Asset Stranding, Climate Credit Risk and Capital Structure Design Under Global Warming

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Abstract. We examine the impact of climate change on the pricing of corporate securities and on the capital structure of firms. The main transmission channel through which global warming has an impact is the stranding of assets upon liquidation. The predictions of our model are consistent with recent empirical evidence. Global warming has a profound impact on debt capacity and the optimal capital structure. We are the first to document a possible disciplinary effect. The higher the exposure, the lower the leverage, which interestingly does not necessarily lead to lower credit spreads. We disentangle the direct and indirect effects of global warming on credit risk management metrics and show how these effects complement each other and which effects dominate and when.

Keywords: Asset stranding, climate credit risk, global warming, capital structure design, structural pricing model.

JEL Classification: G21, G28, G32, G33

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1 Introduction

The last 5 years are the warmest observed since 1850. This phenomenon is primarily caused by human activities, such as the burning of fossil fuels and deforestation, which release large amounts of greenhouse gases, such as carbon dioxide, into the atmosphere. The Intergovernmental Panel on Climate Change (IPCC) reports are now authoritative and informative on the causes and effects of global warming. According to the IPCC report (2019), the equilibrium climate sensitivity (ECS) is likely to be between 1.5 and 4.5°C, it is extremely unlikely to be below 1°C, and it is very unlikely to be above 6°C. The effects of global warming are far-reaching and include more frequent and severe heat waves, droughts, wildfires, and extreme weather events such as hurricanes and floods. The associated direct costs to individuals, insurance companies, states, and communities undoubtedly already amount to hundreds of billions of dollars[1]. The worrying actual and potential impacts on ecosystems, biodiversity, and some economic sectors (such as tourism) are being covered extensively by the media in many countries[2].

Companies are confronted with various concerns associated with climate risks among which the global warming. On the one hand, physical climate risks are of particular concern to businesses because, by their very nature, a company's assets are often tangible and located in specific places, making it difficult, impractical and/or very costly to relocate, reposition and re-purpose them. The public infrastructure that companies rely on for their routine operations and revenue-generating activities (such as export), may also deteriorate or become nonfunctional. Damage to tangible assets due to climate-related events or factors has been investigated by Bernstein et al. (2019), Baldauf et al. (2020) and Murfin and Spiegel (2020) for real estate and by Painter (2020) in connection with municipal bonds, among others. Regarding the impact on businesses, Brown et al. (2021) examine the cash flow shocks due to unexpectedly severe winter weather. And they document that firms increase the size of their credit lines and that banks respond by charging borrowers for this liquidity via higher interest rates. Huang et al. (2018) show that firms located

[1] The "*USD 83.3 billion [...] provided and mobilized jointly by developed countries for climate action in developing countries in 2020*" (source OECD) represents a floor.

[2] Climate change has become a global concern, and the international community has begun to take initiatives to combat global warming. In 1992, the United Nations established a secretariat when countries adopted the United Nations Framework Convention on Climate Change (UNFCCC). In December 2015, the overwhelming majority of the world's countries adopted the Paris Agreement, the central goal of which is to "*strengthen the global response to the threat of climate change [...and] to pursue efforts to limit global temperature rise to 1.5°C*" above pre-industrial levels and resulting global greenhouse gas emissions (source: UNFCCC).

in countries with increased exposure to climate change are more likely to take on long-term debt and hold more cash. Huang et al. (2022) show that physical climate risk increases the cost of (bank) borrowing. On the other hand, companies are exposed to the transition risks of climate change. Transition risks are associated to adverse changes in regulations, rules, standards and more extensively in the functioning of business conditions in terms of products, raw materials, production, transportation, customers, competition, supply chains, etc. It may prove difficult to avoid detrimental government actions in favour of environment and climate; it also may be very costly to reshape the portfolio of activities accordingly. Most carbon-intensive companies and their financiers have already recognized these transition climate risks; other industries and investors should do so in the near future. At the very least, companies should ask themselves whether (and to what extent) the corporate exposure to climate risks could lead to a revaluation of existing assets (Krueger et al. (2020)). Semieniuk et al. (2021) discuss a list of possible transmission channels, one of which is asset stranding.

Asset stranding refers to processes and mechanisms that cause assets to become stranded or lose value.[3] From now on, one will use this terminology as a comprehensive concept to capture the transmission channel through which global warming manifests, leading to the depreciation of corporate assets and consequently impacting the wealth of stakeholders. In this paper, asset stranding is driven by global warming and includes both material and regulatory elements as well as some business aspects, given that these material and regulatory dimensions are not limited to those included in traditional physical and transition climate risks. Global warming (GW) can indeed lead to operational challenges before (physical) damage occurs. Think, for example, of the inability to extract resources in a mine when temperatures rise. Technological issues may also arise, as some electronic devices may suffer dramatically from the heat. Economic and business challenges

[3]The term became popular in environmental discourse in the 2010s. The Stranded Assets Programme at the University of Oxford's Smith School of Enterprise and the Environment, launched in 2012, has been instrumental in establishing and disseminating the concept in academia. Caldecott et al. (2015) point out that the term can reconcile different concepts used by different communities (such as economic losses in economics, impairment in accounting, stranded costs in regulation and financial losses in finance). In this way, it can promote collaboration and understanding of the big picture. Caldecott et al. (2015) define stranded assets as those that experience "*unanticipated or premature write-downs, devaluations, or conversion to liabilities*". However, there are definitions and examples of asset stranding in the literature with slightly different meanings and understandings. E.g., for Semieniuk et al. (2022), asset stranding is "*the process of collapsing expectations of future profits from invested capital (the asset) as a result of disruptive policy and/or technological change*". Cahen-Fourot et al. (2021) cite the premature retirement of assets, the reduced utilization of assets and the costs associated with the retrofitting of assets, as examples of asset stranding.

include shifts in relative costs and benefits and/or alterations in customer preferences that render a business valueless, before regulation or public intervention intervene. Banks are expected to play a central role in facilitating ecological transition. With the development and implementation of green banking principles and regulatory frameworks in various regions, it is clear that the financing and refinancing of carbon-intensive ("brown") activities will become increasingly difficult in the coming decades.[4]

In our model, the main channel through which the global warming impacts the corporate metrics is the materialization of asset stranding risk at liquidation. This occurs when firms declare bankruptcy and liquidate assets. The exposure to global warming stands for the company's overall vulnerability to asset stranding upon liquidation. Global warming also impact regular short-term cash flows, as explained by the extensive research on climate credit risk. In this paper, the impact on a firm's cash flows, revenues, and operating costs does exist; however, it is embedded within the business-as-usual shocks on the firm's asset value.[5] The restructuring, closure, or liquidation of distressed companies presents unique opportunities to reshape the business landscape at a reduced (strategic, social, and political) cost. The bankruptcy of a company is a unique moment for the authorities to revoke (property) rights, expropriate the company, impose new regulations that restrict the business because the social costs can be considered minimal. The liquidation of a company is also a unique moment for buyers to push down the price of assets for sale, arguing the risk of stranding and regardless of the level of cash flows and profits. Environmental activists can be influential throughout the life of brown companies, but their pressure on stakeholders can be particularly high and effective during financial difficulties. Surprisingly, the stranding of corporate assets is still neglected in the academic literature beyond the specialised research focusing on fossil and carbon-intensive industries, given that these latter almost exclusively focused on stranded assets. To date there are very few quantitative studies on stranded assets and asset stranding[6]. One

[4]In countries where the banking sector is a key pillar of the economy (see Becker and Josephson (2016) for some statistics), banks are under pressure from regulators to help businesses transition. The Basel Committee clarifies how the existing framework of banking supervision can deal with climate-related financial risks. The industry as a whole is clearly concerned about climate risks arising "*from policy action taken to transition the economy off of fossil fuels*," as explained by the White House. Holding a portfolio of brown assets may become very expensive in the medium term, and very risky if the portfolio can be suddenly worthless. The academic literature has only recently highlighted this critical issue. For example, Karydas and Xepapadeas (2019), Hambel et al. (2020) reconsider traditional portfolio management decisions with brown or green assets in an environment where climate policies are uncertain.

[5]through the standard Brownian motion.

[6]Early studies mostly struggle to classify stranded assets and clarify the nature of the risk (see Caldecott et al.

of the notable exceptions, Semieniuk et al. (2022), reports large losses for investors at the macro level. Representative of the many economic studies assessing the devaluation risk associated with fuel reserves, Hansen (2022) points out that fossil fuel companies have "*remained significantly more profitable than renewable energy firms*", but notes that their profitability should be considered at risk.

With regard to the design of the capital structure of companies under imperfections, trade-off theory suggests that climate risk should influence corporate decisions. Indeed, companies tend to adjust their capital structure in response to external conditions (including the climate events and the associated impacts mentioned above), as well as financing and the legal macro environment (some of which being under the influence of climate policy). All these dimensions actually represent potential channels through which climate risks can influence and affect capital structure at the enterprise level. By the way, Ginglinger and Moreau (2023) provide some first empirical evidences that the (physical) climate risk exposure correlates negatively with leverage and positively with credit spreads. To establish this, the authors use a forward-looking proxy for physical climate risks and show that the larger the proxy, the lower the leverage of firms in the post-2015 period. Hence, exposure to physical climate risks reduces corporate leverage. They also evidence for the same period that high climate risk firms increase their net equity offerings, suggesting that firms endogeneize climate in their financing decisions. As a conjecture, Ginglinger and Moreau (2023) write that "*physical climate risk affects leverage via larger expected distress costs*". As far as we know, there is no quantitative corporate finance model to date that endogeneizes the climate risk so as to explain how to optimally design capital structure, appropriately price corporate debt, and set the compensation investors in the face of global warming. This article essentially fills this gap by formalizing one channel through which a firm's exposure to global warming affects its leverage ratio, corporate debt prices, and related credit risk management metrics.

This article examines the impact of climate change (and global warming in particular) on the firm value, its capital structure and the values of corporate securities. We develop a continuous-time quantitative corporate model *à la* Leland (1994) for corporate financing with taxes, liquidation costs and corporate climate exposure where asset stranding at default is the main channel through which GW materializes and impacts the stakeholders' wealth. We therefore extends the framework of Leland (1994) and draws on a standard macro model of temperature perturbation

(2016)). The 2017 special issue of *Journal of Sustainable Finance & Investment* bears witness to this.

(see Hassler et al. (2016)).[7] The dynamics of global warming affect bankruptcy costs and make them be time-varying. Our structural model relies on an IPCC global warming scenario and a couple of firm-specific parameters for modelling exposure to global warming. We can derive closed-form formulas for valuing corporate debt, equity, and the firm as a whole. As a byproduct, we can quantify and explore conventional metrics useful for managing capital structure and credit risk (leverage, credit spreads, probability of default, etc.). In our model, equity financing does not bear a direct effect of global warming. But the overall indirect effect may be substantial. The absence of direct effect is clearly a consequence of the limited liability, as asset stranding materializes precisely at liquidation whose costs are not supported by the shareholder. The indirect effect has nevertheless significant and somewhat complex impact, as it depend on the level of company climate exposure and the considered IPCC scenario. Regarding the level of exposure, equity financing may increase with it in absolute terms, meaning that shareholders of more exposed companies must invest more money. In relative term and in some scenarios, the equity financing may increase or decrease depending on whether the amount of investment decreases with leverage.

Predictions of our model are consistent with recent yet scarce empirical evidences, and some of them may stimulate new empirical research. Among other things, we show that the firm's debt capacity deteriorates with firm-specific exposure to global warming. Consistent with Ginglinger and Moreau (2023), we find that the leverage decreases and that more exposed firms tend to substitute equity for debt. We evaluate the specific compensation that investors require for climate risk in addition to the traditional compensation for default risk. We find that credit spreads do not necessarily increase with exposure to global warming. This means that firms exposed to global warming may not have credit spreads that are ranked by their degree of exposure to climate risk. Moreover, although the leverage systematically declines with exposure to global warming, the credit spreads do not necessarily decrease with it. These somewhat surprising results come from endogenizing the dimension of climate credit risk into the design of capital. Actually credit spreads materialize the *disciplinary effect* that climate risk can have on corporate decisions. We show that the term structure of default probabilities of firms endogenizing the consequences of asset stranding and global warming decreases with their exposure to climate risk. For illustration, we finally fit

[7]Thus, in our model, cash flows and the resulting profits are maintained, which is consistent with the observation of Hansen (2022). It may also be noted that the perpetual nature of corporate debt in the Leland framework fits well with the slowness sometimes associated with transition climate risks.

the temperature model to some IPCC (2023) predictions and compute key financial figures related to firms with different profiles. One finds that the level of business risk may exacerbate the impact of the global warming exposure, among which the disciplinary effect.

Our paper relates to the burgeoning asset pricing literature, which aims to incorporate climate issues into corporate bond pricing.[8] All these studies have in common to examine the effects of global warming on cash flows. Agliardi and Agliardi (2019) develop an EBIT-based model where brown firms face an *ad hoc* penalty for pricing some green bonds and valuing greenium. In the structural model of Agliardi and Agliardi (2021), the value of corporate assets can jump downwards due to climate-related measures and policies. The arrival rate of the random shocks reflects the intensity of the transition risk and their average size can differentiate bonds of green and brown firms. Le Guenedal and Tankov (2022) extend Leland and Toft (1996) in order to price corporate bonds issued by carbon-intensive firms exposed to transition risk. In their setting, the progressive observation of the carbon price path contributes to the resolution of uncertainty. All these models focus on the pricing of corporate bonds and, by design, cannot examine the design of the capital structure. As far as we know, no previous study mentions that companies can proactively incorporate and in particular endogenize climate risks into their financing decisions. The relationship between corporate decisions and climate risks still needs to be explored in depth, especially from a theoretical perspective. This is exactly where our study comes in.

Several empirical papers have examined the impact of climate issues on the market prices of corporate bonds, credit risk metrics (such as bond credit rating, issuer credit rating, Merton's Distance to Default and so) and/ or the cost of debt. In general, the bond prices decline with the level of climate risk and/or the level of corporate exposure to climate risk. This literature is nowadays extensive and, to quote a few, we refer to Seltzer et al. (2020), Capasso et al. (2020), Nguyen et al. (2023), Ramos-García et al. (2023) and the references therein. Of course, our paper belongs to the climate finance literature paved by research such as Bansal et al. (2016), Karydas and Xepapadeas (2019), Hambel et al. (2020), Gregory (2021), Ardia et al. (2023) and Bolton and Kacperczyk (2023). All of these publications stress the importance of climate change, global warming and long-run temperature shifts on financial assets, markets and decisions[9].

