tau)^2\sigma_g^2 + A'(\tau) + B'(\tau)g_t\right] \tag{A.45}$$

$$\sigma_{\tau,g} = \frac{1}{P(\tau)}\left[\frac{\partial P}{\partial g}\sigma_g\right] = B(\tau)\sigma_g \tag{A.46}$$

Nominal instantaneous expected excess returns are given by

$$E_t\left[\frac{dP_\tau}{P_\tau}\right] - \frac{\partial P_\tau}{\partial \tau} - E_t^{\mathbb{Q}}\left[\frac{dP_\tau}{P_\tau}\right] = -E_t\left[\frac{d\Lambda_t}{\Lambda_t}\frac{dP_\tau}{P_\tau}\right] \tag{A.47}$$

$$\frac{1}{dt}BRP(\tau) = -E_t\left[\frac{d\Lambda_t}{\Lambda_t}\frac{dP_\tau}{P_\tau}\right] \tag{A.48}$$

$$= \Theta_c B(\tau)\sigma_g\rho_{cg} + \Theta_g B(\tau)\sigma_g \tag{A.49}$$

D. The Subjective Measure

Here, we use superscripts $\mathbb{P}$ to indicate the physical SDF which is that of an econometrician who has full information about the data generating process and the parameters which govern its distribution. The relationship between the econometricians measure and the subjective measure is given by $E_t^{\mathbb{P}}[z_s] = E_t^{\mathbb{S}}\left[\frac{d\mathbb{P}}{d\mathbb{S}}z_s\right]$where from Girsanov[16]

$$\frac{d\mathbb{P}}{d\mathbb{S}} = \exp\left(-\int_t^s \phi_z dW_{z,u}^{\mathbb{P}} - \frac{1}{2}\int_0^t \phi_z^2 du\right), \tag{A.50}$$

$$dW_{z,t}^{\mathbb{P}} = dW_{z,t}^{\mathbb{S}} + \phi_z dt \tag{A.51}$$

We call ϕ_z a BELIEF DISTORTION. Now we have two changes of measure which satisfy

$$dW_{z,t}^{\mathbb{Q}} = dW_{z,t}^{\mathbb{P}} + \Theta_z^{\mathbb{P}}dt \quad , \quad dW_{z,t}^{\mathbb{Q}} = dW_{z,t}^{\mathbb{S}} + \Theta_z^{\mathbb{S}}dt, \tag{A.52}$$

$$dW_{z,t}^{\mathbb{P}} = dW^{\mathbb{S},z} + \phi_z dt \quad \rightarrow \quad \phi^z = \Theta_z^{\mathbb{S}} - \Theta_z^{\mathbb{P}} \tag{A.53}$$

It what follows we assume that a belief distortion only exists on the distribution of g_t

$$dg_t = [a_{g0}^{\mathbb{S}} + a_{g1}^{\mathbb{S}}g_t]dt + \sigma_g dW_{g,t}^{\mathbb{S}}, \tag{A.54}$$

$$a_{g0}^{\mathbb{S}} = a_{g0} + \phi_g\sigma_g \tag{A.55}$$

$$a_{g1}^{\mathbb{S}} = a_{g1} \tag{A.56}$$

[16]We assume $\mathbb{S}$ is absolutely continuous with respect to $\mathbb{P}$.

The subjective SDF is given by

$$\frac{d\Lambda_t^{\mathbb{S}}}{\Lambda_t^{\mathbb{S}}} = -r_t dt - \Theta_c^{\mathbb{S}} dW_{c,t}^{\mathbb{S}} - \Theta_g^{\mathbb{S}} dW_{g,t}^{\mathbb{S}} - \Theta_Q^{\mathbb{S}} dW_{Q,t}^{\mathbb{S}} \tag{A.57}$$

$$\Theta_c^{\mathbb{S}} = \Theta_c^{\mathbb{P}} \tag{A.58}$$

$$\Theta_g^{\mathbb{S}} = \Theta_g^{\mathbb{P}} + \phi_g \tag{A.59}$$

$$\Theta_Q^{\mathbb{S}} = \Theta_Q^{\mathbb{P}} \tag{A.60}$$

Note that bond prices can equivalently be solved for under the subjective or physical measure via the changes of measure defined above:

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] \tag{A.61}$$

$$= E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] \tag{A.62}$$

$$= E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\frac{d\mathbb{S}}{d\mathbb{P}}\right] \tag{A.63}$$

$$= E_t^{\mathbb{S}}\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\right]$$

$$= E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right] = \exp\Big(A_\tau + B_\tau g_t\Big). \tag{A.64}$$

E. Recursive HJB equation

The HJB equation implied by the normalised aggregator is given by

$$\sup_{\{C\}} f(C, J) + \mathcal{D}J(\cdot) = 0 \tag{A.65}$$

which spelling out is equal to

$$\frac{1}{dt}\sup_{\{C\}} f(C, J)dt + J_g E(dg) + J_C E(dC) + \frac{1}{2}\left[J_{gg} d\langle g, g\rangle + J_{CC} d\langle C, C\rangle\right] + J_{gC} d\langle g, C\rangle = 0 \tag{A.66}$$

which after inserting the states is given by

$$\sup_{\{C\}} f(C, J) + J_g(a_{g0} + a_{g1} g) + J_C C g + \frac{1}{2}\left[J_{gg}\sigma_g^2 + J_{CC} C^2 \sigma_c^2\right] + J_{gC} C \sigma_g \sigma_c \rho_{cg} = 0 \tag{A.67}$$

and we denote the maximised aggregator f^*. In the following we will solve for an exact and an approximate solution based on the following ansatz

$$J(W, g) = F(g)\frac{W^{1-\gamma}}{1-\gamma} \tag{A.68}$$

$$F(g) = \exp(a_{F0} + a_{Fg} g) \tag{A.69}$$

F. Log-linear solution

For $\psi \neq 0$ the consumption-wealth ratio again follows from the envelope condition

$$\frac{C}{W} = \beta^{\psi} F^{\frac{1-\psi}{1-\gamma}} = \beta^{\psi} F^{\alpha} \tag{A.70}$$

where we have defined

$$\alpha \equiv \frac{1-\psi}{1-\gamma} \tag{A.71}$$

The HJB becomes

$$\begin{aligned} &f^* J^{-1} + a_{Fg}\psi(a_{g0} + a_{g1}g) + (1-\gamma)g + \frac{1}{2}\Big[(a_{Fg}\psi)^2\sigma_g^2 + (-\gamma)(1-\gamma)C^{-2}C^2\sigma_c^2\Big] \\ &+ a_{Fg}\psi C^{-1}(1-\gamma)C\sigma_g\sigma_c\rho_{cg} = 0 \end{aligned}$$

From $f = \theta J(\beta Z - \beta)$ we can re-write the first term as

$$f^* J^{-1} = \theta J(\beta Z - \beta)J^{-1} = \theta\beta Z - \theta\beta = \theta\frac{C}{W} - \theta\beta \tag{A.72}$$

Where we obtain the last equality by nothing that $Z(C, J)$ can be re-written in terms of the consumption wealth ratio since

$$\begin{aligned} Z =& (1-\gamma)^{-1/\theta}\beta^{\psi-1}\exp\left([a_{F0} + a_{Fg}g]\left(\frac{1-\psi}{1-\gamma}\right)\left(\frac{\psi-1}{\psi}\right)\right) W_t^{\frac{\psi-1}{\psi}} \qquad (A.73) \\ &\times \exp\left([a_{F0} + a_{Fg}g]\left(\frac{1/\psi - 1}{\gamma - 1}\right)\right)\frac{W_t^{1/\psi-1}}{(1-\gamma)^{-1/\theta}} \\ Z =& \beta^{\psi-1}\exp\left([a_{F0} + a_{Fg}g]\alpha\right) W_t^0 \qquad (A.74) \\ \beta Z =& \frac{C_t}{W_t} \qquad (A.75) \end{aligned}$$

Following Chacko and Viceira (2005), define $\mu_{cw} = E[c_t - w_t]$ where lowercases indicate logs, and take a Taylor expansion of the consumption wealth ratio around its long run mean

$$\begin{aligned} \frac{C}{W} &= \exp(c-w) \sim \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})(c-w) = h_0 + h_1[a_{F0} + a_{Fg}g]\alpha \qquad (A.76) \\ \frac{C}{W} &\sim h_0 + h_1[a_{F0} + a_{Fg}g]\alpha \qquad (A.77) \end{aligned}$$

$h_0 = \exp(\mu_{cw})(1 - \mu_{cw} + \psi \ln\beta)$ and $h_1 = \exp(\mu_{cw})$ are determined endogenously in equilibrium. Using $f = \theta J(\beta Z - \beta)$ we linearise the non-homogeneity in the Bellman equation. Then substituting into the Bellman gives

$$\begin{aligned} &\theta h_0 + \theta h_1\alpha(a_{F0} + a_{Fg}g) - \theta\beta + a_{Fg}\psi(a_{g0} + a_{g1}g) + (1-\gamma)g \\ &+ \frac{1}{2}\Big[(a_{Fg}\psi)^2\sigma_g^2 + \gamma(\gamma-1)\sigma_c^2\Big] + a_{Fg}\psi(1-\gamma)\sigma_g\sigma_c\rho_{cg} = 0 \end{aligned}$$

Nothing that

$$\theta\alpha = \left(\frac{1-\gamma}{1-1/\psi}\right)\left(\frac{1-\psi}{1-\gamma}\right) = \frac{1-\psi}{1-1/\psi} = \frac{\psi(1-\psi)}{\psi-1} = -\psi \tag{A.78}$$

via separation of variables we have

$$\begin{aligned} a_{F0} =& \frac{1}{\psi h_1}\left(\theta(h_0-\beta) + a_{Fg}\psi a_{g0} + \frac{1}{2}\Big[(a_{Fg}\psi)^2\sigma_g^2 + \gamma(\gamma-1)\sigma_c^2\Big] + a_{Fg}\psi(1-\gamma)\sigma_g\sigma_c\rho_{cg}\right) \\ a_{Fg} =& \frac{1-\gamma}{\psi(h_1-a_{g1})} \end{aligned}$$

Now we see the effect of separating risk aversion with EIS. The sign of $h_1 a_{Fg}\alpha$ determines whether the consumption-wealth ratio (also the dividend-price ratio) rises or falls on good news (positive $dW_{g,t}$ shocks

$$\frac{\partial C/W}{\partial g} = h_1\frac{1-\gamma}{\psi(h_1-a_{g1})}\frac{1-\psi}{1-\gamma} \tag{A.79}$$

$$= \left(\frac{h_1}{h_1-a_{g1})}\right)\frac{1-\psi}{\psi} \tag{A.80}$$

which is negative for $\psi > 1$ meaning that prices (wealth) rise faster than consumption. We now have to solve for h_1. Spelling things out explicitly

$$E\left[\frac{C}{W}\right] \sim \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})E[(c-w)] \tag{A.81}$$

$$= \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})\mu_{cw} \tag{A.82}$$

$$= \exp(\mu_{cw}) = h_1 \tag{A.83}$$

$$E\left[\frac{C}{W}\right] \sim h_1 \tag{A.84}$$

So

$$h_1 = E\left(\frac{C_t}{W_t}\right) = E\left[\beta^\psi \exp\left((a_{F0} + a_{Fg}g_t)\alpha\right)\right] \tag{A.85}$$

$$= \beta^\psi e^{a_{F0}\alpha} E\left[e^{a_{Fg}\alpha g_t}\right] \tag{A.86}$$

$$= \beta^\psi e^{a_{F0}\alpha} E\left[e^{u g_t}\right] \quad \text{where } u = a_{Fg}\alpha \tag{A.87}$$

We first need the *conditional* Laplace transform for g_t

$$\phi(t,h;u) = E_t\left(e^{u\cdot g_{t+h}}\right) \tag{A.88}$$

which Feynman-Kac says this function satisfies the following partial differential equation

$$0 \equiv \mathcal{D}\phi(t,h;u) + \frac{\partial\phi}{\partial t}(t,h;u) \tag{A.89}$$

with initial condition $\phi(t,0;u) = e^{u \cdot g_t}$ and where $\mathcal{D}$ is the infinitesimal generator for g_t. Spelling out the partials

$$\frac{\partial P}{\partial g}(a_{g0} + a_{g1} g_t) + \frac{1}{2}\frac{\partial^2 P}{\partial g^2}\sigma_g^2 + \frac{\partial P}{\partial t} = 0 \tag{A.90}$$

We conjecture an exponentially affine solution

$$\phi(t,h;u) = e^{A_{h_1}(h,u) + B_{h_1}(h,u) g_t} \tag{A.91}$$

with $A_{h_1}(0,u) = 0$ and $B_{h_1}(0,u) = u$. After substituting in we obtain

$$B_{h_1 g}(a_{g0} + a_{g1} g_t) + \frac{1}{2} B_{h_1 g}^2 \sigma_g^2 = \frac{\partial A_{h_1}}{\partial h} + \frac{\partial B_{h_1}}{\partial h} g_t \tag{A.92}$$

Collecting terms we obtain the system of equations

$$\frac{\partial A_{h_1}}{\partial \tau} = B_{h_1 g} a_{g0} + \frac{1}{2} B_{h_1 g}^2 \sigma_g^2 \tag{A.93}$$

$$\frac{\partial B_{h_1}}{\partial \tau} = B_{h_1 g} a_{g1} \tag{A.94}$$

whose solutions are given by

$$A_{h_1}(h,u) = u\frac{a_{g0}}{a_{g1}}\left[e^{a_{g1} h} - 1\right] + \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}\left[e^{2 a_{g1} h} - 1\right] \tag{A.95}$$

$$B_{h_1}(h,u) = u e^{a_{g1} h} \tag{A.96}$$

Notice that $\lim_{h\to\infty} B_{h_1}(h,u) = 0$ only in the case that the parameter $a_{g1} < 0$ which is stationarity condition of the OU process. Under this condition the transition at horizon h tends to the stationary distribution (or marginal distribution, i.e., the distributions dependence on g_t has been marginalised out). The marginal distribution admits the Laplace transform

$$E\left[e^{u g_t}\right] = \lim_{h\to\infty} e^{A_{h_1}(h,u)} = e^{-u\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}} \tag{A.97}$$

and so

$$h_1 = \beta^{\psi} e^{a_{F0}\alpha} E\left[e^{u g_t}\right] \tag{A.98}$$

$$= \beta^{\psi} e^{a_{F0}\alpha} e^{-u\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}} \tag{A.99}$$

$$= \beta^{\psi} e^{a_{F0}\alpha - a_{Fg}\alpha\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 a_{Fg}^2 \alpha^2}{a_{g1}}} \tag{A.100}$$

A solution for h_1 follows from a fixed point iteration. Specifically, empirically speaking $\mu_{cw} = E[c_t - w_t] \sim -6 \to h_1 \sim 0.003$ so we take this as an initial value and update for a new h_1 by updating the parameters a_{F0} and a_{Fg} repeat this procedure until convergence.

G. Risk Free Rate

Above we derived

$$f_C = \beta(1-\gamma)JC^{-1}Z(C,J). \tag{A.101}$$

which we can re-express as

$$f_C = \beta^{\gamma\psi} \exp\left((a_{F0} + a_{Fg}g)\chi\right) C^{-\gamma} \tag{A.102}$$

where we have defined another constant

$$\chi = \psi + \alpha = \left(\frac{1-\psi\gamma}{1-\gamma}\right) \tag{A.103}$$

Similarly

$$f_J = (\theta-1)h_0 - \theta\beta - h_1\chi(a_{F0} + a_{Fg}g) \tag{A.104}$$

The diffusions for f_C is given by

$$\frac{df_C}{f_C} = \left(-\gamma g + a_{Fg}\chi(a_{g0} + a_{g1}g) + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 - \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg}\right) dt \tag{A.105}$$

$$- \gamma\sigma_c dW_{c,t} + a_{Fg}\chi\sigma_g dW_{g,t} \tag{A.106}$$

The risk free rate is then given by

$$r_t^* = -\frac{1}{dt}E_t\left(\frac{df_C}{f_C}\right) - f_J \tag{A.107}$$

$$= \left(\gamma g_t - a_{Fg}\chi(a_{g0} + a_{g1}g_t) - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 + \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg}\right) \tag{A.108}$$

$$- (\theta-1)h_0 + \theta\beta + h_1\chi(a_{F0} + a_{Fg}g_t) \tag{A.109}$$

which for convenience we denote

$$r_t^* = r_0 + r_g g_t, \tag{A.110}$$

$$r_0 = +\theta\beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - a_{Fg}\chi a_{g0} - \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 + \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg} - (\theta-1)h_0 + h_1\chi a_{F0} \tag{A.111}$$

$$r_g = \frac{1}{\psi} \tag{A.112}$$

Notice in the limit $\psi \to \frac{1}{\gamma}$ we recover the CRRA solution above $r_x = \gamma$ and $r_0 = \theta\beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2$ since in this case $\theta = 1$ and $\chi = 0$. We now want the diffusion for the SDF which follows

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^* dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} \tag{A.113}$$

$$\Theta_c = \gamma\sigma_c \tag{A.114}$$

$$\Theta_g = -a_{Fg}\chi\sigma_g = \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \tag{A.115}$$

Note the price of long run risk shocks can be positive or negative but for preference for early resolution $a_{Fg} < 0$ and so negative long run growth shocks are bad news. The nominal SDF follows

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.116}$$

$$r_t = r_t^* + \pi - \sigma_Q^2 \tag{A.117}$$

$$\Theta_c = \gamma\sigma_c \tag{A.118}$$

$$\Theta_g = \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \tag{A.119}$$

$$\Theta_Q = \sigma_Q \tag{A.120}$$

The nominal bond risk premium is constant and given by

$$BRP_\tau = \Theta_c B(\tau)\sigma_g\rho_{cg}dt + \Theta_g B(\tau)\sigma_g dt \tag{A.121}$$

$$\frac{1}{dt}BRP_\tau^* = \gamma\sigma_c\sigma_g\rho_{cg}B(\tau) + \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g^2 B(\tau) \tag{A.122}$$

The first term is the standard one arising in a CRRA Lucas tree economy: when $\langle dW_{c,t}, dW_{g,t}\rangle = \rho_{cg} > 0$ bonds have a hedging property against $dW_{c,t}$ shocks. The second term arises when agents have a preference for temporal resolution. The early resolution condition is $\gamma > \frac{1}{\psi}$ in which case $dW_{g,t} < 0$ are bad states of the world and bonds also hedge these shocks.