[8] Agliardi and Agliardi (2021) claim "*there are very few theoretical papers studying the effects of climate-related risks on the bond market*" and later "*a theoretical explanation of the relationship between climate related risks and bond pricing is still an open question*"

[9] Tao et al. (2022) conduct a comprehensive bibliometric analysis of articles published on environmental finance

The rest of the paper is organized as follows: Section 2 introduces our framework, which includes three crucial dimensions: the firm and its business environment, the dynamics of global warming, and our approach to modeling the firm's exposure to global warming. Section 3 characterizes shareholder default policy and presents a comprehensive collection of analytical formulas fundamental to security valuation and capital structure analysis. Section 4 examines numerically how capital structures are affected by global warming and exposure to global warming. Section 5 examines various credit risk metrics that help manage climate credit risk. Here, our main objective is to shed light on the disciplinary effect through the lens of these credit risk metrics and, where possible, disentangle the direct effect and the indirect effect of global warming. Section 6 provides an application of our model. Section 7 concludes the paper.

2 The framework

This section presents and discusses our framework and its three important dimensions, namely the firm and its business environment, climate change, and our chosen specification for firm-specific exposure to global warming.

2.1 The firm and its business environment

Following Leland (1994), consider an economy in which there is a risk-free investment with a constant interest rate r, some taxes with rate equal to $\eta \in (0, 1)$, a single firm financed by equity and a perpetual bond that promises a continuous coupon, denoted by c. The owner of the equity pays the net cash outflows associated with the choice of leverage (i.e., coupons after tax benefits) out of his own pocket and thus freely determines the timing of default payment. One denotes by τ_B such a default time. Default then leads to immediate bankruptcy and liquidation of the firm's assets, which leads to either the transfer of assets auctioned piecemeal or the transfer of the business sold in one piece. After the liquidation of assets, the rule of absolute priority applies and the debtors are first compensated with the value of the liquidated assets before the shareholders can receive anything at all. The auction or sale makes it possible for the asset stranding to materialize

since the 1970s and conclude that climate finance remains an emerging topic. Existing empirical findings are mixed, if not inconclusive. For instance, Lanfear et al. (2019) reports both negative and positive effects of weather events on stocks. Very few studies examine the empirical impact of climate risk on corporate ratios and financial performance (see, e.g., Pankratz et al. (2023) and Huang et al. (2018)).

and be reflected in the bids and thus in the liquidation costs. The bankruptcy costs, denoted by $\alpha\left(T^{\circ}\left(\tau_B\right)\right) \equiv \alpha\left(\delta T^{\circ}\left(\tau_B\right)\right)$ depends on T° the temperature prevailing at the liquidation time, namely τ_B, or equivalently on δT° the temperature perturbation relative to an initial steady state. Whatever the definition, the realized value at the liquidation time amounts to $\left(1-\alpha\left(T^{\circ}\left(\tau_B\right)\right)\right.$ and this depends on the temperature prevailing at the default/ liquidation time. We also follow Leland (1994) by considering a company whose activities have risk-neutral value process $V=\left(V_t\right)_t$ described by:

$$dV_t = \mu V_t dt + \sigma V_t dW_t \tag{1}$$

where $W=\left(W_t\right)_t$ is a standard Brownian motion. V is the asset value of the firm and μ and σ are the associated constant risk-neutral drift and volatility. It is assumed that $\mu \leq r$. One denotes by $v(V)$ the total value of the firm, which is the value of the firm's assets plus the present value of the tax deduction of coupon payments and minus the present value of bankruptcy costs. Since there are imperfections in this economy (taxes and bankruptcy costs), the trade-off theory states that there is an optimal capital structure.

For the specification of the bankruptcy costs $\alpha\left(\delta T^{\circ}\right)$, we consider a piecewise affine function of δT° the temperature perturbation relative to the initial steady state. Denoting $\delta T_{\min}$, $\delta T_{\max}$, α_0 and β some firm-specific parameters (we discuss below), one posits

$$\begin{aligned}
\alpha\left(\delta T^{\circ}; \delta T_{\min}, \delta T_{\max}, \alpha_0, \beta\right) &= \begin{cases} 100\% & \text{if } \delta T_{\max} < \delta T^{\circ} \\ \alpha_0 + \beta\left(\delta T^{\circ} - \delta T_{\min}\right) & \text{if } \delta T_{\min} < \delta T^{\circ} \leq \delta T_{\max} \\ \alpha_0 & \text{if } \delta T^{\circ} \leq \delta T_{\min} \end{cases} \\
&= \alpha\left(\delta T^{\circ}; \delta T_{\min}, \alpha_0, \beta\right).
\end{aligned} \tag{2}$$

The parameter α_0 stands for the bankruptcy costs suffered by the creditor if the liquidation of the firm's assets intervenes when the actual temperature perturbation stands below the limit $\delta T_{\min}$, that is the minimum temperature perturbation to which the firm is exposed. This parameter is the reference Leland bankruptcy costs. The parameter β stands for the sensitivity of bankruptcy costs to global warming, The parameter $\delta T_{\max}$ represents the level of temperature perturbation beyond which there will be no recovery upon liquidation for the creditor. As shown by the second equality, $\delta T_{\max}$ actually is not an input parameter, because it is the minimal temperature perturbation that implies the largest bankruptcy costs, namely $\alpha\left(\delta T_{\max}; \delta T_{\min}, \delta T_{\max}, \alpha_0, \beta\right) = 1$ and, as a result, it is a function of other structural parameters, that is more formally $\delta T_{\max} = \delta T_{\min} + \frac{1-\alpha_0}{\beta} \equiv \delta T_{\max}\left(\delta T_{\min}, \alpha_0, \beta\right)$. If the sensitivity β is close to zero (ultimately equal to zero), then $\delta T_{\max}$ is very large (ultimately equal to ∞), the most extreme level of bankruptcy costs is hardly attained and the above specification is Leland's. When the sensitivity β is very large (ultimately equal to ∞), then $\delta T_{\max}(\delta T_{\min}, \alpha_0, \beta)$ is close to $\delta T_{\min}$ and the bankruptcy costs immediately reach their maximum once the temperature exposure $\delta T_{\min}$ is surpassed.

The above specification deserves few final comments. First, the pair $(\beta, \delta T_{\min})$ admits some interpretations. With a risk management perspective, the pair $(\beta, \delta T_{\min})$ introduced by our bankruptcy costs model informs about the overall firm's vulnerability to global warming and asset stranding, the coefficient β being related more to the (marginal) sensitivity. We expect brown firms to have a larger β and smaller $\delta T_{\min}$ than green firms may have. Climate credit risk, just like all other risks, results from a hazard, an exposure and a vulnerability. In our setting, hazard relates to the default event (and τ_B), exposure to the firm's assets value upon liquidation time (i.e., V_{τ_B}) and vulnerability to the pair of parameters $(\beta, \delta T_{\min})$[10]. Second, since the temperature dynamics will be considered a nonlinear function of time, the above linear specification makes bankruptcy costs a nonlinear function of time.

2.2 Climate Change and global warming dynamics

Here we present one of the simplest possible global warming models, taken from Hassler et al. (2016). If we denote the temperature perturbation with respect to the initial state (at time t) by $\delta T^\circ(t) = T(t) - T_{init}$, the perturbation satisfies the following ordinary differential equation

$$\frac{d(\delta T^\circ(t))}{dt} = \kappa(\theta^\circ - \delta T^\circ(t)) \tag{3}$$

where θ° represents the long-run temperature perturbation and κ the speed of convergence. The solution of this O.D.E. is

$$\delta T^\circ\left(t\right) = \theta^\circ - \left(\theta^\circ - \delta T^\circ\left(0\right)\right) e^{-\kappa t}.$$

[10] We do not pursue this risk management perspective further, but the pair puts forward a couple of dimensions the firm can consider to manage climate risk.

for a known contemporaneous temperature pertubation $\delta T^{\circ}(0)$. One can verify that $\delta T^{\circ}(\infty) = \theta^{\circ}$ and therefore the interpretation is justified[11]. The solution is a deterministic function of time that is bijective on the appropriate intervals; thus, it is a bijection that can be inverted. It also has an important effect on the (absolute) value of the temperature. By the observation that $\frac{d(\delta T^{\circ}(t))}{dt} = \lim_{\delta t \to 0} \frac{(T^{\circ}(t+\delta t) - T_{init}) - (T^{\circ}(t) - T_{init})}{\delta t} = \lim_{\delta t \to 0} \frac{T^{\circ}(t+\delta t) - T^{\circ}(t)}{\delta t} = \frac{dT^{\circ}(t)}{dt}$, one may write $\dot{T}^{\circ}(t) = \kappa\left(\{\theta^{\circ} + T_{init}\} - T^{\circ}(t)\right)$. Thus, a (global) temperature perturbation model may produce a local temperature model that is more appropriate for a particular enterprise. Experts and the IPCC agree on some scenarios for temperature change relative to pre-industrial levels. These numbers can then be easily translated in terms of the long-term limit of the dynamics. The consensus identifies some very optimistic scenarios where the temperature perturbation could be below 1.5 (thanks to some stringent measures) $\theta^{\circ}_{opt} = 1.5$, some pessimistic scenarios that see extreme perturbations above $\theta^{\circ}_{pess} = 4.4$, the average scenario refers to $\theta^{\circ}_{av} = 2.5$.

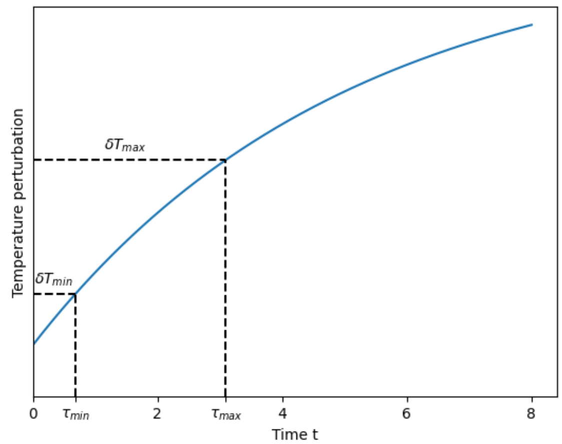
Figure 1 illustrates the dynamics of the temperature perturbation with respect to time in years, with two hypothetical thresholds $\delta T_{\min}$ and $\delta T_{\max}$ for a given sensitivity to global warming, $\beta = 0.5$. From the previous subsection, we know that they correspond respectively to the fixed level at which warming begins to affect the firm's bankruptcy costs and the fixed level at which the impact is greatest. Since the dynamics of the temperature perturbation is deterministic and the perturbation is a bijective function on the appropriate interval, we can assign some initial hit

[11] This dynamics for temperature pertubation is proposed by many experts. This type of model is common in environmental macroeconomics, where researchers attempt to link global warming (i.e., the dynamics of the temperature perturbation) to the consumption of an energy budget. This is one way to emphasize the endogeneity of global warming with industrial and economic activities. As temperature increases, the outgoing energy flux increases because a warmer object radiates more energy, all else being equal. As an approximation, let this increase be proportional to the increase in temperature relative to the baseline. If we denote the temperature perturbation relative to the initial steady state at time t by δT° and the proportionality factor between energy fluxes and temperature by λ, we can summarize these relationships in the following equation:

$$\frac{d(\delta T^{\circ}(t))}{dt} = \kappa(\theta^{\circ} - \lambda \delta T^{\circ}(t))$$

$\theta^{\circ} = F + \delta T^{\circ}{}_{pre}$, where F is commonly referred to as forcing parameter and is defined as the change in the energy budget caused by human activities and $\delta T^{\circ}{}_{pre}$ is the reference temperature in pre-industrial times. The parameter κ is (inversely) related to the heat capacity of the system for which the energy budget is defined, and determines the rate at which the temperature changes for a given imbalance in the energy budget. To reach steady state, the energy budget must be balanced so that the term in parentheses in the equation has become zero. Let the steady-state temperature associated with a forcing parameter F be $\delta T^{\circ}(F)$. At $\delta T^{\circ}(F)$, the temperature is constant, which requires that the energy balance is balanced, i.e., $\delta T^{\circ}(F) = \frac{\theta^{\circ}}{\lambda}$. Thus, $\delta T^{\circ}(t) = e^{-\lambda\kappa t}\left(\delta T^{\circ}(0) - \frac{\theta^{\circ}}{\lambda}\right) + \frac{\theta^{\circ}}{\lambda}$ In the scientific literature, the value of λ is often equal to 0.3^{-1} and the forcing parameter F to 3.7.

Figure 1: Temperature perturbation : Dynamics



Temperature perturbation δT° as a function of time t.

times to these two thresholds. From now on denote these two times

$$\begin{aligned}\tau_{\min} &= \inf\left\{t > 0 : \delta T^{\circ}(t) = \delta T_{\min}\right\} = \delta T^{\circ -1}\left(\delta T_{\min}\right) \\ &= -\frac{1}{\kappa}\ln\left[\frac{\theta^{\circ} - \delta T_{\min}}{\theta^{\circ} - \delta T^{\circ}(0)}\right] 1_{\{\delta T_{\min} < \theta^{\circ}\}} + \infty 1_{\{\delta T_{\min} \geq \theta^{\circ}\}}\end{aligned}$$

and similarly $\tau_{\max} = \inf\left\{t > 0 : \delta T^{\circ}(t) = \delta T_{\max}\right\} = \delta T^{\circ -1}\left(\delta T_{\max}\right)$.