A.4. Simulated Method of Moments

We estimate the model via simulated method of moments (SMM) which is analogous to the generalized method of moments (GMM) estimator, but allows us to estimate the parameters even if latent long-run growth factors are not directly observable. Moreover, SMM avoids difficulties of computing analytical moment conditions which given the number of moment conditions is a tedious procedure.

We collect the structural parameters of interest in a $q \times 1$ parameter vector β. Given β, we simulate the counterpart $\tilde{x}(\beta)$ of the observed data sample x using the model specification. Given a simulation of length $\tau \times T$ where $\tau > 1$ we obtain the $p > q$ vector of moments $M(\tilde{x}(\beta))$. Under the assumption that the model $\tilde{x}(\beta)$ is correctly specified and β_0 is the true structural parameter vector, the moment conditions $M(\tilde{x}(\beta_0))$ converge asymptotically to the sample moments $M(x)$. Thus, SMM proceeds in a similar fashion to GMM by choosing $\widehat{\beta}$ to minimize the weighted sum of squared moments errors $G_T(\hat{\beta}) = \left[M(\tilde{x}(\hat{\beta})) - M(x)\right]$

$$\widehat{\beta} = \min_{\hat{\beta}} \; G_T^\top \; W \; G_T \tag{A.123}$$

where W is a $q \times q$ positive definite weighting matrix. Under the regularity conditions set out in Duffie and Singleton (1993), $\widehat{\beta}$ is a consistent estimate of β_0.

For a general weighting matrix W, the asymptotic distribution of the parameters is given by

$$\sqrt{T}(\widehat{\beta} - \beta_0) \sim \mathcal{N}[0, (1 + 1/\tau)V], \tag{A.124}$$

$$\text{where } V = (d^\top W d)^{-1} d^\top W S W d (d^\top W d)^{-1} \quad \text{and} \quad d = E\left[\frac{\partial M}{\partial \beta}\right]. \tag{A.125}$$

Diffusions are discretized using a Milstein scheme (Kloeden and Platen (2013)). Our sample moments are estimated with $T = 20$ years of data, we set $\tau = 100$ and discard the first 5-years of each path to avoid sensitivity to initial conditions. Random number streams are held constant for each simulation path to avoid introducing sampling error. Moments $\widehat{M_S}$ are computed on data sampled at monthly frequency consistent with the sampling frequency of the empirical moment vector M_D. $\widehat{S}$ is computed using 100 block bootstrap samples of moments from the data and a Newey and West (1987) variance covariance estimate with lag length $K = 12$. The Jacobian d is computed using 100 replications in line with the recommendations of Gouriéroux and Monfort (2000), and we use the identity matrix as weighting matrix in the estimation, but replace (i, i) elements corresponding to subjective moments with 10's, i.e., giving these moments 2 times the influence in estimation, to counteract the fact that we have more moments related to the physical probability distribution. Note that 2/3's of all moments are physical (term structure moments) and 1/3 subjective (subjective risk premia and forecast errors).

A.5. Tables

Panel (a): Summary Statistics	Yields		Returns	
	Mean	Std Dev	Mean	Std Dev
Par	3.87	1.64	4.53	1.49
Zero	3.98	1.67	4.79	1.55
Difference	−0.13	0.09	−0.28	0.21
Panel (b): Frequency Comparison	Yields			
	Mean	Std Dev		
Continuous	3.98	1.67		
Annualy	4.07	1.74		
Semi-Annual	4.02	1.70		

Table A.1. Summary Statistics: Par Yields and Zero Yields
This table reports the means and standard deviations for (1) US 10-year par yields obtained from the Fed, (2) US 10-year zero yields obtained from Bloomberg, and (3) their differences. Panel (b) contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Sample period is monthly observations from between 1995.1 and 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): 1995.1 to 2007.12.										
Mean	−0.82	0.91	−1.61	−1.02	−0.86	0.58	−2.67	−0.29	−0.04	−0.73
Std	3.77	3.73	2.04	2.03	2.66	2.02	3.22	3.26	3.33	2.96
Skew	1.08	0.62	0.07	0.10	−0.03	0.58	−0.09	0.66	0.51	0.28
AR(1)	0.18	0.43	−0.17	0.03	0.49	0.15	0.63	0.10	0.53	0.23
Panel (b): 2008.1 to 2020.12.										
Mean	−0.85	−1.76	−2.13	−1.33	−0.24	−0.02	−2.78	−1.60	−3.08	−2.07
Std	3.10	3.34	1.93	2.23	3.33	1.34	3.43	3.76	3.13	2.56
Skew	0.41	0.96	0.10	0.77	0.56	0.37	−0.41	0.25	0.11	0.55
AR(1)	−0.27	0.30	0.04	−0.13	0.47	0.12	−0.04	−0.20	−0.08	0.03

Table A.2. Sub Sample Summary Statistics BRP
This table presents the means, standard deviations, skewness and AR(1) coefficients for subjective bond risk premia (BRP) for the sample periods 1995.1 to 2007.12. (panel A) and 2008.1 to 2020.12. (panel B). The reported AR(1) coefficients relate to the yearly autocorrelation.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
					Panel (a): BRP projected					
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19	4.65
Std	2.99	3.21	1.88	2.98	2.82	2.57	3.44	2.49	3.77	1.86
Skew	0.04	0.40	0.29	0.27	0.24	0.44	−0.33	0.10	0.56	0.15
AR(1)	0.28	0.33	0.39	0.48	0.75	0.78	0.27	0.46	0.01	0.59
					Panel (b): XRP projected					
Mean	1.43	0.29	−0.68	−1.32	−0.31	−3.00	−0.89	2.19	−1.20	
Std	4.19	1.15	4.24	3.53	1.40	3.18	3.37	3.79	3.65	
Skew	0.06	−0.11	0.02	0.13	0.25	−0.33	0.58	−0.42	0.29	
AR(1)	0.66	0.29	0.75	0.65	0.62	0.82	0.57	0.51	0.48	

Table A.3. Descriptive Statistics for Projected Risk Premia

This table presents the means, standard deviations, skewness, and AR(1) coefficients for exchange rate risk premia (XRP) and bond risk premia (BRP) where conditional expectations defined in Equations 3 and 4 are replaced by projections. These predictions are obtained by regressing realized ex-post excess returns on the slope of the yield curve (for bond risk premia) and the interest rate differential between the foreign country and the United States (for exchange rate risk premia). The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
					Panel (a): *BRP*					
AUD	1.00	0.54	0.66	0.76	0.44	0.31	0.65	0.75	0.61	0.62
CAD	0.54	1.00	0.48	0.68	0.52	0.52	0.45	0.66	0.53	0.69
CHF	0.66	0.48	1.00	0.72	0.21	0.36	0.66	0.59	0.66	0.54
EUR	0.76	0.68	0.72	1.00	0.54	0.37	0.65	0.68	0.69	0.69
GBP	0.44	0.52	0.21	0.54	1.00	0.28	0.19	0.34	0.21	0.56
JPY	0.31	0.52	0.36	0.37	0.28	1.00	0.32	0.36	0.41	0.32
NOK	0.65	0.45	0.66	0.65	0.19	0.32	1.00	0.59	0.73	0.46
NZD	0.75	0.66	0.59	0.68	0.34	0.36	0.59	1.00	0.65	0.62
SEK	0.61	0.53	0.66	0.69	0.21	0.41	0.73	0.65	1.00	0.49
USD	0.62	0.69	0.54	0.69	0.56	0.32	0.46	0.62	0.49	1.00
					Panel (b): *XRP*					
AUD	1.00	0.72	0.55	0.78	0.53	−0.07	0.54	0.91	0.78	
CAD	0.72	1.00	0.30	0.53	0.40	−0.17	0.47	0.71	0.58	
CHF	0.55	0.30	1.00	0.84	0.50	0.41	0.72	0.49	0.69	
EUR	0.78	0.53	0.84	1.00	0.67	0.15	0.76	0.73	0.88	
GBP	0.53	0.40	0.50	0.67	1.00	0.12	0.55	0.46	0.62	
JPY	−0.07	−0.17	0.41	0.15	0.12	1.00	0.06	−0.17	0.01	
NOK	0.54	0.47	0.72	0.76	0.55	0.06	1.00	0.48	0.77	
NZD	0.91	0.71	0.49	0.73	0.46	−0.17	0.48	1.00	0.69	
SEK	0.78	0.58	0.69	0.88	0.62	0.01	0.77	0.69	1.00	

Table A.4. Cross Country Correlations of Survey Premia

This table shows the correlation coefficients between the measures of subjective risk premia, as defined in Section III in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
				Panel (a): Δx_{t+1}					
Mean	0.09	0.85	−0.11	0.93	−0.26	−0.37	2.92	−0.99	3.70
Std	5.33	2.50	5.08	4.35	2.81	4.81	4.25	5.85	4.72
AR(1)	0.74	0.32	0.59	0.62	0.24	0.32	0.12	0.77	0.58
				Panel (b): IRD					
Mean	−1.64	−0.18	1.59	0.76	−0.47	2.39	−0.63	−2.13	0.22
Std	1.66	0.86	1.53	1.37	1.14	2.06	1.76	1.56	1.70
AR(1)	0.69	0.30	0.77	0.70	0.69	0.81	0.59	0.59	0.53
				Panel (c): $rx_{t+1}^{(11)}$					
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19
Std	8.94	6.65	5.80	6.69	7.33	4.46	7.20	8.29	8.68
AR(1)	−0.28	−0.03	−0.18	−0.30	−0.20	0.25	−0.08	−0.45	−0.15
				Panel (d): Slope					
Mean	0.78	1.26	0.98	1.35	0.93	1.09	0.80	0.55	1.27
Std	0.65	0.96	0.64	0.83	1.21	0.70	1.03	1.12	0.77
AR(1)	0.29	0.37	0.41	0.49	0.75	0.79	0.27	0.46	0.03

Table A.5. Descriptive Statistics Predictive Regressions

This table presents the means, standard deviations, skewness and AR(1) coefficient for log spot rate changes (Δx_{t+1}), interest rate differentials ($IRD_t = (i_t^{(1)} - i_t^{(1),f})$), realized excess bond returns ($rx_{t+1}^{(11)}$), and the slope of the yield curve ($i_t^{(10),f} - i_t^{(1),f}$). The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

*

	(i)	(ii)	(iii)	(iv)
		Panel (a): BRP		
E[con]	−0.57			
	[−0.98 −0.16]			
E[gdp]		−0.55		
		[−1.02 −0.09]		
$BondVar$			2.16	
			[0.87 3.45]	
$rx^{(11)}_{t-12,t}$				−0.09
				[−0.14 −0.04]
R^2 (%)	3.78	3.57	5.85	5.11
		Panel (b): XRP		
E[con]	1.08			
	[0.32 1.83]			
E[gdp]		0.95		
		[0.14 1.75]		
$FXVar$			0.78	
			[0.20 1.36]	
$rx^{FX}_{t-12,t}$				−0.09
				[−0.19 0.01]
R^2 (%)	6.36	4.87	3.58	4.81

Table A.6. Cyclicality Regressions: Subjective Bond & FX Risk Premia

This table reports estimates from pooled OLS regressions of the forms

$$BRP_t = a + b^\top X_t + \epsilon_t, \text{ and}$$
$$XRP_t = a + b^\top X_t + \epsilon_t,$$

in panels (a) and (b), respectively. BRP_t is the survey-implied bond risk premium, XRP_t is the survey-implied currency risk premium, and X_t is a vector of explanatory variables. For the specifications in panel (a), X_t contains the 1-year *expected* consumption (*con*) growth and domestic *gdp* growth, the realized bond variance ($BondVar$), and the excess return on an 11-year bond, realized between $t-1$ year and t. For the specifications in panel (b), X_t contains the 1-year *expected* consumption (*con*) growth and foreign *gdp* growth, the FX option-implied risk neutral variance constructed and discussed by Krohn, Mueller, and Whelan (2024) ($FXVar$), and the excess currency return realized between $t-1$ year and t. A constant is included but not reported. Confidence intervals in [·] are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. The sample period is 1995.1 to 2020.12 except for specification (iii) in panel (b) which only contains data from 1999.1 to 2019.4.

	(i)	(ii)
$E[con^f - con]$	0.72	
	[0.30 1.17]	
$E[gdp^f - gdp]$		0.76
		[0.17 1.35]
R^2 (%)	3.57	2.17

Table A.7. Cyclicality: Subjective FX Risk Premia (Macro Spreads)
This table reports estimates from pooled OLS regressions of the forms

$$XRP_t = a + b^\top X_t + \epsilon_t.$$

where XRP_t is the survey-implied currency risk premium (XRP_t), and X_t is a vector of explanatory variables containing spreads in 1-year *expected* consumption *con* growth and *gdp* growth between the foreign countries and the US. A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. The sample period is 1995.1 to 2020.12.

*

	(i)	(ii)	(iii)
		Panel (a): Bond Return	
Slope	2.81		2.49
	[1.78 3.85]		[1.35 3.63]
BRP		0.52	0.30
		[0.22 0.82]	[−0.03 0.63]
R^2 (%)	13.85	4.96	14.79
		Panel (b): FX Return	
IRD	−1.56		−1.41
	[−2.74 −0.38]		[−2.52 −0.31]
XRP		0.55	0.45
		[−0.03 1.12]	[−0.08 0.99]
R^2 (%)	8.89	5.43	12.54

Table A.8. Bond & FX Return Panel Predictability Regressions
This table reports estimates from pooled OLS regressions of the forms

$$rx^{(11)}_{t,t+1} = a + b_1 Slope_t + b_2 BRP_t \epsilon_{t,t+1},$$
$$rx^{FX}_{t,t+1} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1},$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively. $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ is the slope of the domestic yield curve, BRP_t is the survey-implied bond risk premium, $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$ is the one-year interest rate differential, and XRP_t is the survey-implied currency risk premium. We also report univariate regressions with the same predictors in columns (i) and (ii). A constant is included but not reported. Confidence intervals in [·] are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. The sample period is 1995.1 to 2020.12.

	Yields	FX
$Level_t$	0.08	
	[0.01 0.15]	
$Slope_t$	0.08	
	[−0.06 0.22]	
IRD_t		−0.95
		[−1.48 −0.42]
R^2 (%)	5.71	5.73

Table A.9. Forecast Error Predictability

This table shows pooled OLS regressions of (a) 10-year yields and (b) foreign exchange forecast errors, both at an annual horizon, on either the level and slope of the yield curve or the 1-year interest rate differential:

$$FE_{t,t+1} = a + bX_t + \eta_{t+1},$$

where for interest rate forecast errors X_t includes $Level_t = i_t^{(1)}$ and $Slope_t = (i_t^{(10)} - i_t^{(1)})$ and for exchange rate forecast errors it includes $IRD_t = (i_t^{(1)} - i_t^{(1),f})$. Confidence intervals in $[\cdot]$ are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. The sample period is 1995.1 to 2020.12

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
3m mean	4.13	2.52	0.82	1.70	2.92	0.12	3.18	4.68	2.18	2.33
3m delta std	0.87	0.76	0.66	0.64	0.78	0.25	1.20	1.04	0.74	0.73
5y mean	4.65	3.48	1.42	2.50	3.57	0.68	3.59	5.00	3.10	3.37
10y mean	4.00	4.01	1.96	3.16	3.97	1.28	4.00	5.25	3.62	3.98

Table A.10. Summary Statistics: Term Structure Moments

This table shows the means of 3 month, 5 year and 10 year bond yields for all countries in our sample. It also reports the annualised standard deviations of month-on-month differences in 3 month yields. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
mean	8.63	6.83	2.18	5.62	8.10	0.66	8.35	8.05	4.72	9.17
std	1.72	1.41	1.67	1.35	1.63	1.97	1.93	2.36	2.26	1.95
skew	0.62	0.38	0.07	−0.18	0.41	0.94	−0.24	0.38	0.19	−0.62
kurt	2.89	2.98	3.24	3.23	3.81	4.71	4.13	2.90	2.54	4.30
AR(1)	0.86	0.66	0.85	0.82	0.84	0.83	0.66	0.83	0.87	0.82
AR(5)	0.32	0.02	0.14	−0.00	0.28	0.04	−0.14	0.12	−0.02	0.04
AR(10)	−0.02	0.03	0.20	−0.25	0.03	−0.07	0.07	0.19	−0.24	0.02