2.3 Climate Change and bankruptcy costs

In subsection 2.1, we assume that global warming has an impact on bankruptcy costs. In subsection 2.2, we specify the deterministic dynamics of the temperature perturbation. Since temperature is a deterministic function of time, the dynamics of temperature make bankruptcy costs time-dependent $\alpha(t)$ and one will prefer a simpler notation for bankruptcy costs, namely $\alpha\left(T^{\circ}\left(\tau_B\right)\right) = \alpha\left(\tau_B\right)$. The nonlinear time dependent bankruptcy costs are

$$\alpha\left(t; \delta T_{\min}, \alpha_0, \beta, \theta^{\circ}, \kappa\right) = \begin{cases} 100\% & \text{if } \tau_{\max} < t \\ \alpha_0 + \beta\left(\left(\theta^{\circ} - \delta T_{\min}\right) - \left(\theta^{\circ} - \delta T^{\circ}(0)\right) e^{-\kappa t}\right) & \text{if } \tau_{\min} < t \leq \tau_{\max} \\ \alpha_0 & \text{if } t \leq \tau_{\min} \end{cases} \tag{4}$$

Now, the parameter α_0 is viewed as the bankruptcy costs suffered by the creditor if the liquidation of the firm's assets intervenes before the temperature perturbation attains the limit $\delta T_{\min}$. If ever

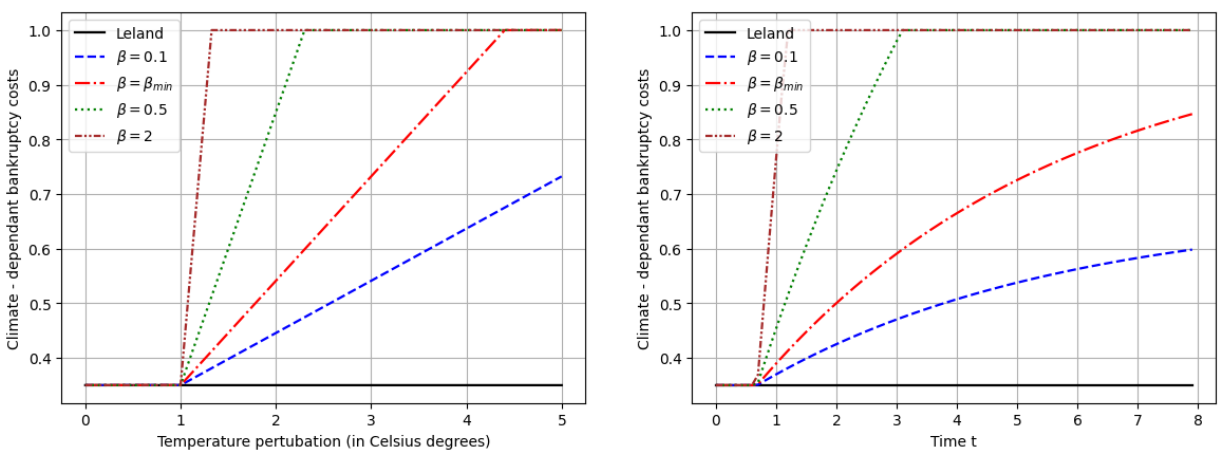
$\delta T_{\min} = \delta T^{\circ}(0)$, then

$$\alpha\left(t;\alpha_0,\beta\right) = \begin{cases} 100\% & \text{if } \tau_{\max} < t \\ \alpha_0 + \beta\xi\left(1-e^{-\kappa t}\right) & \text{if } 0 < t \leq \tau_{\max} \\ \alpha_0 & \text{if } t = 0 \end{cases} \tag{5}$$

with $\xi = \theta^{\circ} - \delta T^{\circ}\left(0\right) = T^{\circ}_{max} - T^{\circ}\left(0\right)$ the coming global warming. The bankruptcy costs hence depend on time, on some firm-specific parameters, and on some global warming parameters. Figure 2 illustrates the response of bankruptcy costs to temperature rise in the left graph and some possible dynamics over time for different parameters in the right graph. As illustrated by the left

Figure 2: Climate-Dependent Bankruptcy Costs: Specification & Dynamics

a) α as a function of δT° b) α as a function of time t



This figure illustrates in graph a) how climate-dependent bankruptcy costs may depend on Global Warming and in graph b) the resulting dynamics. it is assumed that speed of convergence $\kappa = 20\%$, the long-run temperature perturbation $\theta = 4.4°C$, bankruptcy costs are 35 percent ($\alpha_0 = 35\%$), and $\delta T_{\min} = 1$.

graph of Figure 2, the larger the global warming sensitivity β, the closer $\tau_{\max}$ to $\tau_{\min}$ and for an infinite sensitivity parameter, one has

$$\alpha\left(t;\alpha_0,\infty\right) = \begin{cases} 100\% & \text{if } \tau_{\min} < t \\ \alpha_0 & \text{if } t \leq \tau_{\min} \end{cases}$$

On the contrary, the bankruptcy costs may never reach 100% for some small values of β,. The following proposition elaborates on this feature.

Proposition 2.1 *The worst-case scenario for the bankruptcy costs ($\alpha = 100\%$) will not be experienced by firms whose $\beta \in \left[0; \beta_{\min}\left(\theta^{IPCC}\right)\right[$ with*

$$\beta_{\min}\left(\theta^{IPCC}\right) = \frac{100\% - \alpha_0}{\theta^{IPCC} - \delta T_{\min}}.$$

And the set of firms the most exposed to global warming increases non linearly with the IPCC scenario about θ.

It is clear that this number of companies may decrease rapidly if the IPCC changes its mind and possibly increases the average global warming scenario (θ^{IPCC}). In addition, one has $\delta T_{\max}(\beta_{\min}\left(\theta^{IPCC}\right)) = \theta^{IPCC}$.

In our setting, climate-induced bankruptcy costs are time-varying. The time-dependent deterministic function we consider for global warning does not means imply that these costs are deterministic. On the contrary, bankruptcy costs depend on the timing of the default decided by the shareholder. This time is modeled by a random default time τ_B, which is the first time at which the value of the firm's assets V becomes too low. τ_B is the first time at which the value of the assets reaches a default threshold and this threshold is itself deterministic, as it reflects the shareholder's strategic default policy . The following section clarifies what such a default policy should be.

3 Analytical Formulae for Pricing Securities and Analyzing the Capital Structure

In this section, we first derive an important result related to the default policy of the equityholder and then, as a by-product, provide the full list of analytical formulas needed to value corporate securities and the firm subject climate risk.

Proposition 3.1 *The equityholder chooses the constant default threshold identified by Leland (1994) to design the optimal default policy.*

Demonstrations are given in the appendix. Proposition (3.1) implies the level of the bankruptcy threshold chosen by the shareholder is time-homogeneous. This result can be viewed as counter-intuitive because bankruptcy costs are time-dependent in our setting. The demonstration in the appendix proves the equityholder wealth does not depend on these costs. As a result, there is no reason to deviate from the Leland policy. In details, the analysis of the partial differential equations associated with the various securities shows that the value of equity is time-homogeneous, as in Leland (1994), while the value of debt is not (the same is true for the value of the firm). Thus, equity does not depend directly on bankruptcy costs, and Leland's analysis (that equityholders decide to file for bankruptcy when a constant default threshold is reached) holds.

The default time thus corresponds to the time when the value of the company's assets reaches the constant default threshold identified by Leland (1994) for the first time, namelly $V_B(c) = \frac{(1-\eta)c}{r}\frac{X}{X+1}$, the term X in our setting being $X = \frac{1}{\sigma^2}\left(\left(\mu - \frac{\sigma^2}{2}\right) + z_r\right)$ with $z_r = \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}$. The default time satisfies $\tau_B = \inf\{t : V_t \leq V_B\}$ and the default threshold V_B is the (theoretical) value of assets at the time of default (when the firm is still operating, i.e., just before bankruptcy costs are incurred). The specification for the default threshold again has an important implication: the value of equity is not directly affected by global warming, but it can be indirectly affected. Indeed, the size of the coupon can be affected by this risk if the firm designs the capital structure to take advantage of its debt capacity or to maximize the value of the firm (the values of debt and the firm are sensitive to bankruptcy costs and therefore depend on global warming).

Armed with Proposition (3.1), we can now derive the analytical pricing formulae. The debt value is

$$\begin{aligned} D(V) &= E^{\mathbb{Q}}\left[\int_0^{\tau_B} ce^{-rs}ds + \left(1 - \alpha\left(T^{\circ}(\tau_B)\right)\right)V_{\tau_B}e^{-r\tau_B}\right] \\ &= E^{\mathbb{Q}}\left[\frac{c}{r}\left(1 - e^{-r\tau_B}\right) + V_B e^{-r\tau_B} - \alpha(\tau_B)V_B e^{-r\tau_B}\right] \\ &= \frac{c}{r} + \left(V_B - \frac{c}{r}\right)E^{\mathbb{Q}}\left[e^{-r\tau_B}\right] - V_B E^{\mathbb{Q}}\left[\alpha(\tau_B)e^{-r\tau_B}\right] \end{aligned} \tag{6}$$

where $\alpha(\tau_B)$ is specified in equation (4). Following similar lines of reasoning, the firm's value equals the current firm's assets value V plus the tax benefits $TB(V) = \eta\frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right)$ and minus the expected bankruptcy costs $BC(V) = E^{\mathbb{Q}}\left[\alpha(\tau_B)V_B e^{-r\tau_B}\right]$ so that $v(V) = V + TB(V) - BC(V)$ or more explicitly

$$v(V) = V + \eta\frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right) - V_B E^{\mathbb{Q}}\left[\alpha(\tau_B)e^{-r\tau_B}\right] \tag{7}$$

and the corresponding market value of equity is the firm's market value minus the market value of debt, or $Eq(V) = V - (1-\eta)\frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right) - V_B E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]$. One therefore needs to evaluate the building blocks $E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]$ and $E^{\mathbb{Q}}\left[\alpha\left(\tau_B\right)e^{-r\tau_B}\right]$. The following proposition summarizes the results.

Proposition 3.2 *The debt, firm and equity values satisfy*

$$\begin{aligned} D\left(V;c,\alpha_0,\beta\right) &= D_L\left(V;c,\alpha_0\right)+\Psi_\beta V_B(c), \\ \upsilon\left(V;c,\alpha_0,\beta\right) &= \upsilon_L\left(V;c,\alpha_0\right)+\Psi_\beta V_B(c) \end{aligned}$$

and $Eq\left(V;c,\alpha_0,\beta\right)\equiv Eq\left(V;c\right)=Eq_L\left(V;c\right)$ *where* D_L*,* υ_L *and* Eq_L *are the Leland's pricing formulae and*

$$\begin{aligned} \Psi_\beta &= \beta\left(\theta^\circ-\delta T^\circ(0)\right)\left(G_{\tau_B}^{r+\kappa}\left(\tau_{max}\right)-G_{\tau_B}^{r+\kappa}\left(\tau_{min}\right)\right) \\ &- \beta\left(\theta^\circ-\delta T_{min}\right)\left(G_{\tau_B}^{r}\left(\tau_{max}\right)-G_{\tau_B}^{r}\left(\tau_{min}\right)\right) \\ &+ \left(1-\alpha_0\right)\left(G_{\tau_B}^{r}\left(\tau_{max}\right)-\left(V_0/V_B\right)^{-X}\right), \end{aligned}$$

with

$$G_{\tau_B}^{r}\left(t\right)=\left(\frac{V_0}{V_B}\right)^{-X+2\frac{z_r}{\sigma^2}}N\left[-d_2\left(V_0,V_B;z_r,t\right)\right]+\left(\frac{V_0}{V_B}\right)^{-X}N\left[d_2\left(V_B,V_0;z_r,t\right)\right],$$

where $d_2\left(V_0,V_B;z_r,t\right)=\frac{\ln(V_0/V_B)+z_r t}{\sigma\sqrt{t}}$ *and* N *stands for the Gaussian cumulative distribution function.*

More detailed formulas are provided in the appendix for readers who are not familiar with the Leland (1994) model.

The design of the capital structure of course depends on the criterion the firm retains. Classically, two types of decisions are distinguished. In the first, the firm dimensions debt (coupon in our framework) so as to exhaust its debt capacity. In the second, the firm dimensions debt (coupon in our framework) so as to maximize its overall value. In the first scenario, the capital structure is determined by solving

$$C_{\max}=\underset{c\in[0,\bar{C}]}{\arg\max}D\left(V;c\right)$$

and the debt capacity amounts to $D_{\max}\left(V\right)=D\left(V;C_{\max}\right)$. Under the second scenario, the optimal capital structure is determined by solving by

$$C^* = \underset{c\in[0,\bar{C}]}{\arg\max} v\left(V;c\right)$$

where $\bar{C} = \frac{rV_B}{(1-\eta)}\frac{1+X}{X}$ and the debt value is $D^*\left(V\right) = D\left(V;C^*\right)$. The complexity of the function $G^r_{\tau_B}$ makes it impossible to derive the solutions of the to above optimization problems. We therefore recourse to numerical optimization techniques.

Once the above formulas are available, on may calculate a number of important metrics for analyzing the capital structure. The leverage ratio is given by $L = \frac{D(V)}{v(V)}$, the yield to maturity which is the interest rate paid on risky debt is derived by dividing the coupon level c by the debt price $D\left(V\right)$, and the credit spread is

$$CS = c/D(V) - r, \tag{8}$$

whatever the level of coupon c. Although we consider a financing without maturity, it is worhtwhile computing the probability of default associated with time horizon T. These probabilities result from the cumulative distribution function of the random default time and admit in our context an analytical expression, namely:

$$\begin{aligned} PD\left(V_B,T\right) &:= \mathbb{P}\left[\tau_B \leq T\right] \\ &= N\left[-d_2\left(V_0,V_B;m,T\right)\right] + \left(\frac{V_0}{V_B}\right)^{-2m/\sigma^2} N\left[d_2\left(V_B,V_0;m,T\right)\right]. \end{aligned} \tag{9}$$

Hence, the exposure to global warming impacts the probabilities of default through the default threshold V_B that depends on the coupon level, one has $PD\left(V_B\left(c\right)\right)$, $PD\left(V_B\left(C_{\max}\right)\right)$ and $PD\left(V_B\left(C^*\right)\right)$, depending on the sort of coupon the firm selects. The insurance cost provides an alternative perspective on the consequences of climate risk. An insurance contract enables the insured to avoid all adverse consequences of certain risks. Since the insured portfolio is a risk-free debt, one has

$$Ins_0^{Tot}(c) = \frac{c}{r} - D\left(V;c\right) \tag{10}$$

where c is the considered coupon. In our framework, the firm and stakeholders are exposed to both financial risk and climate exposure. To evaluate the relative contribution of these dimensions. it is interested to split this total insurance cost into components. For an arbitrary level of coupon, one may consider:

$$
\begin{aligned}
Ins_0^{Tot}(c) &= \left[\frac{c}{r} - D_L\left(V; c, \alpha_0\right)\right] + \left[D_L\left(V; c, \alpha_0\right) - D\left(V; c\right)\right] \\
&= ins_0^{Lel}(c) + \left[D_L\left(V; c, \alpha_0\right) - D\left(V; c\right)\right]
\end{aligned}
$$

where ins_0^{Lel} is the insurance cost the firm would pay in absence of exposure to climate-related asset stranding. Actually, the above decomposition is relevant and the term $D\left(V; \alpha_0\right) - D\left(V; \alpha\right)$ quantifies (in numeraire) the marginal value of the firm's climate risk, to the extent the coupon c applies equally (i.e. with the same meaning) to both frameworks. The two objective functions above show that this is not the case, since the solutions will depend on climate.

The following two sections will illustrate our discussion numerically. To produce numerical results, we use $V_0 = 100$, $\eta = 35\%$, $r = 5\%$, $\sigma = 25\%$ and $\alpha_0 = 35\%$ for the firm's structural parameters. Regarding the parameters capturing the company exposure to global warming, the sensitivity β will change in the graphs, whereas $\delta T_{\min} = 1.15$ is chosen for illustrative purposes. The current level of global warming, i.e. the contemporaneous temperature perturbation, is $\delta T^{\circ}(0) = 1$. IPCC scenarios are characterized by their convergence rate, κ, and long-term long-term temperature, θ. One will distinguish two different situations. Firms of Panel A face a net-zero scenario, in which the long-term temperature perturbation is $\theta_A^{\circ} = 1.5^{\circ}C$ and the rate of convergence is $\kappa_A = 10\%$. Firms of Panel B face a more pessimistic scenario, in which the long-term temperature perturbation is $\theta_B^{\circ} = 4.4^{\circ}C \equiv \theta$ and the convergence speed is $\kappa_B = 20\%$. Our "Panel A: Net-Zero" scenario conforms to the net-zero *emission* scenario that was identified by the IPCC (2018) and discussed in-length by the IEA (2021), whereas our "Panel B: Currently Most Pessimistic" scenario conforms to the SSP5-8.5 scenario[12]. In the following sections, we may occasionally depart from this base case.