Table A.11. Summary Statistics: Inflation Moments

This table shows the annualised means, annualised standard deviations, skewness and kurtosis as well as AR(1), AR(5) and AR(10) coefficients of realized inflation for Australia, Switzerland and the United States. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
AUD	1.00	0.99	0.63	0.96	0.99	0.15	0.98	0.99	0.94	0.98
CAD	0.99	1.00	0.71	0.98	0.00	0.26	0.99	0.99	0.96	0.99
CHF	0.63	0.71	1.00	0.82	0.73	0.83	0.77	0.68	0.81	0.76
EUR	0.96	0.98	0.82	1.00	0.99	0.39	0.99	0.97	0.97	0.99
GBP	0.99	0.00	0.73	0.99	1.00	0.28	0.00	0.99	0.95	0.99
JPY	0.15	0.26	0.83	0.39	0.28	1.00	0.32	0.19	0.40	0.29
NOK	0.98	0.99	0.77	0.99	0.00	0.32	1.00	0.98	0.97	0.00
NZD	0.99	0.99	0.68	0.97	0.99	0.19	0.98	1.00	0.97	0.98
SEK	0.94	0.96	0.81	0.97	0.95	0.40	0.97	0.97	1.00	0.96
USD	0.98	0.99	0.76	0.99	0.99	0.29	0.00	0.98	0.96	1.00

Table A.12. Correlation of realized inflation

This table shows the correlation coefficients between realized inflation rates between any two countries. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
mean	13.24	11.52	6.34	4.91	9.45	2.87	12.22	13.93	9.60	5.33
std	2.28	2.35	1.32	1.68	4.33	3.16	3.49	3.57	2.96	3.13
skew	0.17	−0.72	−0.36	−0.43	−1.64	−0.67	−0.54	−0.67	−0.06	−1.32
kurt	2.93	4.31	2.49	3.10	6.15	3.18	4.31	4.90	4.05	5.22
AR(1)	0.79	0.79	0.80	0.70	0.90	0.60	0.68	0.75	0.75	0.89
AR(5)	0.10	−0.20	0.07	0.24	0.40	−0.08	0.07	0.28	−0.15	0.39
AR(10)	0.02	0.15	−0.13	0.12	0.46	−0.19	0.11	0.00	−0.15	0.04

Table A.13. Summary Statistics: Consumption Growth Moments

This table shows the annualised means, annualised standard deviations, skewness and kurtosis as well as AR(1), AR(5) and AR(10) coefficients of realized consumption growth for Australia, Switzerland and the United States. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
AUD	1.00	0.98	0.98	0.91	0.87	0.61	0.98	0.99	0.97	0.74
CAD	0.98	1.00	0.00	0.97	0.94	0.72	0.00	0.00	0.99	0.85
CHF	0.98	0.00	1.00	0.97	0.94	0.73	0.99	0.99	0.99	0.85
EUR	0.91	0.97	0.97	1.00	0.99	0.85	0.96	0.96	0.97	0.95
GBP	0.87	0.94	0.94	0.99	1.00	0.90	0.95	0.94	0.96	0.97
JPY	0.61	0.72	0.73	0.85	0.90	1.00	0.74	0.73	0.78	0.95
NOK	0.98	0.00	0.99	0.96	0.95	0.74	1.00	0.00	0.00	0.85
NZD	0.99	0.00	0.99	0.96	0.94	0.73	0.00	1.00	0.99	0.84
SEK	0.97	0.99	0.99	0.97	0.96	0.78	0.00	0.99	1.00	0.87
USD	0.74	0.85	0.85	0.95	0.97	0.95	0.85	0.84	0.87	1.00

Table A.14. Correlation of realized consumption growth

This table shows the correlation coefficients between realized consumption growth rates between any two countries. Based on monthly observations from between 1995.1 to 2020.12.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.05	-0.06	0.16
$\theta_g(\%)$	1.31	1.10	1.52
$\sigma_c(\%)$	0.62	0.57	0.68

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	1.36	1.36	1.15	1.57
std (%)	0.67	0.67	0.61	0.73
skew	-0.40	-0.00	-0.00	0.00
kurt	4.01	2.86	2.85	2.87
AR(1)	0.60	0.11	0.07	0.15
AR(5)	0.25	0.08	0.05	0.12
AR(10)	0.10	0.03	0.00	0.06

Table A.15. US Consumption Growth Estimates

This table reports estimation results for the consumption growth process given by

$$
\begin{aligned}
\frac{dC_t}{C_t} &= g_t dt + \sigma_c dW_{c,t}, \\
dg_t &= \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.
\end{aligned}
$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.50$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.20	0.09	0.31
$\theta_g(\%)$	1.55	1.50	1.59
$\sigma_c(\%)$	0.53	0.51	0.56

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	1.57	1.57	1.53	1.61
std (%)	0.56	0.56	0.53	0.59
skew	0.00	-0.01	-0.01	-0.00
kurt	3.09	2.88	2.88	2.88
AR(1)	0.19	0.07	0.05	0.10
AR(5)	0.10	0.05	0.03	0.07
AR(10)	-0.05	0.01	-0.01	0.02

Table A.16. Swiss Consumption Growth Estimates

This table reports estimation results for the consumption growth process given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_c dW_{c,t},$$
$$dg_t = \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.50$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.06	-0.12	0.24
$\theta_g(\%)$	3.22	2.80	3.63
$\sigma_c(\%)$	0.92	0.83	1.01

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	3.29	3.29	2.87	3.71
std (%)	0.99	0.99	0.90	1.08
skew	0.20	-0.00	-0.01	0.00
kurt	2.75	2.86	2.84	2.88
AR(1)	0.08	0.11	0.05	0.17
AR(5)	0.05	0.08	0.02	0.13
AR(10)	0.06	0.03	-0.01	0.07

Table A.17. Australian Consumption Growth Estimates

This table reports estimation results for the consumption growth process given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_c dW_{c,t},$$
$$dg_t = \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.50$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