[12] As Roncalli (2023) explains, "*Net-zero emissions refers to a state in which the greenhouse gases going into the atmosphere are balanced by removal out of the atmosphere. This is a condition to stop global warming. According to IPCC (2018), global temperature increase needs to be limited to 1.5°C pre-industrial levels in order to mitigate the worst impacts of climate change and preserve a livable planet. Generally, we assume that netzero emissions must be achieved by 2050 IEA (2021), otherwise multiple tipping points could be triggered with irreversible impacts*". It also must be precised that, unfortunately, our most pessimistic scenario is not the most extreme one. Actually, the long-term temperature perturbation $4.4^{\circ}C$ is only the best estimate within the range $[3.3^{\circ}C, 5.7^{\circ}C]$ whose values are likely to occur in case GHG emissions remain very high. Interested readers may consult the recent 2023 "AR6 Synthesis Report" of IPCC on climate change downloadable at https://www.ipcc.ch/report/ar6/syr/.

4 Global Warming and Capital Structure

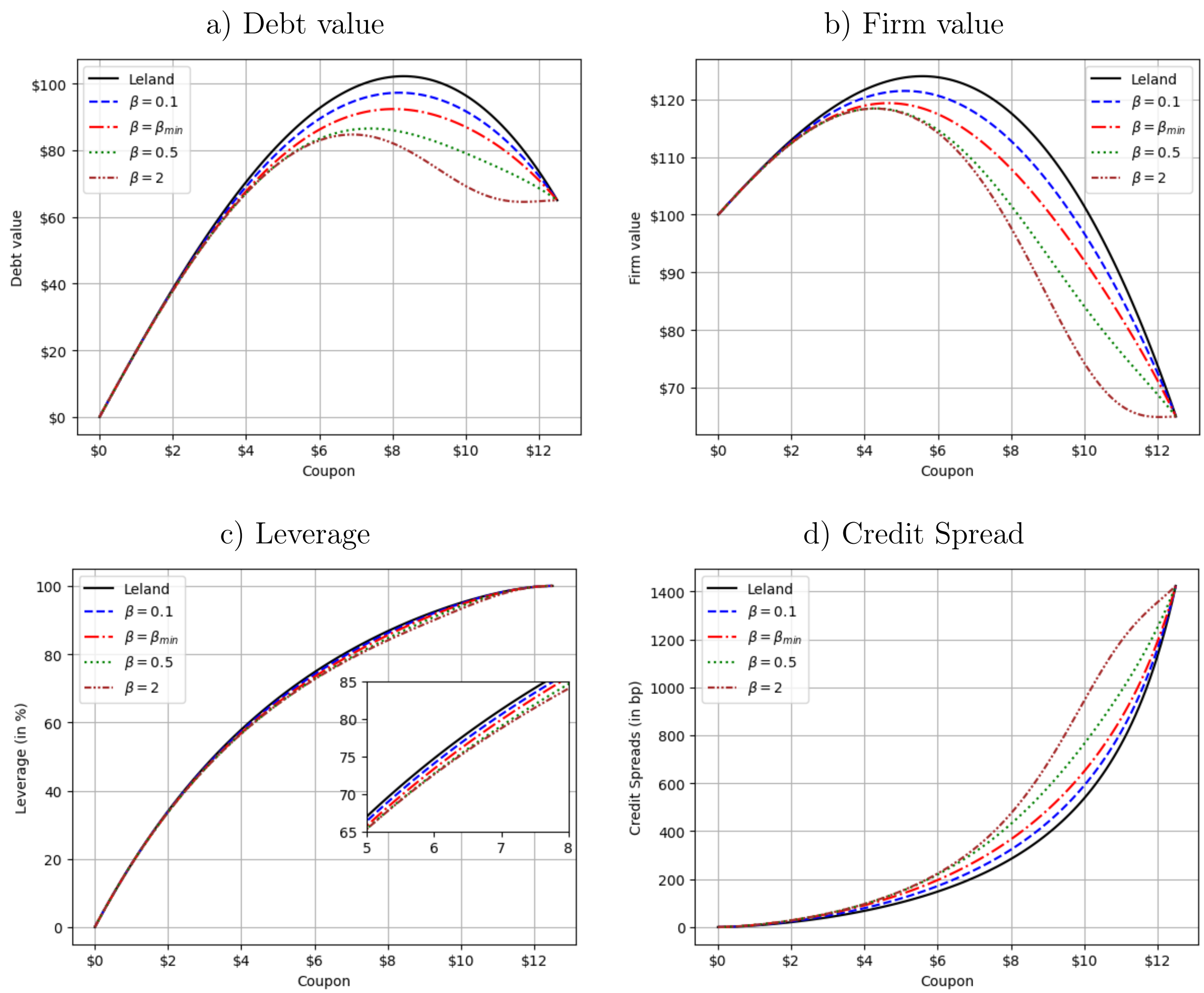
The purpose of this first numerical section is to investigate the impact of global warming on existing capital structure (subsection 4.1) and on the design of the capital structure (subsections 4.2) and (4.3)). We investigate different firms under two different scenarios, with regards to global warming (cf. the "Panel A: Net-Zero" scenario and the "Panel B: Currently Most Pessimistic" scenario described earlier). These figures reveal that the set of firms able to avoid the worst-case liquidation scenario is significantly narrower in panel B compared to panel A. This disparity illustrates that our model specification can capture the fact that very few firms anticipate concretely and internalize in their decisions the effects of global warming. It is worth noting that, under the zero-net scenario, one has $\beta_{\min}(\theta_A^\circ) \approx 1.86$, which highly contrasts with the most pessimistic scenario in which $\beta_{\min}(\theta_B^\circ) \equiv \beta_{\min}(\theta^\circ) \approx 0.2$.

4.1 Climate Exposure and existing Capital Structure

This sub-section investigates whether our model predictions are consistent with empirical observations. It is of great interest to see how the climate risk exposure impacts a given (not chosen) capital structure, because existing empirical studies may have collected data on firms that have designed their capital structure months or years before observations, that is before the climate risk becomes a concern.

Figure 3, inspired by Leland (1994), plots under the IPCC's most pessimistic scenario (Panel B) the value of debt in the graph (a), the firm value in the graph (b), the leverage in the graph (c) and the credit spread in the graph (d) for some *arbitrary* coupon levels and different levels of exposure to global warming β. We therefore simultaneously consider firms with different coupon levels and different levels of exposure to global warming, namely β equal to 0, 0.1, $\beta_{\min}$, 0.5 and 2). The Leland case (i.e. β) is depicted by a black solid line. Note that this figure does not represent equity because equity depends on the exposure β only when the coupon level is chosen endogenously. Moreover, we place in appendix the graphs for the Net Zero Scenario (Panel A).

Figure 3: Climate exposure & existing capital structure



Graphs are plotted with base case parameters.

The upper left graph (a) in Figure 3 shows the debt value for different coupon levels c and five levels of exposure β. The peak of each curve indicates the debt capacity (i.e., the largest debt value that the exposed firms could achieve by adjusting the coupon level). The upper left graph shows that, when the coupon level is given, the larger the climate risk β, the smaller the value of debt. The upper right graph (b) plots the total value similarly. The peak of each curve points to the optimal firm value i.e., the greatest value the exposed firms could achieve by adjusting the level of the coupon. This graph shows that, when the coupon level is given, the greater the climate risk β, the lower the firm value. Since both debt value and firm value decline when climate risk β increases, it is of interest whether and how leverage (i.e., their ratio) changes. The bottom left graph (c) in Figure 3 explores this question and shows first that leverage increases with coupon whatever the level of climate risk exposure and second that, for a given coupon level, the higher the β, the lower the leverage. So, regardless of their capital structure, companies with higher exposure have lower leverage. The debt value and the leverage appear to be respectively non-monotonic and monotonic functions of the level of coupon. The bottom right graph (d) in Figure 3 examines the credit spreads which result from this debt value and leverage. It shows that, given the size of the coupon, greater exposure to global warming leads to a larger credit spread. In this graph, the coupon amount is fixed and therefore does not depend on the amount of climate risk β, which in turn implies that the default barrier V_B does not depend on β. Here, a larger exposure simply means higher bankruptcy costs, and this quite logically imply higher credit spreads.

Overall, the results of Figure 3 are consistent with Ginglinger and Moreau (2023) and to some extent and Brown et al. (2021), which document that debtors and investors indeed require a compensation for climate risks. Our investigation can also explore the situations of firms with optimal capital structure or that exploit their debt capacity. This is precisely the topic of the following sub-sections.

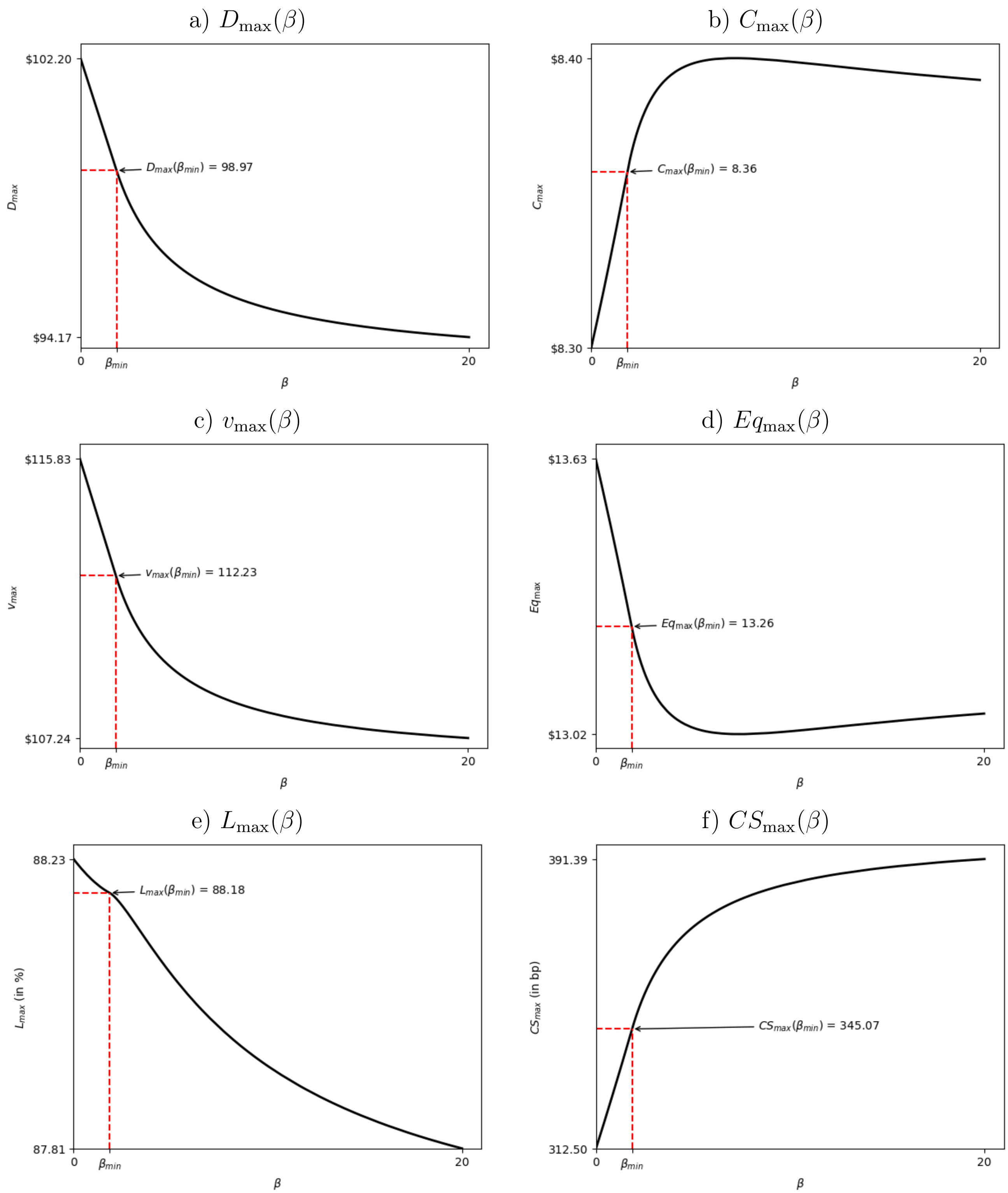
4.2 Climate Exposure and Debt Capacity

In our setting, a firm exhausts its debt capacity when it chooses the coupon level $C_{\max}$ so as to issue the debt with the largest value $D_{\max}$. Figure 4 shows, for each of our reference scenarios Panel A and B, a set of six graphs for examining firms that choose debt capacity as the criterion for designing its capital structure. Panels A and B both display the debt capacity, the level of the coupon associated to this maximum debt value and the corresponding firm value, equity value, leverage and credit spread as functions of the exposure to climate risk β. Note that we choose in abscissa the range of values for the firm's exposure to global warming (β) for illustrative purposes. It is important to stress that these graphs plot together firms with different profiles in terms of climate risk exposure (β, $\delta T_{\min}$), with a similar $\delta T_{\min}$ strictly larger than $\delta T^{\circ}(0)$ and that companies with relatively low β are less exposed to global warming, for short 'greener', than those having large β.

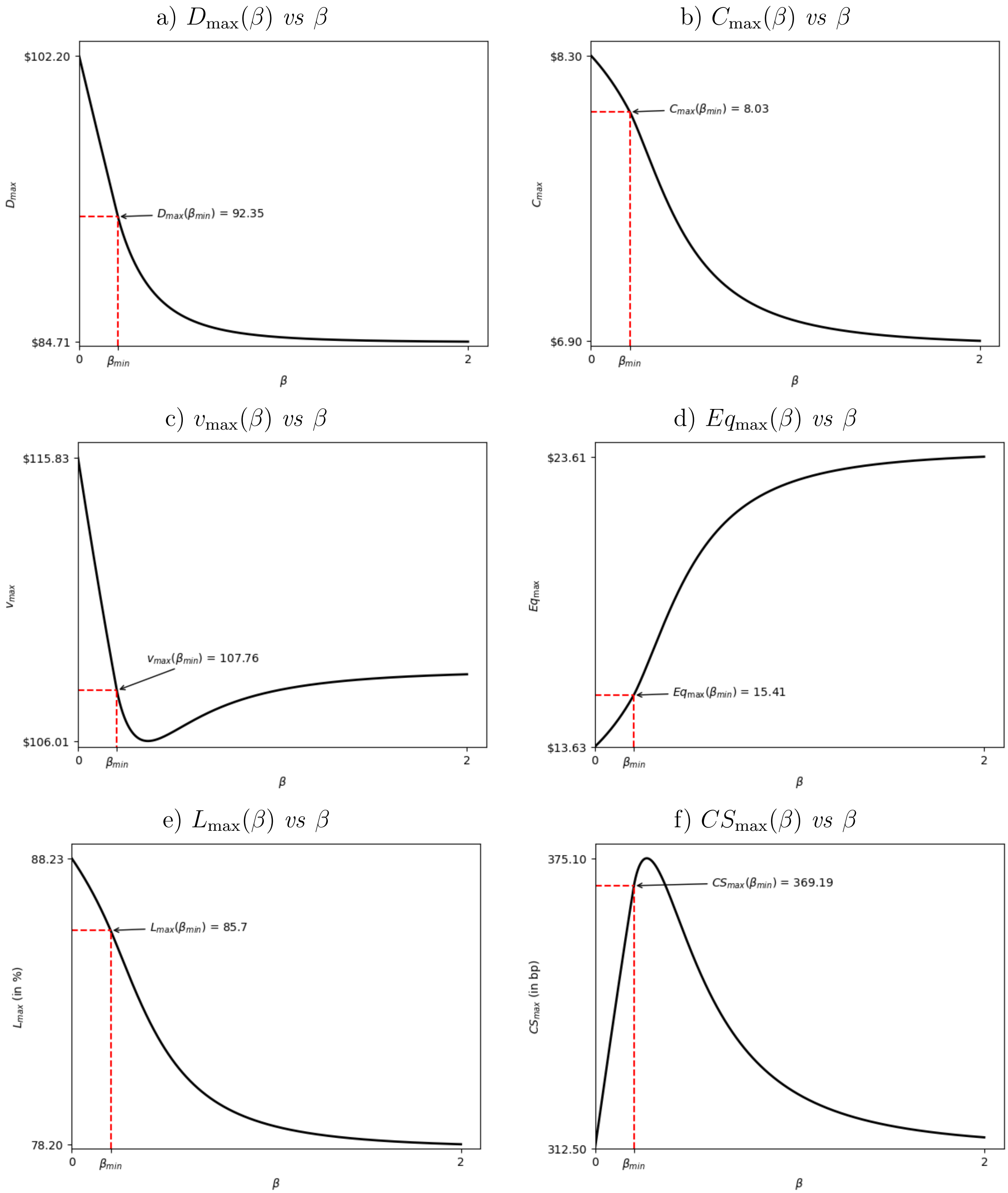
The six graphs of Panel A examine the situation of firms facing or anticipating the net-zero scenario, where the long-term temperature perturbation is $\theta^{\circ} = 1.5^{\circ}C$. The upper left graph (a) shows the firm's debt capacity as a function of its exposure to climate risk. The debt capacity

Figure 4: Capital Structure and Climate Exposure under Debt Capacity

Panel A: Net-Zero Scenario ($\theta_A^\circ = 1.5°C$)



Panel B: Current Most Pessimistic Scenario ($\theta^{\circ} = 4.4°C$)



The various graphs illustrate the effects of a change in global warming exposure on the debt capacity, coupon, firm value, equity value, leverage and credit spread.

of companies exposed to climate risk is lower than that of Leland's. It also indicates that an increase in global warming exposure β leads to a decrease in debt capacity. The graph specifically reveals that the loss of debt capacity is roughly -7.86% ($\approx (94.17 - 102.2)/102.2$), when the exposure attains 20. Interestingly, almost half of this loss is realized for an exposure as small as $\beta_{\min}(\theta_A^\circ) = 1.86$. In the upper right graph (b), the level of coupon, which corresponds to debt capacity, shows the debt service of companies exposed to climate risk is larger than that of Leland's. The level of the coupon however does not mimic the significant and monotonic decline in the maximum debt value observed in graph (a). Rather, the coupon $C_{\max}$ changes only slightly at first, at most 10 basis points. As the coupon level determines the default barrier, this graph shows the equityholder of an exposed firm tends to default slightly earlier than in Leland. Second, the coupon initially increases, reaches a maximum, and then decreases as larger exposures are considered. This implies that the coupon level (viewed as a driver of the default barrier) does not explain the decline in debt value observed in graph (a). As can be seen from graph (c) in Figure 4, the value of companies exposed to climate risk is lower than that of Leland's. When the exposure β increases, the value of the firm decreases as does the debt capacity. This is confirmed by the very small change in leverage ratio in graph (e). Graph (d) shows interesting features for equity. First, in a Net-Zero scenario, companies exposed to climate risk require almost as much equity as in Leland's. Equity is the difference between two monotonic and decreasing functions, firm value and debt, whose ratio is approximately constant, so this was to be expected. Graph (d) nevertheless shows that the value of equity slightly decreases, then reaches a minimum (namely 13.02), and then increases up to 13.17 for $\beta = 20$ (and same for larger values of β). In graph (e), the leverage ratio of companies exposed to climate risk is slightly lower than that of Leland and it decreases as the exposure increases. This means that the decline in firm value and debt value is not strictly proportional and that the decline in debt value is only slightly larger than the decline in firm value. The bottom right graph (f) on credit spreads shows that the compensation required for credit risk is greater for firms exposed to global warming than for Leland's. This compensation increases monotonically with exposure to climate risk, but the increase marginally decreases as β becomes large. Credit spreads tend to converge towards a limit that expresses both the finiteness of liquidation costs and the control of leverage by the exposed companies. For the most exposed companies, the decline in leverage observed in graph (e) relates (roughly speaking) to a similar coupon level, i.e. a similar default barrier and a similar probability of default. This clearly contributes to the similarity of credit spreads for the greatest exposure. Overall, our model of debt capacity predicts that, *under the net-zero scenario*, the exposure to global warming and asset stranding makes firms reduce the amount of debt they choose to issue. This decision, however, leads to only a slightly smaller leverage because the firm's value simultaneously decreases. The credit spreads, which are larger than Leland's, strictly increase with the exposure. This means that, under the net-zero scenario and for the climate risk profiles we consider (namely, $(\delta T_{\min}, \beta)$ with $\delta T_{\min} > \delta T^{\circ}(0)$), a larger exposure increases the recovery risk upon liquidation and this deserves an extra compensation.

The six graphs of Panel B examine the situation of firms that are facing or anticipating the currently most pessimistic scenario, where the long-term temperature perturbation is $\theta^{\circ} = 4.4°C$. The upper left graph (a) shows the firm's debt capacity as a function of its exposure to climate risk. The debt capacity of companies exposed to climate risk here again is lower than that of Leland's. Compared to firms facing or anticipating a net-zero scenario, exposed firms show a more dramatic decline in debt capacity as their exposure to global warming worsens. the loss in debt capacity is larger, and this loss occurs at far smaller βs.[13] The loss in debt capacity amounts to -17.1% ($\approx (84.71 - 102.2)/102.2$) for $\beta = 2$. Here again, more than half of the loss is realized for $\beta_{\min}$ (although this beta differs from previous case). Under the most pessimistic scenario, in the upper right graph (b), the debt service of companies exposed to climate risk is smaller than that of Leland's. Since the coupon level determines the default barrier, the equityholder of an exposed firm in this context tends to default slightly later than in Leland. Here, the coupon level correlates positively with the debt value and mimics the monotonic decline. Convexity of the curve is however different. In Graph (c), the value of exposed firms is smaller than Leland's and does not respond monotonically to the climate risk exposure. It first falls sharply, as β increases, up to a certain level (smaller than the smallest one found under the zero-net scenario) and then it slightly rises. For an exposure as small as $\beta_{\min} = 20\%$, the loss attains 82.2% ($\approx$ $(115.83 - 107.76)/(115.83 - 106.01)$) of the maximum loss. Hence, the exposure to climate risk has first a strong negative impact on the value of the firm. Then, the firm value appears slightly greater, meaning that high levels of β pushes the exposed firms to take decisions worthy for the firm. Graph (d) displays interesting features. In contrast to the net zero scenario, the equity value of companies exposed to climate risk in a pessimistic scenario is greater than that of Leland,

[13]Remind the range of βs in the abscissa.

which means that they require more equity financing. And the difference in equity financing can be as significant as 73.2% ($\approx (23.61 - 13.63)/13.63$) for $\beta = 2$. This need for additional equity financing means that exposed companies bear some additional risks to which debtors do not want to be so exposed. With no surprise, in graph (e), the leverage decreases with the exposure. More interesting, decline is far more significant (than under the net zero scenario) as it amounts to -11.4% ($\approx (78.20 - 88.23)/88.23$) for $\beta = 2$. The credit spreads in the bottom right graph (f) also show interesting features. First, here again, the compensation for credit risk is larger for firms exposed to global warming than for Leland's. More interestingly, the credit spread rises sharply at first, attains a maximum and then declines. Hence, the exposure to climate risk initially has first a negative direct effect and increases credit spread and, then, at a higher level, it affects the design of the capital structure in such a way that the compensation required by creditors marginally decreases with the level of exposure. In other words, it could be that the exposure to climate risk has a disciplining effect that makes the exposed companies less credit risky (and also more valuable, as we have observed with firm value). The relative reduction in credit risk, as evidenced by the comparatively lower credit spreads, corresponds with the strategic decision to issue less debt, the increased necessity for equity financing, and the lower coupon rates, which subsequently result in a lower default barrier. The credit spread required to a firm with a modest $\beta_{\min} = 20\%$ almost attains the maximum. Overall, our model of debt capacity predicts that, *under the currently most pessimistic scenario*, the exposure to global warming and asset stranding makes firms reduce the amount of debt they choose to issue. This decision leads to a reduction in leverage, a larger recourse to equity financing, given that the equity value increases with the exposure. Both the level of debt to issue, the leverage, the debt service and the default barrier systematically decrease. Exposure to climate risk first adversely affects credit risk (captured by the credit spread) and then affects debt capacity and financing, allowing credit risk to decline.

At total, firms that are exposed to climate risk and exhaust their debt capacity, will lower debt capacity and leverage. Such declines may suggest that exposure to global warming and asset stranding have a systematic disciplinary effect. Our simulations show that the potential disciplinary effect on credit risk critically depends on the scenario considered by the firm (net-zero vs. most pessimistic scenario) and the structural firm's exposure to global warming, in particular $\delta T_{\min}$. Under the net zero scenario, the larger the exposure to climate risk, the larger the credit spread and the earlier the default decided by shareholders (relative to Leland). Firm values and credit spreads are negatively and positively correlated with exposure, respectively. Under the most pessimistic scenario, the leverage then decreases much more sharply and the sign of the correlation between the exposure and the firm value and the credit spread may change. Firm value correlates negatively at first and then positively with global warming exposure, as this exposure increases. The opposite is found for the credit spread.

Finally, it should be emphasized that in the above simulations we consider companies that are not *yet* affected by global warming (or in the notation $(\beta, \delta T_{\min})$ with $\delta T_{\min} > \delta T^{\circ}(0)$). We perform (but do not report) additional simulations for firms that are *already* exposed to global warming (namely $\delta T_{\min} \leq \delta T^{\circ}(0)$). We find no significant difference compared to Figure 4, panel B (even for the net zero scenario). The qualitative implications are the same, the quantities can (of course) be different. For the net zero scenario, this is a significant change. And this shows that the distance between the long-term temperature (θ°) and $\delta T_{\min}$ is an important factor in making the disciplinary effect real. And if $\delta T_{\min} = \delta T^{\circ}(0)$, this distance is of course the largest possible.

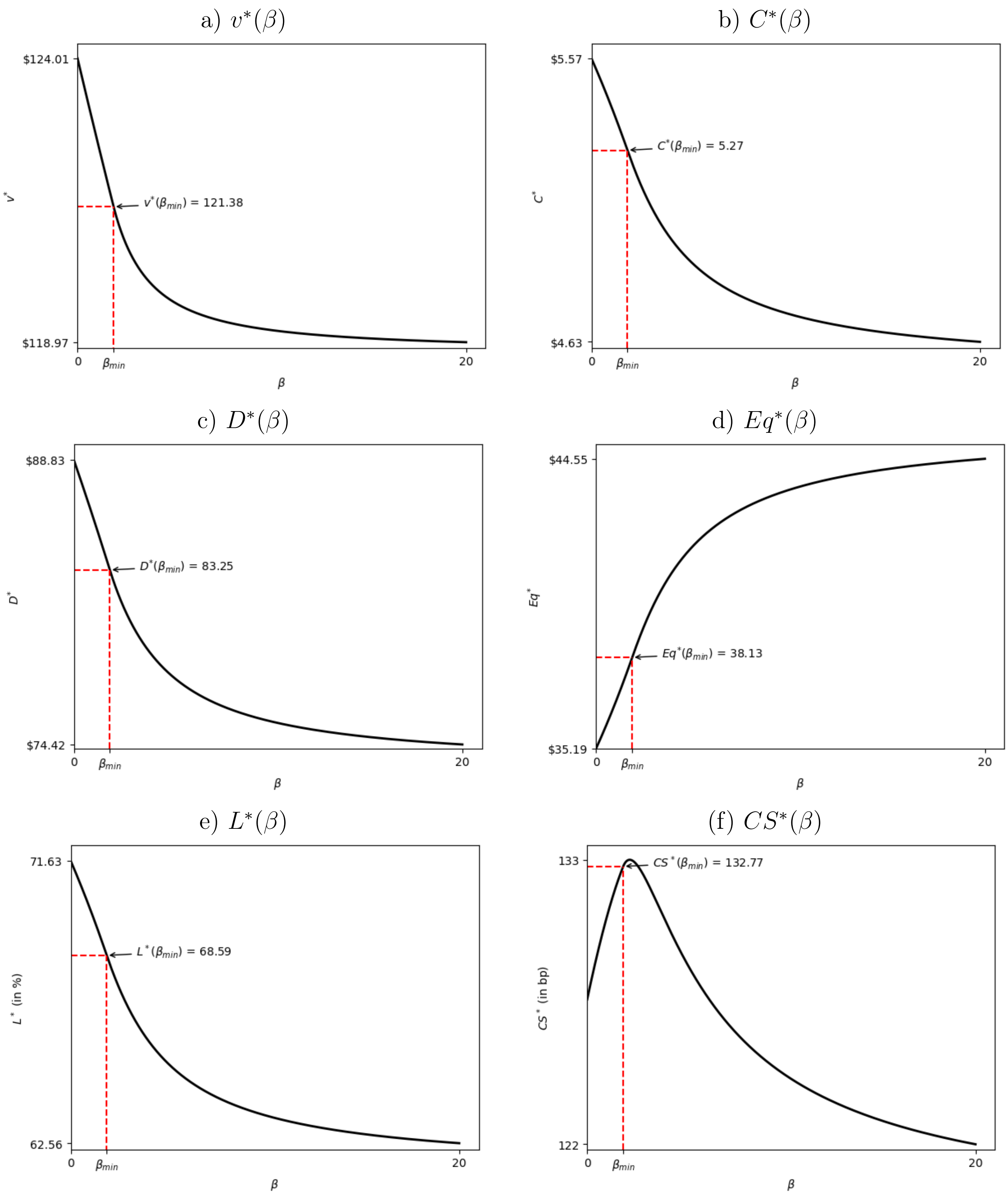
4.3 Climate Exposure and Optimal Capital Structure

The optimal capital structure arises when the firm determines the appropriate coupon C^* that maximizes the value of the firm v^*. Figure 5 shows, for each of our reference scenarios Panel A and B, a set of six graphs for examining a firm that chooses maximization of firm value as the criterion for designing its capital structure.