A.6. Figures

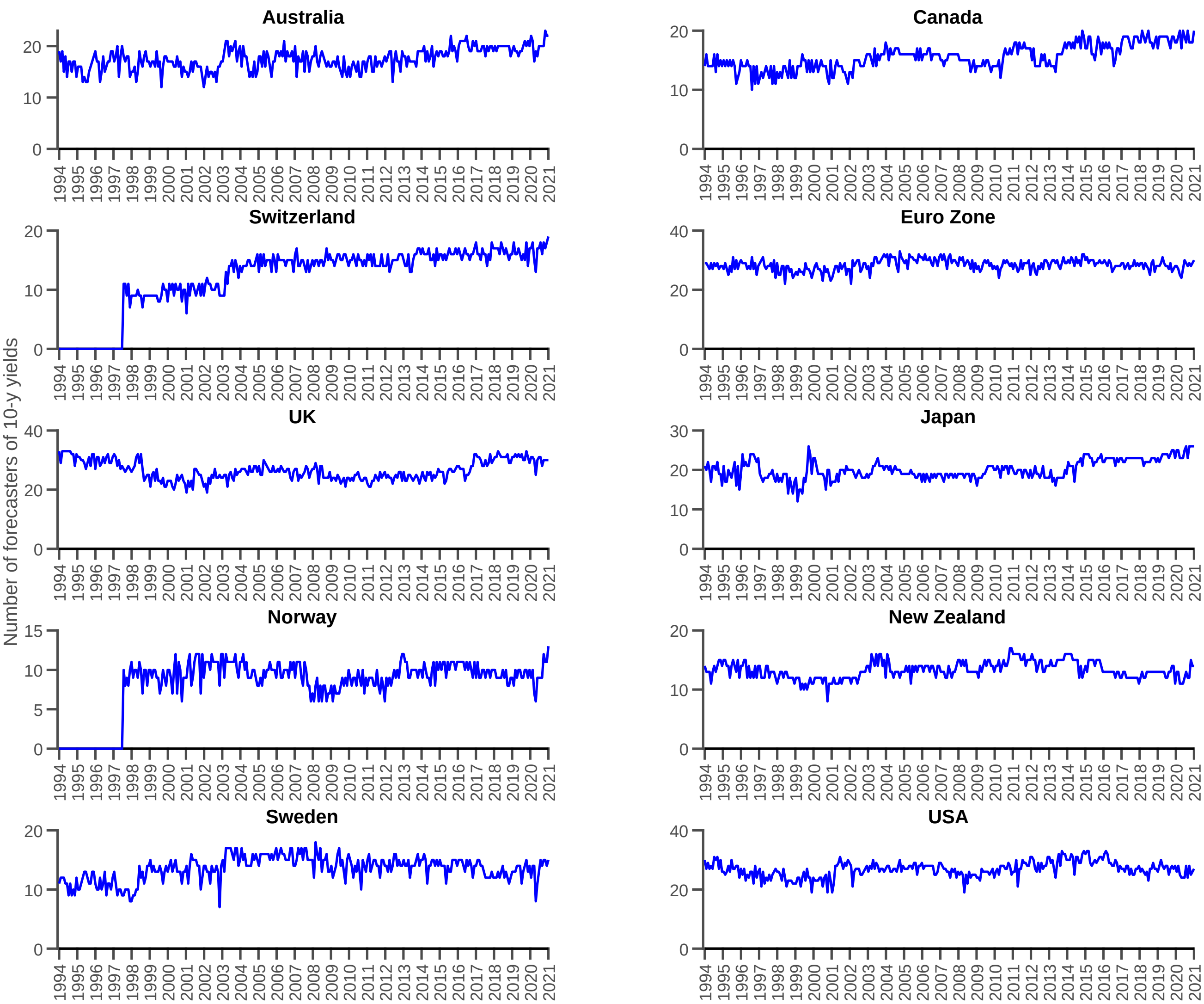
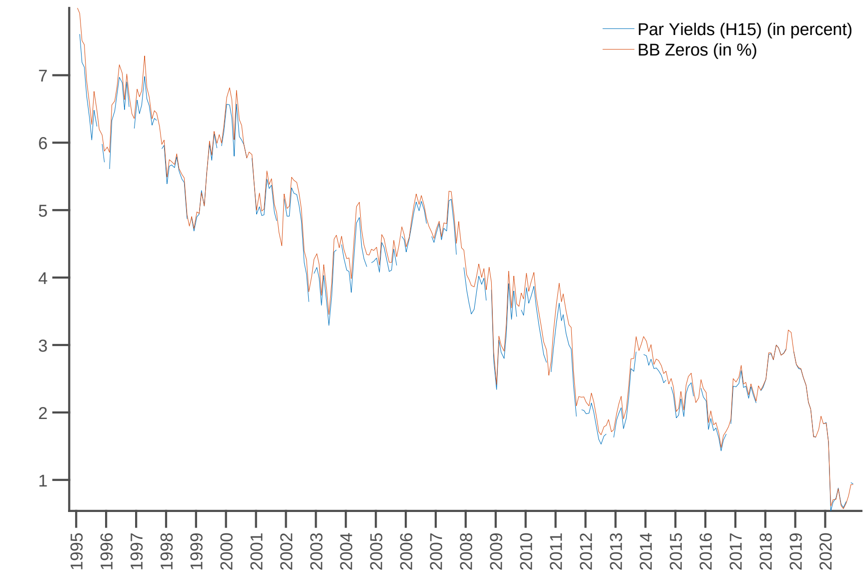
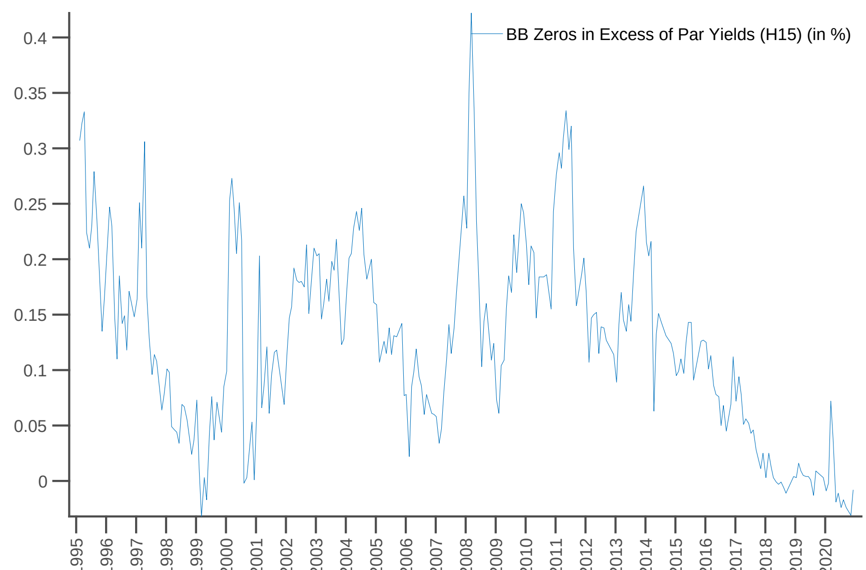


Figure A.1. Number of long-term bond yield forecasters by country
This figure displays number of forecasters predicting 10-year as part of the Consensus Economics surveys for the given country. The sample period is 1995.1 - 2020.12.



(a)



(b)

Figure A.2. Comparison of H15 and BB yields

The above figures show the time series of US 10-year par yields obtained from the Fed and US 10-year zero yields obtained from Bloomberg (Panel (a)) as well as the difference between the two series (Panel (b)). Data is available for the sample period 1995.1 to 2020.12.

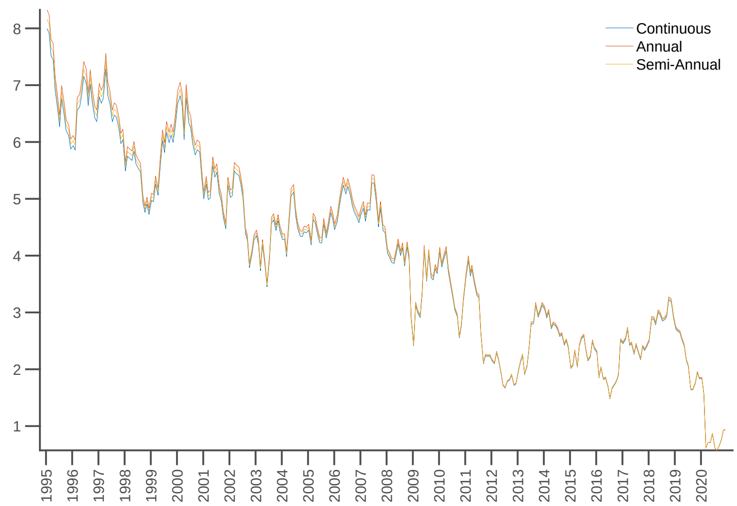


Figure A.3. Comparison of compounding frequencies
The above figure shows the time series of US 10-year zero log yields obtained from Bloomberg that have been generated assuming continuous, annual, and semi-annual compounding. Data is available for the sample period 1995.1 to 2020.12.

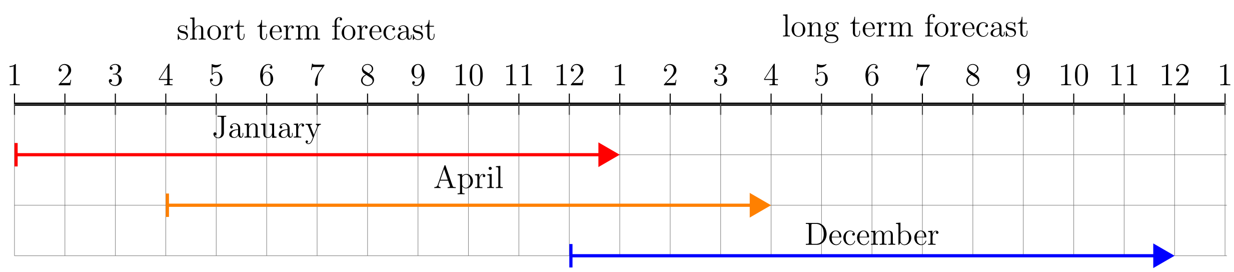
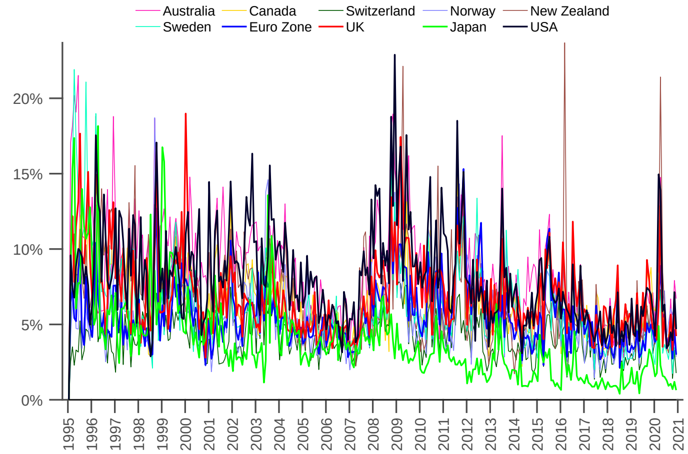
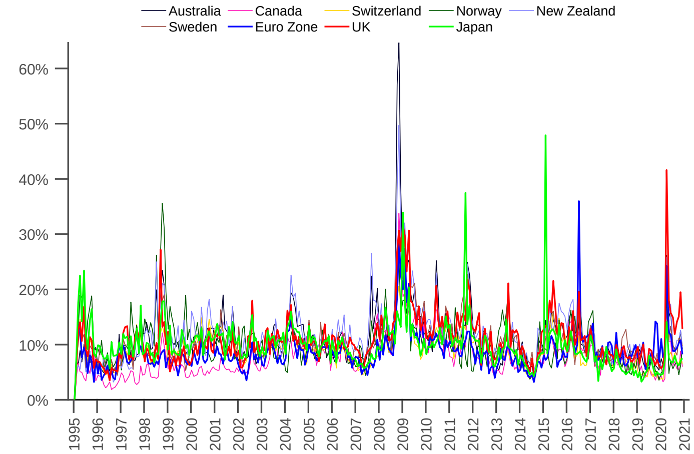