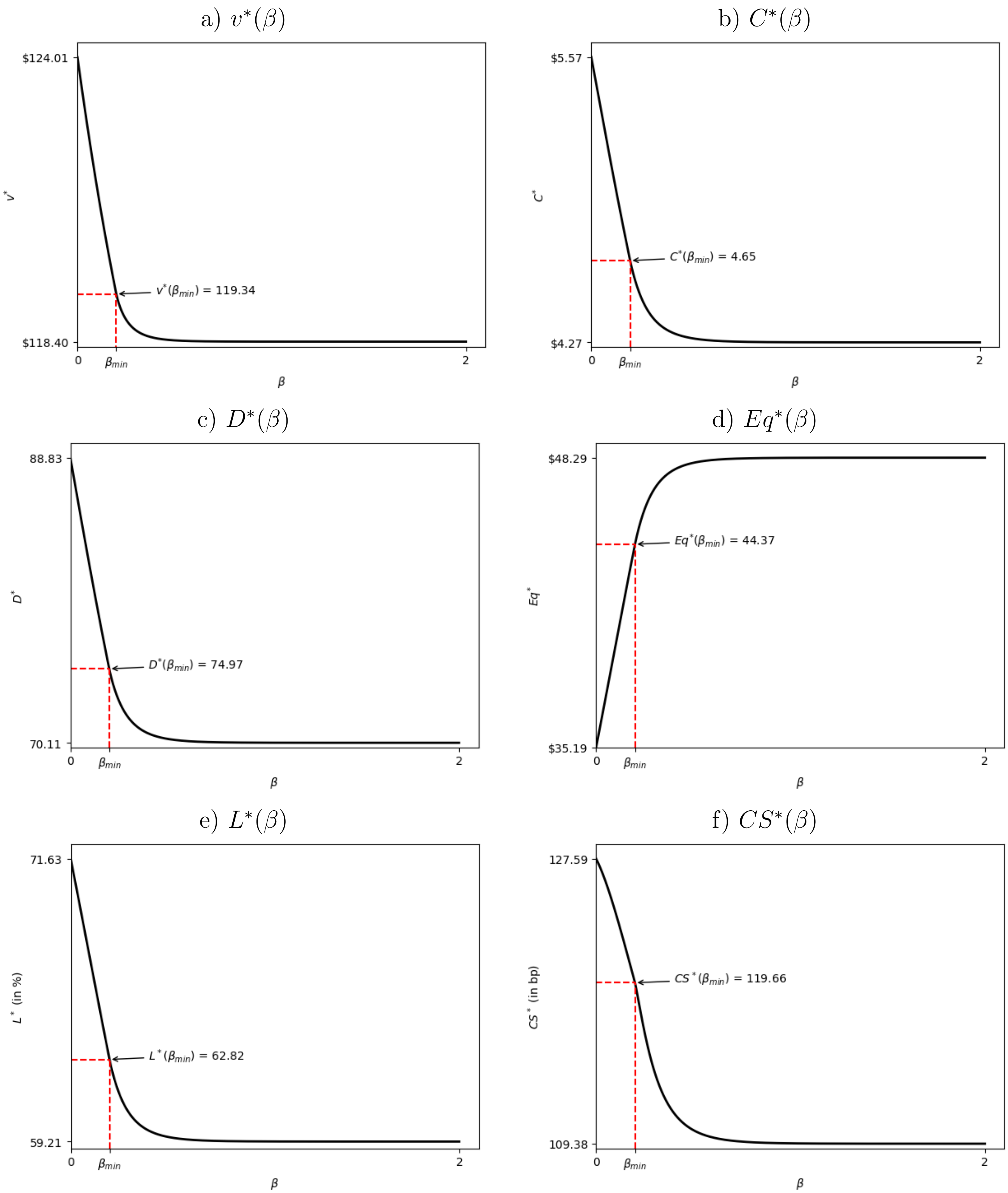
The six graphs of Panel A examine the situation of firms facing the net-zero scenario, where the long-term temperature perturbation is $\theta^{\circ} = 1.5^{\circ}C$. The upper left graph (a) shows the firm's value as a function of its exposure to climate risk. The value of companies exposed to climate risk is lower than that of Leland's. It also indicates that an increase in global warming exposure β leads to a decrease in firm value. The loss in value is -4.06% ($\approx (118.97 - 124.01)/124.01$) for a company with $\beta = 20$. Interestingly, almost half of this loss is realized for an exposure as small as $\beta_{\min}(\theta^{\circ}_A) = 1.86$. In the upper right graph (b), the debt service of companies with optimal capital structure and exposed to climate risk is smaller than that of Leland's. The coupon level decreases, resulting in a reduced debt service as the exposure increases. While the change in coupon may appear modest, its decline correlates positively with the huge negative impact of the exposure on the debt value observed in graph (c). The loss in debt value due to exposure to global warming is -16.2% ($\approx (74.42 - 88.83)/88.83$) for a firm with $\beta = 20$. The equity value plotted in graph (d) increases, meaning that more equity financing is required compared to Leland. And the increase amounts to 26.6% ($\approx (44.55 - 35.19)/35.19$) for $\beta = 20$. Clearly, this contrasts with the situation of firms that anticipate the same scenario but decide to maximize their debt capacity. Graph (e) shows leverage decreases significantly with exposure. This indicates that the decline in debt value is larger than the decline in firm's value. Quite interestingly, the graph (f) first shows that credit spreads associated to firms exposed to climate risk may be larger or smaller than the Leland's. The firm's exposure to global warming first causes credit spreads to be above the one of Leland, then the credit spreads attain a peak and then decline while remaining strictly larger than the Leland. There exists a threshold for the exposure beyond which the credit spread becomes smaller than Leland's. With climate credit risk in mind, this graph shows that global warming and asset stranding may have a strong disciplinary effect on firms maintaining optimal capital structure. The six graphs of Panel B examine the situation of firms facing the currently most pessimistic scenario, where the long-term temperature perturbation is $\theta^{\circ} = 4.4°C$. Despite the different range of values for β on the abscissa, most of these graphs do not differ significantly from those in Panel A. Graphs (a, b, c, d, e) show similar curves, differing only in the range and magnitude of the numbers. We therefore seize the opportunity to add some insights about the potential convergence. The optimal coupon declines sharply. It is smaller here for $\beta_{\min} = 20\%$ than it is in Panel A for $\beta = 20$. Graph (a) shows a greater exposure to global warming leads to a monotonic response in firm value that declines sharply at first and then slowly. Using the Appendix 2 and the optimal coupon C^* found numerically, one finds the firm value converges to $\lim_{\beta \to \infty} v^*(\beta) = 118.4$. And, when the sensitivity of the firm has a modest value of $\beta_{\min} = 20\%$, the loss of firm value represents 73% ($\approx (124.01 - 119.34)/(124.01 - 118.4)$) of the maximum decline. Graph (c) shows the optimal debt values stand between 89 and 70 (to compare with 74 in Panel A). Here again, the limit of the debt value can be determined analytically and the value of debt converges, as β becomes large, to $\lim_{\beta \to \infty} D^*(\beta) = 70.1$. Graph (d) shows equity values larger than the Leland's and a positive correlation between the equity value and global warming exposure. The firm's equity value grows by $37.2\% \approx (48.29 - 35.19)/35.19$ when $\beta = 2$ which is close to the limit $\lim_{\beta \to \infty} Eq^*(\beta) = \lim_{\beta \to \infty} (v^*(\beta) - D^*(\beta)) = 48.3$. Graph (e) shows leverage is in line with expectations. For a modest $\beta_{\min} = 20\%$, the decrease of leverage reaches 68.76% ($\approx (71.63 - 62.82)/(71.63 - 59.21)$) of the total decline. Far more interesting (compared to Panel

Figure 5: Optimal Capital Structure and Climate Exposure

Panel A: Net-Zero Scenario ($\theta_A^\circ = 1.5°C$)



Panel B: Current Most Pessimistic Scenario ($\theta^{\circ} = 4.4°C$)



The various graphs illustrate the effects of a change in global warming exposure on the firm value, coupon, debt value, equity value, leverage and credit spread given that the firms have an optimal capital structure.

A), the graph (f) shows here, in the pessimistic scenario, that credit spreads are all smaller than Leland's and strictly decreasing with β. The credit spread sharply declines first and then more slowly. As lower credit spreads represent lower credit risk, one may conclude that firms with higher exposure reduce their credit risk when setting an optimal capital structure. The exposure to climate risk affects the financing decision and management so much that a disciplinary effect emerges. Firms that are more exposed to global warming may be (in relative terms) less credit risky.

In summary, firms that are exposed to climate risk and optimize their value will lower leverage relative to Leland. Corresponding credit spreads indicate the existence of a disciplinary effect of global warming and asset stranding. Our simulations show that the potential disciplinary effect on credit risk depends critically on the scenario that the firm considers (net zero scenario vs. most pessimistic scenario). In the net zero scenario, exposure to climate risk may result in a wider or narrower credit spread (compared to Leland). In the most pessimistic scenario, exposure leads to a smaller credit spread. In both scenarios, the leverage decreases with exposure, as do the firm value, the debt service and default threshold, while the equity value increases. Thus, the shareholder decides to default later than in Leland.

Our simulations have illustrated the impact of global warming and asset stranding on key dimensions of corporate finance when companies incorporate these issues into their financing decisions. We find a potential disciplinary effect of global warming and asset stranding on these dimensions and show that this effect is much more pronounced for firms with an optimal capital structure than for firms that utilize their debt capacity. The next section examines a number of well-known credit risk management metrics to disentangle the direct and indirect effects of global warming on credit risk.

5 Climate Credit Risk Management

The purpose of this second numerical section is to examine the extent to which global warming affects common credit risk management metrics. For illustration purposes, we concentrate on the most pessimistic scenario in the core text and relegate the net-zero scenario into appendix. Our ultimate goal here is to isolate climate credit risk, i.e., the climate component of credit risk. Actually, this is not an easy task due to the disciplinary effect of global warming. In this section, we review credit spreads, insurance costs, default probabilities, and losses given default, the latter two dimensions being core components of the former.

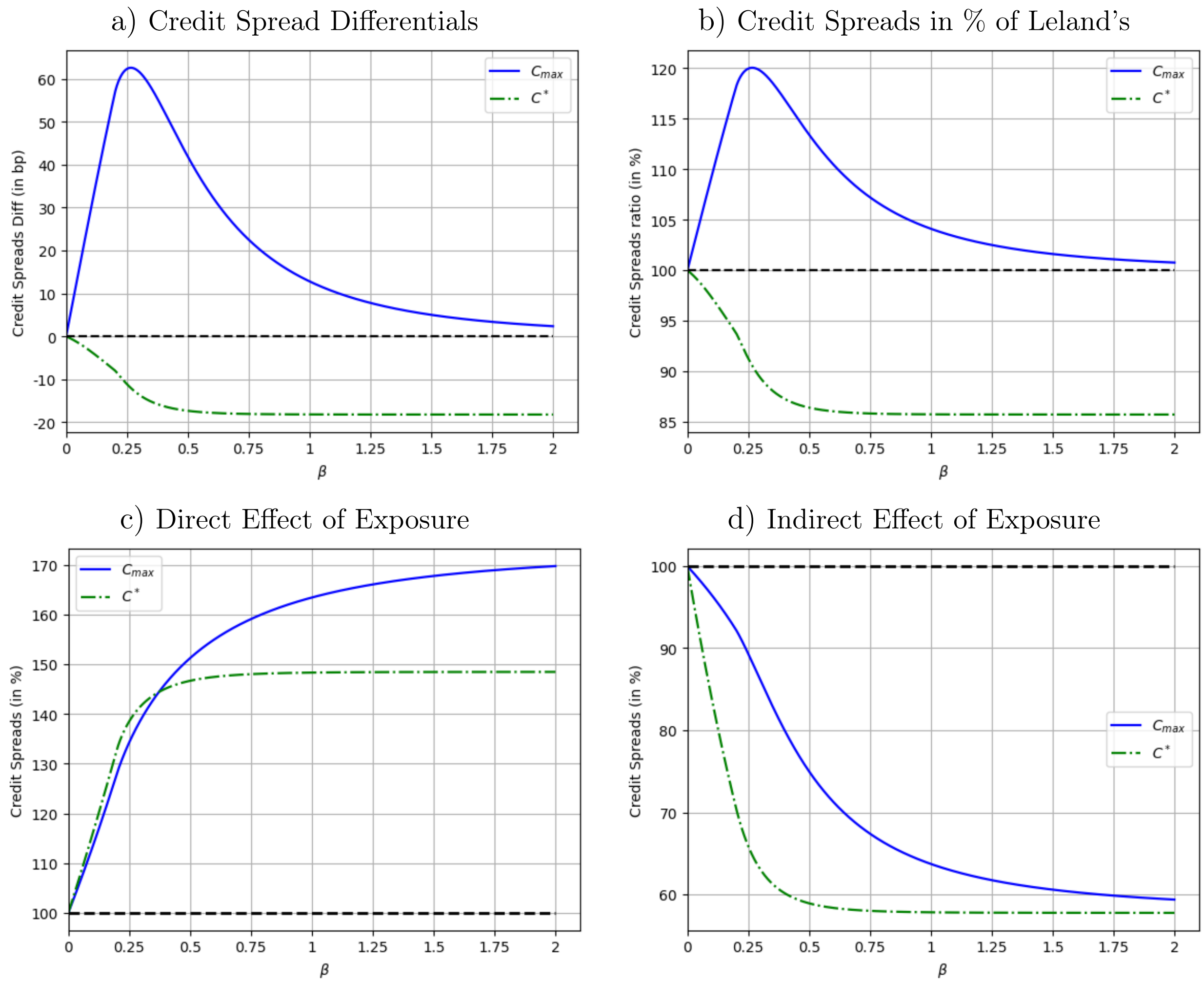
5.1 Credit Spreads & Climate Credit Risk

Computing a simple difference of credit spreads, namely $CS(c;\beta) - CS(c;0)$, between firms exposed to global warming and Leland's ones (given arbitrary level of coupons) is not an adequate way to isolate and quantify the overall climate credit spreads for at least three reasons. First, any arbitrary level of coupon c may have a completely different meaning for firms and it is not that easy to control how different the arbitrary level of coupon is from the coupon levels that lead to optimal capital structure or debt capacity. Second, credit spreads increase with the level of c (cf. Figure 3.d)), so that the additional compensation to investors is not very meaningful in absolute terms. These two problems imply that calculating a marginal credit spread as a percentage of Leland (namely, the ratio $CS(c;\beta)/CS(c;0)$) is neither very helpful in this regard. Third, an overall climate credit spread (considered as a natural proxy for climate credit risk) should take into account not only the direct compensation for climate risk, but also the indirect potential disciplinary effect on the amount of debt. At minimum, one should compute the differences $CS(C^*(\beta);\beta) - CS(C^*(0);0)$ and $CS(C_{\max}(\beta);\beta) - CS(C_{\max}(0);0)$ or the ratios $\frac{CS(C^*(\beta);\beta)}{CS(C^*(0);0)}$ and $\frac{CS(C_{\max}(\beta);\beta)}{CS(C_{\max}(0);0)}$ as a function of the sensitivity to global warming. Here, $CS(C^*(0);0)$ is the credit spread of the optimal debt issued by a Leland's firm (with no climate exposure). Of course, the values of these differences or ratios can be derived naturally from Figures 4 and 5 (see Panel B). However, it is not clear how they break down into the direct and indirect effects.

Figure 6 shows in upper graphs how credit spreads diverge to Leland benchmarks when firms that are differently exposed to global warming design their capital structure so as to maximize firm value or debt capacity. The left upper graph plots the credit spread differential in basis points, while the right one plots the credit spread ratio in percentage. It is important to note that in both graphs the single dashed abscissa stands for two different reference Leland's value (one associated with firms with optimal capital structure and one associated with maximum debt capacity).

Upper graphs show that, under the most pessimistic scenario, a firm that maximizes debt capacity will have a larger compensation to investors compared to benchmark. This was expected given 4 and 5. The additional reward to pay is not however monotonic with the level of exposure

Figure 6: Credit Spreads & Climate Credit Risk



This figure illustrates the effects on the credit spreads of a change in β the firm's exposure to global warming. The credit spread differential in basis points is computed with $CS(C(\beta);\beta) - CS(C(0);0)$. The credit spread in % of Leland's is computed with $\frac{CS(C(\beta);\beta)}{CS(C(0);0)}$. The direct effect is measured by $\frac{CS(C(0);\beta)-CS(C(0);0)}{CS(C(0);0)}$, the indirect effect by $\frac{CS(C(\beta);\beta)-CS(C(0);\beta)}{CS(C(0);\beta)}$. Beware that these quantities do not sum.

to global warming and the existence of a peak signals the disciplinary effect discussed earlier. The new information compared to Figure 4 and Figure 5 concerns the magnitude of this difference. For the firms that exploit their debt capacity, the credit spread is (relatively speaking) at most 20% larger than the Leland benchmark, it then declines with the level of their exposure β, the limit of the incremental cost being as small as 1%. In contrast, companies with an optimal capital structure experience a stronger disciplinary effect, resulting in the credit spread (relative to Leland) falling to slightly less than -14% for many of them.[14]

[14]Readers could be surprised by the seemingly irreconcilable magnitudes in the left and right graphs. Actually, absolute and relative magnitudes are consistent, because they refer to different values. For firms with optimal capital

The preceding simulations shed light on the overall effect of climate exposure, but they do not isolate the two different components of this overall effect, namely the direct effect of exposure to global warming and the indirect effect resulting from the change in capital structure. Observing that[15] $\frac{CS(C^*(\beta);\beta)}{CS(C^*(0);0)} = \frac{CS(C^*(\beta);\beta)}{CS(C^*(0);\beta)} \times \frac{CS(C^*(0);\beta)}{CS(C^*(0);0)}$, it is possible to investigate the two new ratios separately. The ratio $\frac{CS(C^*(0);\beta)}{CS(C^*(0);0)}$ can be used to assess the direct effects of greater exposure to global warming *ceteris paribus*, i.e. holding the capital structure constant. The ratio $\frac{CS(C^*(\beta);\beta)}{CS(C^*(0);\beta)}$ can be used to assess the indirect effects of a change in capital structure *ceteris paribus*, i.e., for a given level of exposure. This indirect effect is, of course, related to the aforementioned disciplinary effect. Without loss of generality, we have chosen to plot ratios

$$\frac{CS(C^*(0);\beta) - CS(C^*(0);0)}{CS(C^*(0);0)} \quad \text{and} \quad \frac{CS(C^*(\beta);\beta) - CS(C^*(0);\beta)}{CS(C^*(0);\beta)} \tag{11}$$

in graphs c) and d) of Figure 6 as representative of the direct and indirect effects of corporate exposure to global warming. These are just the aforementioned ratios above minus one. It is important to stress that as they do not share the same denominator, summing them is not appropriate. The left graph shows that the direct effect implies an increase in credit spreads, and that firms that favor debt capacity are affected by the direct effect as much (approximately) or more than firms with an optimal capital structure (in relative terms). The right graph shows that the indirect effect reduces credit spreads and that companies with an optimal capital structure are affected by the indirect effect significantly more than companies with maximum debt capacity.

5.2 Climate Credit Risk and Credit Insurance Costs

Paralleling the analysis of the credit spreads, Figure 7 investigates credit insurance costs for different levels of corporate exposure to global warming. It could be shown that the insurance costs increase with the levels of coupon and the magnitude of the exposure. This is no surprise. The considerations we discuss in depth with credit spreads apply to the insurance costs, except now insurance costs nevertheless call for an analysis in absolute rather than in relative terms. We

structure, for illustration, one finds $CS(C^*(\beta = 2); \beta = 2) \approx 109.4$ bp and $CS(C^*(0); 0) \approx 127.6$ bp, so that the difference $CS(C^*(2); 2) - CS(C^*(0); 0) \approx -18.2$ bp and the ratio $CS(C^*(2); 2)/CS(C^*(0); 0) \approx 85.75\%$. For firms with debt capacity, one finds $CS(C_{\max}(2); 2) \approx 314.87$ bp, $CS(C_{\max}(0); 0) \approx 312.5$ bp, so that $CS(C_{\max}(2); 2) - CS(C_{\max}(0); 0) \approx 2.37$ bp and $CS(C_{\max}(2); 2)/CS(C_{\max}(0); 0) \approx 100.8\%$.

[15]We focus here on the 'optimal capital structure' case, but similar considerations apply to the 'debt capacity' case.

therefore favor the following equations that parallel expressions (11),

$$[Ins(C^*(\beta);\beta) - Ins(C^*(0);0)] \quad \text{and} \quad [Ins(C_{\max}(\beta);\beta) - Ins(C_{\max}(0);0)] \tag{12}$$

and plot them in the upper right graph (a) of Figure 7 as a function of the sensitivity to global warming. In broad terms, this graph only partly agrees with our discussion of the upper right graph (a) of Figure 6 on credit spreads. For firms with optimal capital structure, the smaller relative credit spreads translate into smaller insurance costs. For firms with maximal debt capacity, the rise and then fall of the relative credit spreads translates into similar patterns for insurance costs. However, more interestingly, the greater absolute credit spreads *do not* systematically translate into greater absolute insurance costs. To see why this counter-intuitive result is possible, consider a couple of firms A and L with $\beta_A > 0$ and $\beta_L = 0$ and denote $CS^A_{\max} = CS_{\max}(C(\beta_A), \beta_A)$ and $CS^L_{\max} = CS_{\max}(C(\beta_L), \beta_L)$. By virtue of (b) of Figure 6, one has $CS^A_{\max} > CS^L_{\max}$ or equivalently $CS^A_{\max} - CS^L_{\max} > 0$. Given the respective definitions of the credit spread and insurance costs (cf. expression (8) and (10)) and the fact that $r(Ins(c) + D(V;c)) = c$, then

$$\begin{aligned}
CS^A_{\max} - CS^L_{\max} > 0 &\iff \frac{C^A_{\max}}{D^A_{\max}} - \frac{C^L_{\max}}{D^L_{\max}} > 0 \\
&\iff r\frac{Ins^A_{\max} + D^A_{\max}}{D^A_{\max}} - r\frac{Ins^L_{\max} + D^L_{\max}}{D^L_{\max}} > 0 \\
&\iff \frac{Ins^A_{\max}}{D^A_{\max}} - \frac{Ins^L_{\max}}{D^L_{\max}} > 0 \\
&\iff Ins^A_{\max} - Ins^L_{\max}\frac{D^A_{\max}}{D^L_{\max}} > 0
\end{aligned}$$

with $\frac{D^A_{\max}}{D^L_{\max}} < 1$. Hence, the ordering of credit spreads (namely $CS^A_{\max} > CS^L_{\max}$) is not a sufficient condition to warranty the ordering of the insurance costs (i.e. $Ins^A_{\max} - Ins^L_{\max} > 0$) because the debt values to which these credit spreads refer are different. This means that the disciplinary effect affects the capital structure of firms with maximum debt capacity so much that the insurance costs finally become smaller than the benchmark Leland insurance costs. Observing that[16] the

[16]We focus on one difference, the one related to the optimal capital structure, but similar considerations apply to the other difference, which is related to the debt capacity of firms.

difference $Ins(C^*(\beta);\beta) - Ins(C^*(0);0)$ satisfies

$$Ins(C^*(\beta);\beta)-Ins(C^*(0);0) = [Ins(C^*(\beta);\beta) - Ins(C^*(0);\beta)]+[Ins(C^*(0);\beta) - Ins(C^*(0);0)]\,,$$

it is possible to investigate the two new differences separately. The difference $Ins(C^*(0);\beta) - Ins(C^*(0);0)$ informs on the direct effects of greater exposure to global warming *ceteris paribus*, i.e. holding the capital structure constant, while the difference $Ins(C^*(\beta);\beta) - Ins(C^*(0);\beta)$ informs on the indirect effects of a change in capital structure *ceteris paribus*, i.e. for a given level of exposure. Here also, the indirect effect is related to the aforementioned disciplinary effect. We plot these two differences in graphs (c) and (d) of Figure 7 respectively, as representative of the direct and indirect effects of corporate exposure to global warming.

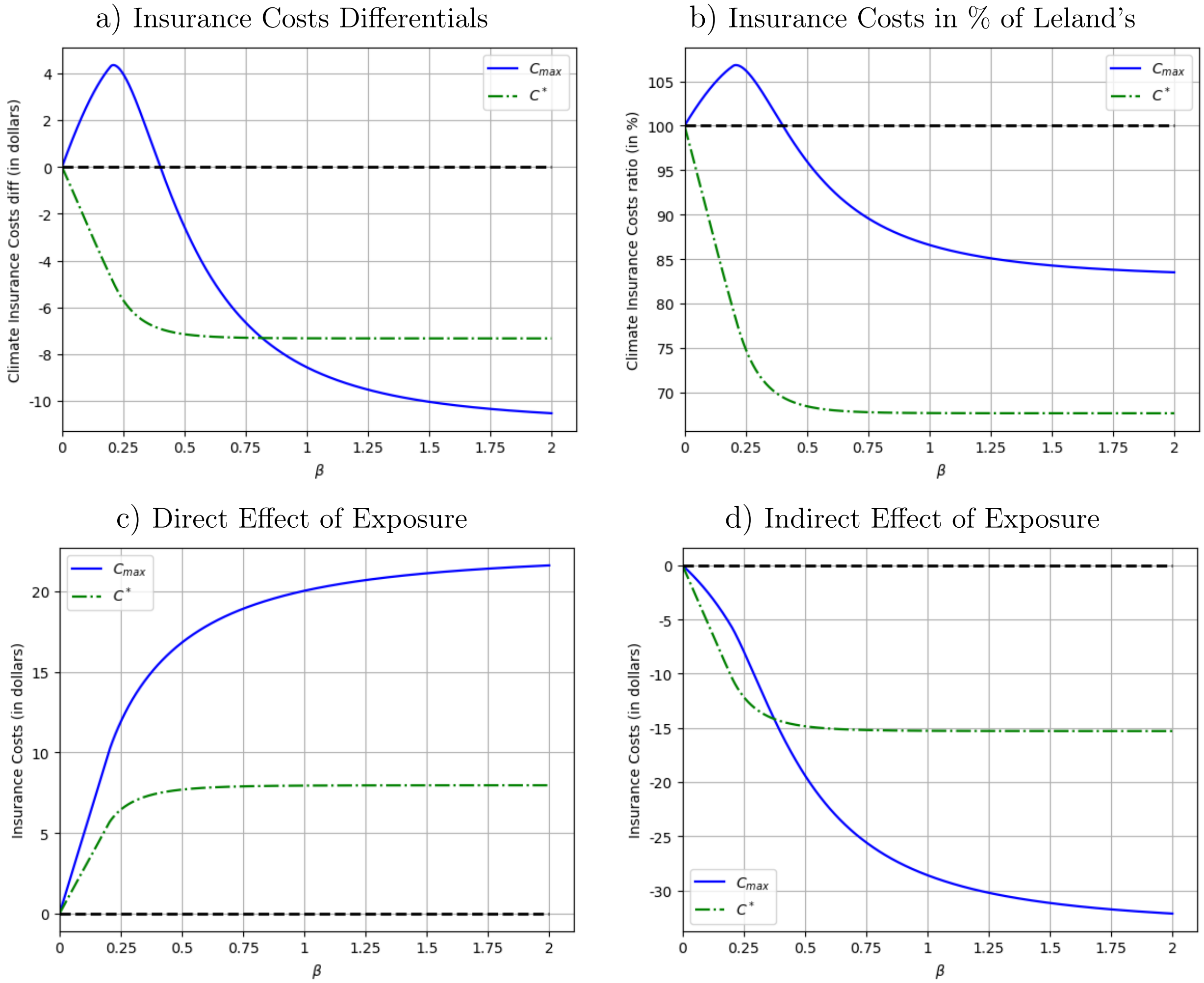
The left graph of Figure 7 shows that the direct effect of the exposure to warming increases the insurance costs, and that firms that favor debt capacity are affected by the direct effect far more than firms with an optimal capital structure. In both cases, there exists a cap to the additional insurance costs. The cap for firms with optimal capital structure is attained for a modest value of β. The right graph of Figure 7 shows, as expected, that the indirect effect reduces the insurance costs and, less expectantly, that companies with an optimal capital structure are not necessarily affected by the indirect effect the most (in absolute value...). The disciplinary effect (as measured by the differential of insurance costs) appears to be particularly pronounced for firms that exhaust their debt capacity and have a significant exposure to warming.