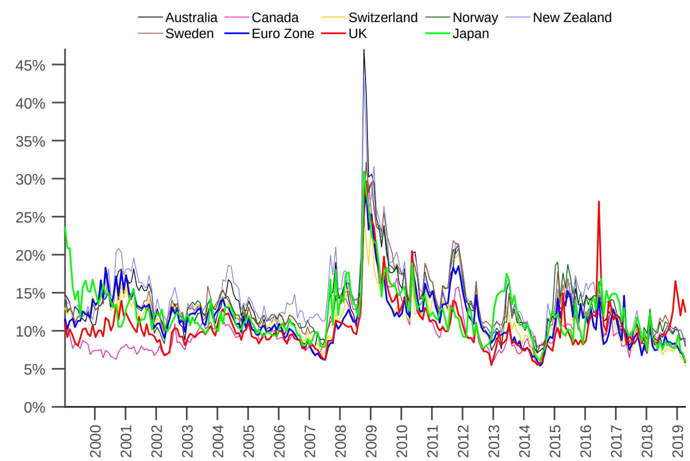
Figure A.4. Constant Maturity Macro Expectations
Figure displays a visual explanation to the construction of the constant maturity proxy. Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2, \ldots, 12$. A constant maturity expectation is formed taking as weight $(1 - \frac{j}{12})$, for the short term projection (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term projection (the forecast for the following year).



(a) Bond returns

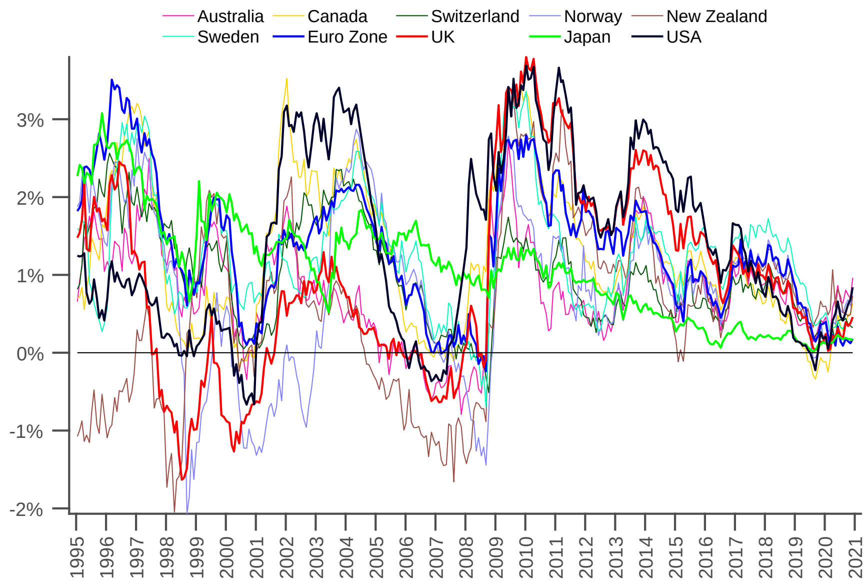


(b) Spot exchange rates

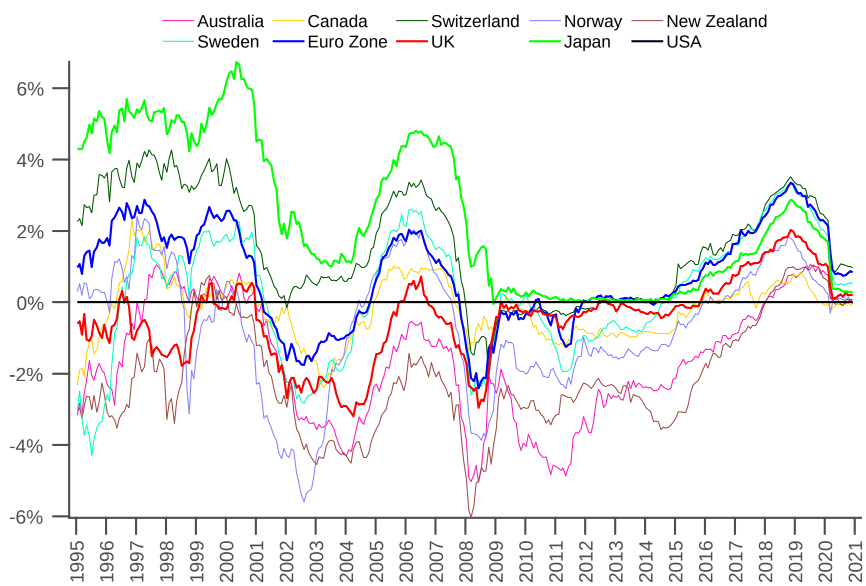


(c) Option-implied risk neutral exchange rate volatility

Figure A.5. Volatility of realized bond returns and realized and risk-neutral spot exchange rates
Panel (a) displays the volatilities of ten year sovereign bond returns, panel (b) displays the volatilities of spot exchange rates changes and panel (c) displays option-implied risk neutral exchange rate volatilities for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP (and USD for panel (a)). Volatilities in panels (a) and (b) are measured as the sum of squared differences of log prices in the 22 days preceding a sampled date. Dates are sampled as the survey dates of the Consensus Economics forecasts.



(a) Slopes



(b) IRDs

Figure A.6. Interest Rate Spreads

Figure displays term structure slopes (panel a) and 1-year interest rate differentials (panel b) for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The slope of the yield curve is defined as the difference between the respective country's ten year bond yield and its one year bond yield. The sample period is 1995.1 to 2020.12.

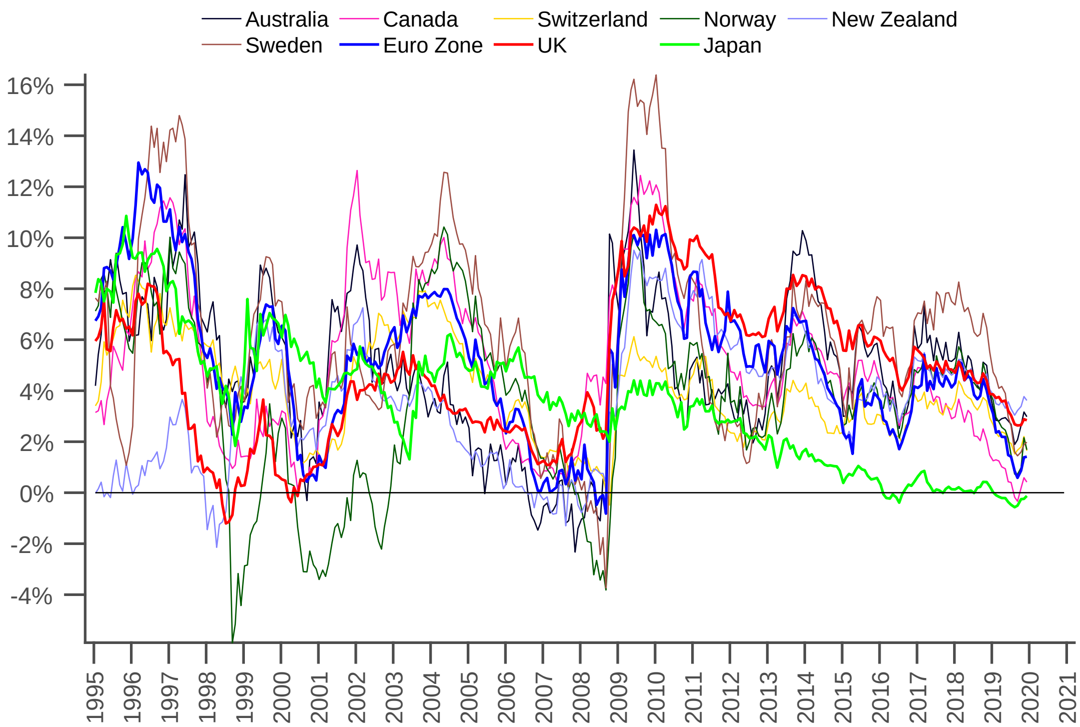


Figure A.7. Projected Bond Risk Premia

Figure displays projected bond risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projections for the bond risk premium are obtained by regressing realized ex-post premia on the slope of the yield curve and then forming 12-month ahead projections. Sample period is monthly observations from between 1995.12 and 2019.12.

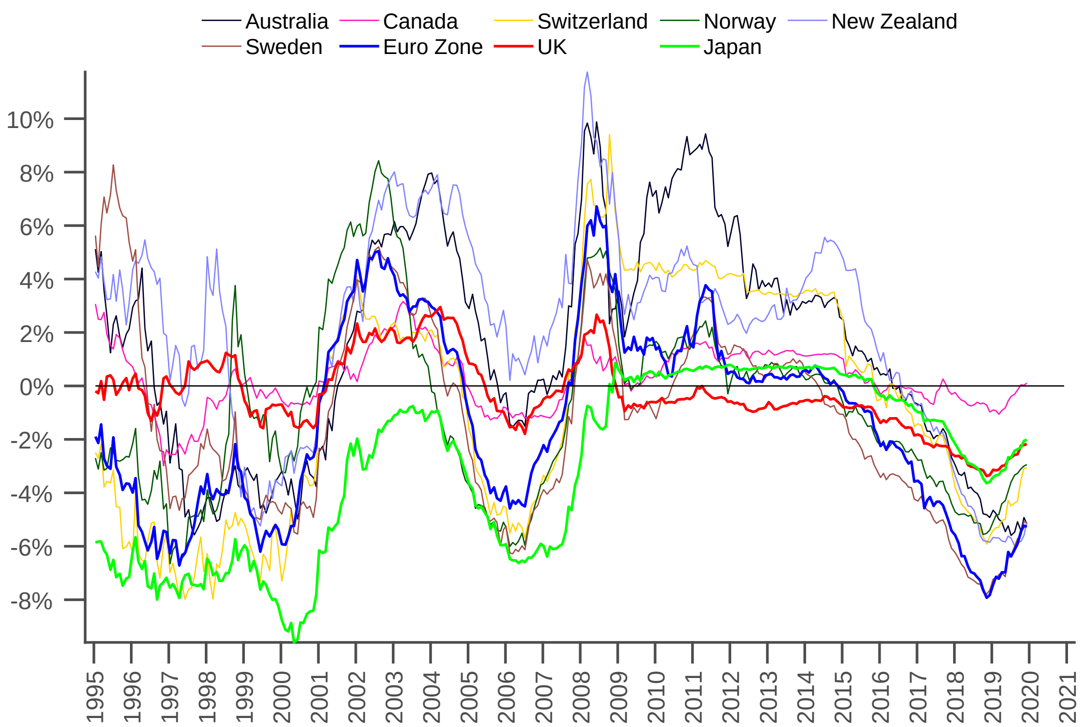
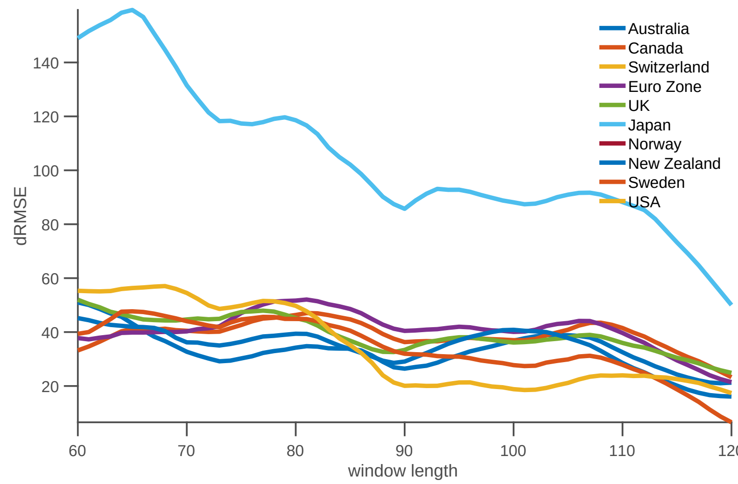
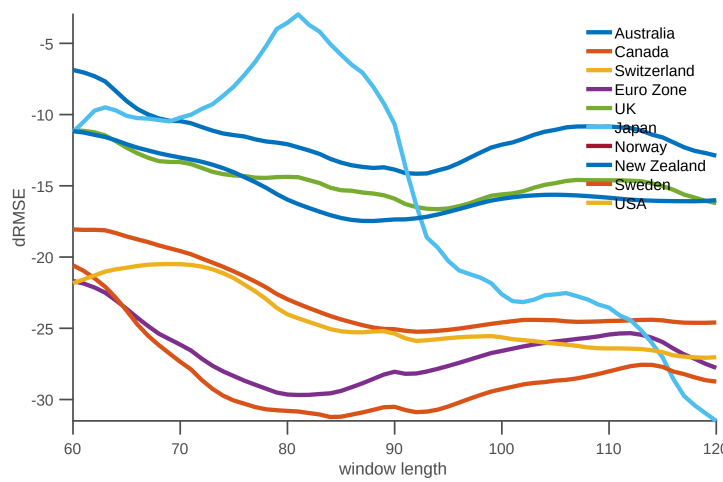


Figure A.8. Projected Exchange Rate Risk Premia

Figure displays subjective exchange rate risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projection for the bond risk premium are obtained by regressing realized ex-post premia on the interest rate differential between the foreign country and the United States and then forming 12-month ahead projections. The sample period is 1995.1 to 2020.12.



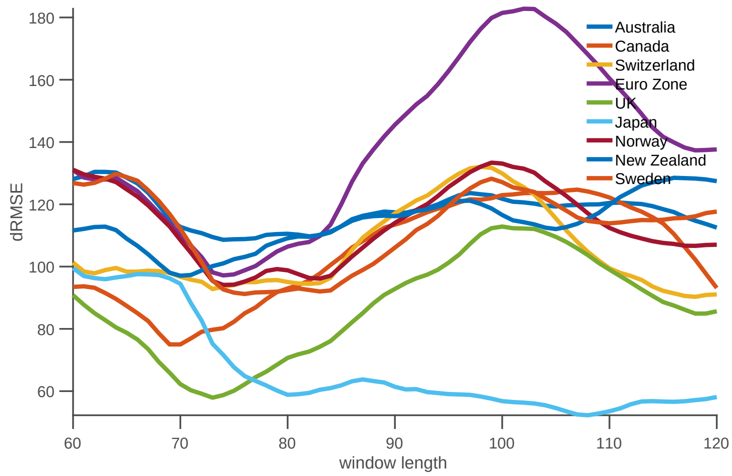
(a)



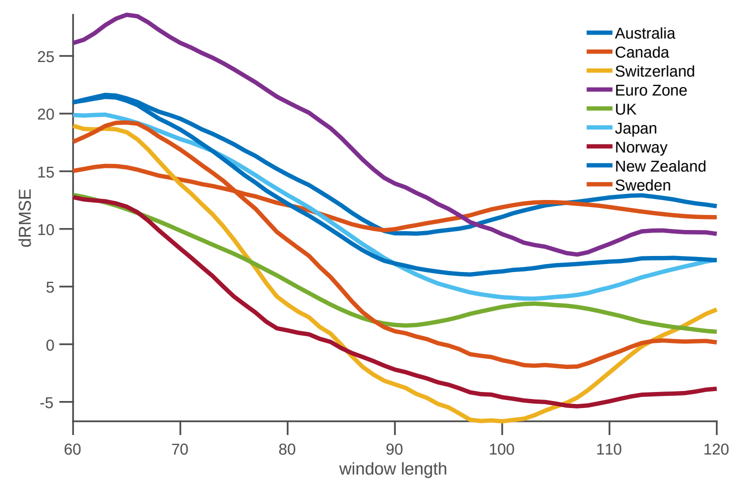
(b)

Figure A.9. Correcting Errors: Δ RMSE in the Fixed Income market

This figure displays the exploitation of predictable forecast errors using Equation (A.3). Panel (a) shows the difference in root mean squared error (Δ RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation (12) is used to predict the error term of Equation (A.3). Panel (b) shows the same difference in RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation (A.3). The horizontal axes shows the behaviour of Δ RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the Δ RMSE in percent ($\frac{RMSE_{corrected} - RMSE_{survey}}{RMSE_{survey}} * 100$). Data is available for the sample period 2000.1 to 2020.12.



(a)



(b)

Figure A.10. Correcting Errors: Δ RMSE in the Foreign Exchange market
This figure displays the exploitation of predictable forecast errors using Equation (A.3). Panel (a) shows the difference in root mean squared error (Δ RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation (12) is used to predict the error term of Equation (A.3). Panel (b) shows the same difference in RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation (A.3). The horizontal axes shows the behaviour of Δ RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the Δ RMSE in percent ($\frac{RMSE_{corrected} - RMSE_{survey}}{RMSE_{survey}} * 100$). Data is available for the sample period 2000.1 to 2020.12.