5.3 Probability of Default and Loss Given Default

The probability of Default (PD) and the Loss Given Default (LGD) are standard metrics in credit risk management, and they are also key parameters in Bank regulation. Both determine the level of credit spreads.

The Probability of Default over a certain period is analytical and given by equation (9). Figure 8 plots some term structures of the default probabilities for different values of β. The curves are with no surprise upward-sloping and we consider ultra-long horizon to figure out how the probabilities of default converge to the one of the perpetual debt. From equation (9), the probability of default converges to $\left(\frac{V_0}{V_B}\right)^{-2m/\sigma^2}$, when $T \to \infty$. It follows that $\lim_{T\to\infty} \mathbb{P}\left[\tau_B \leq T\right] = \left(\frac{(1-\eta)C}{r} \cdot \frac{X}{X+1}\right)^{\frac{2m}{\sigma^2}}$, which of course is well defined because $\frac{(1-\eta)C}{r} \cdot \frac{X}{X+1} > 0$ and such that $C < \bar{C}$. Clearly, a greater

Figure 7: Insurance Costs & Climate Credit Risk

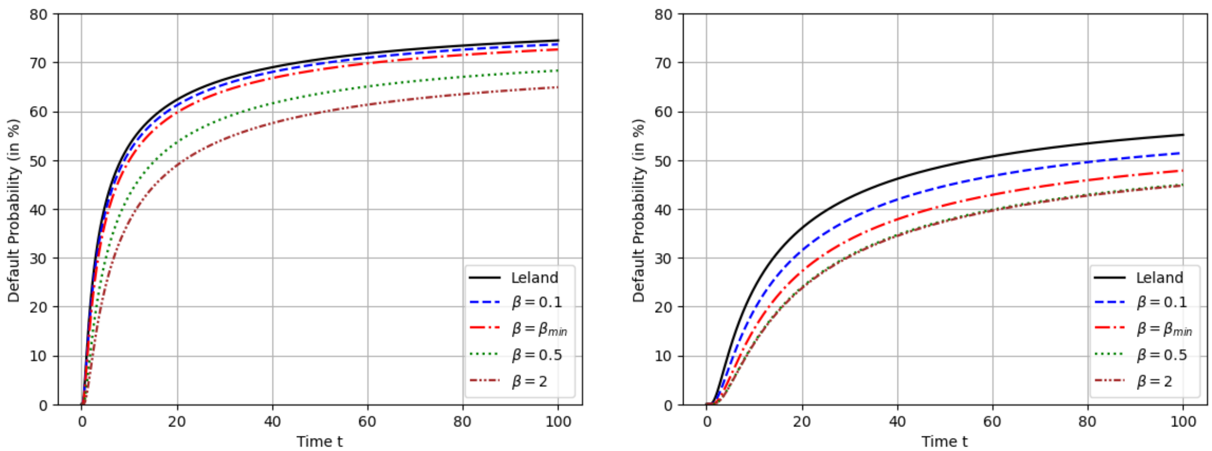


This figure illustrates the effects on the insurance costs of a change in β the firm's exposure to global warming. The insurance costs differentials are computed with $Ins(C(\beta);\beta) - Ins(C(0);0)$. The direct effect of Exposure is measured by $Ins(C(0);\beta) - Ins(C(0);0)$, the indirect effect by $Ins(C(\beta);\beta) - Ins(C(0);\beta)$.

coupon C leads to a larger probabilities and, from Figure (4c) and Figure (5c), one knows that $C_{\max}$ is larger than C^*. Hence, the probabilities of default in graph (8a) are logically greater than the ones in graph (8b). The graphs show that the default probabilities decrease with the sensitivity to global warming. However, the sensitivity parameter affects the probabilities differently and according to the way the capital structure is designed. Actually, this is again the consequence of the disciplinary effect we have already mentioned. Global warming influences the level of coupons, that in turn affects the default threshold and hence the probabilities of defaulting at any horizon. Firms with optimal capital structure adjust downside their coupon, hence the size of their debt, in response of the global warming "faster" than firms exploiting debt capacity and this translates into default probabilities accordingly. To see this, one may use the long-run probability of default as representative of the others. The probability is proportional to $(C)^{\frac{2m}{\sigma^2}}$ so that one may get a cross-sectional view of (long-run) probabilities of default by transforming Figure 4c) and Figure 5c).

Figure 8: Probability of Default: Term Structures

a) For $C = C_{\max}$ b) For $C = C^*$



This figure illustrates the effects of a change of β on term structures of probabilities of default.

Loss Given Default (LGD) is generally defined as the rate of loss incurred by a lender on credit risk in the event of default, the lender then recovering $1 - LGD$ percent of the exposure. An alternative approach is to determine the recovery upon default, which captures the uncertainty about the actual financial recovery that will occur given a Credit Event of a borrower. In our setting with perpetual debt, the expected loss can be proxied by the present value of risk neutral expected loss which is exactly the value of the insurance costs. Hence, the LGD can be proxied by the risk-neutral quantity

$$LGD_0 = ins_0^{Tot} \left(\frac{V_0}{V_B} \right)^{2m/\sigma^2}. \tag{13}$$

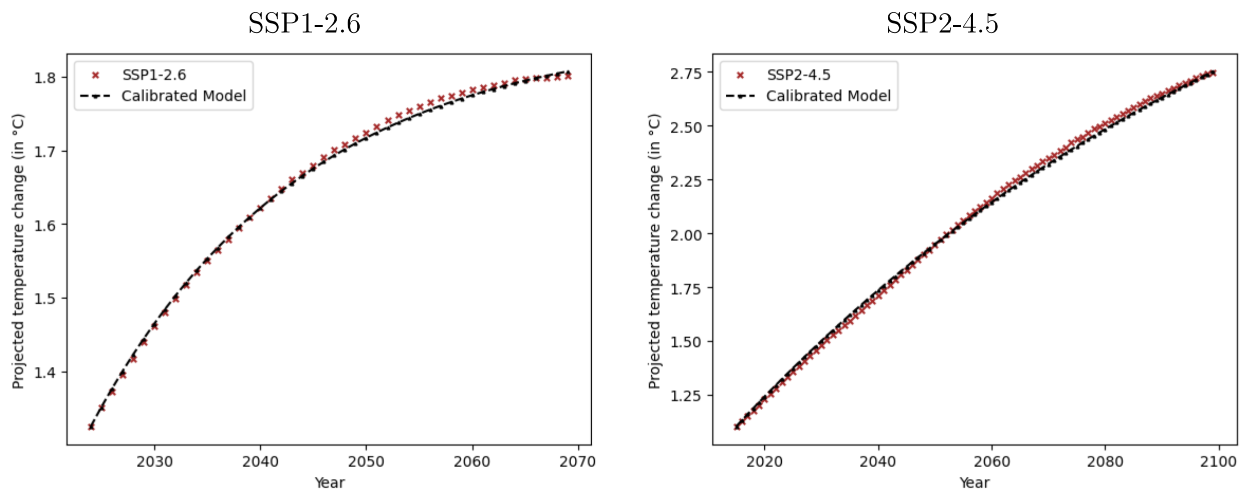
Equation (13) permits to evaluate the LGD for any firm, among which those having an optimal capital structure and those exploiting their debt capacity. Isolating the various impacts of global warming on a LGD (using Leland as a benchmark) is therefore not trivial at all, for the same reasons as the ones mentioned earlier. For instance, the coupon that maximizes the firm or debt value differs for the climate-sensitive firms and the Leland.

6 Application

The objectives of this section are twofold. Firstly, we aim to illustrate the capability of our temperature model to fit paths of temperature change predicted by IPCC. Secondly, we seek to quantify how the company's exposure to global warming (β) influences corporate (financing) key figures by comparing firms with different business risk profiles.

To demonstrate the accuracy of our model in capturing the IPCC scenarios, we fitted the temperature model (equation (3)) to the predictions of IPCC (2023), referred to as SSP1-2.6 and SSP2-4.5 scenarios (see the IPCC (2023) report for extensive details on these two scenarios and appendix 1 for a summary). The calibration results, depicted in Figure 9, appear to be rather satisfactory.[17] We will now restrict our analysis to the SSP1-2.6 scenario.

Figure 9: Temperature Change model calibrated on IPCC scenarios



This figure illustrates the capability of the temperature model to accurately capture two IPCC scenarios, specifically SSP1-2.6 and SSP2-4.5.

Table 1 presents two panels of firms characterized by differing asset volatilities and distinct

[17] All the data we use are available at https://www.ipcc-data.org/ (select Global surface temperature changes in °C relative to 1850–1900). One may alternatively consult the CEDA website https://data.ceda.ac.uk/badc/ar6_wg1/data/spm/spm_08/v20210809/panel_a maintained by Fyfe et al. (2021). We use the files `tas_global_SSP1_2.6.csv` and `tas_global_SSP2_4.5.csv` and censor data in order to consider predictions from 2024 only. Predictions of the `SSP1_2.6` scenario for global warming have been censored from 2069 ownward, because this scenario surprisingly predicts a global cooling beyond that horizon. To fit our temperature model, we minimize the sum of squared errors between IPCC predictions and our model predictions, while constraining the parameters of interest to stay positive (for κ) or larger than the 2024 temperature (for θ). We find

$$\hat{\kappa}_{\text{SSP1}} = 0.05 \qquad \hat{\kappa}_{\text{SSP2}} = 0.01$$
$$\hat{\theta}_{\text{SSP1}} = 1.87 \qquad \hat{\theta}_{\text{SSP2}} = 4.13$$

levels of exposure to global warming (GW). Each panel features companies with identical business risk (i.e., identical σ values), with 'Panel 1' comprising firms that are less risky than those in 'Panel 2.' In both panels, there is one Leland company and three companies exposed to global warming. The labels A, B, and C correspond to different assessments of GW exposure, representing β values of 1, 5, and 10, respectively. The current analysis assumes companies currently are impacted by global warming. This is reflected in setting $\delta T_{\min} = \delta T^{\circ}(0)$, with time 0 corresponding to the year 2024.[18] Other parameters are those employed in previous sections.

Table 1: Climate Risk Exposure vs Business Risk

	Panel 1				**Panel 2**			
Firm	Leland	A	B	C	Leland	A	B	C
σ (in %)	25	25	25	25	40	40	40	40
β	-	1	5	20	-	1	5	20
C^*	5.57	5.25	4.43	4.27	6.44	5.88	4.25	3.88
Eq^*	35.23	38.3	46.62	48.32	41.65	45.45	57.66	60.7
D^*	88.78	83.6	72.05	70.09	75.67	69.4	53.17	49.76
v^*	124.01	121.9	118.67	118.4	117.33	114.85	110.84	110.45
L^* (in %)	71.59	68.58	60.71	59.19	64.5	60.43	47.97	45.05
CS^* (in bp)	127.37	128.02	114.83	109.25	351.02	347.24	299.26	279.8
BC^*	4.28	5.68	6.44	6.13	5.55	7.16	7.61	6.96
$C_{\max}$	8.30	8.26	7.85	6.98	12.21	12.29	12.23	9.85
$Eq_{\max}$	13.65	13.9	16.6	23.0	13.03	12.75	12.96	22.57
$D_{\max}$	102.2	98.71	89.72	84.91	93.96	90.46	80.61	71.75
$v_{\max}$	115.86	112.62	106.31	107.91	106.99	103.21	93.57	94.32
$L_{\max}$ (in %)	88.22	87.65	84.39	78.69	87.82	87.64	86.15	76.08
$CS_{\max}$ (in bp)	312.12	336.76	374.98	322.07	799.48	858.65	1017.16	872.79
$BC_{\max}$	12.07	15.4	22.53	21.72	15.70	19.36	29.09	30.34

Table 1 details for the two panels the capital structure firms select by maximizing firm value or exhausting debt capacity. Of course, the Leland company serves as a benchmark. Within each panel, numbers reproduce quantitatively the qualitative results we found earlier, including the

[18]Note that this contrasts with previous sections where companies were exposed to the effects of global warming only when the actual temperature difference exceeded $\delta T_{\min}$(strictly above $\delta T^{\circ}(0) = 1$). Also, this adjustment simplifies several expressions as $\tau_{\min} = 0$.

disciplinary effect of Global Warming. In both panels, exposure to GW incites firms to limit, actually decrease leverage (computed with market values). Interestingly, this common result hides far different situations. In Panel 1, the decline in leverage is accompanied by the recourse to more equity financing (as shown by Ginglinger and Moreau (2023)). Under optimal capital structure, the gearing (equal to 88.78/35.23 = 2.5 under Leland) declines to 2,18, 1,55 and 1,45 respectively when β increases from 1 to 20. The size of debt (proxied by the value of the debt $D_{\max}$ and D^* or the corresponding level of coupon) systematically decreases but the credit spread (the sole relevant measure of compensation) does not. Actually, the credit spreads do not even behave monotonically with respect to the exposure to global warming. The rise of exposure first increases the detholder's compensation until the lowering of the leverage and of the level of coupon (which drives the default threshold) makes the firm (and corporate debt) less credit risky. By design, the exposure β influences negatively the recovery upon liquidation and therefore pushes the bankruptcy costs up, but the lowering of the coupon level influences the probability of default downward and therefore pushes bankruptcy costs down. Figures for BC in Panel 1 of Table 1 highlight how the two forces balance. In Panel 2 of Table 1, firms have greater business risk. Here again, firms limit their leverage as the exposure to global warming increases. However, this decline in leverage is not systematically accompanied by the recourse to more equity financing (see firms with debt capacity). When companies maintain an optimal capital structure, the known results are exacerbated by volatility. The gearing of a Leland company (1.82) declines to 1.53 and then becomes lower than one, namely 0.92 and 0.82. This indicates that equity financing tends to prevail in companies that maintain an optimal capital structure, particularly those with significant business risk. Companies that maximize their debt capacity exhibit distinct characteristics compared to their less risky counterparts. Contrary to expectations, the coupon paid to creditors does not uniformly decrease as the exposure β increases; instead, it initially rises slightly before eventually declining. This behavior directly influences the default threshold. Remarkably, these subtle variations substantially affect the present value of bankruptcy costs and the magnitude of the credit spread.

.

7 Conclusion

This research studies the impact of climate change on the pricing of corporate liabilities and on the capital structure of firms. We develop a structural model, where the stranding of assets at bankruptcy is the transmission channel through which global warming impacts. Our model for global warming can fit the paths of temperature change predicted by IPCC. We show that the firm's exposure to global warming has both direct and indirect implications for traditional firm financial decisions and related metrics such as the compensation investors require in addition to the traditional default premium. We are the first to highlight the sort of disciplinary effect the exposure to climate risk can have on financing and credit risk. We can assess the extent to which firms reduce their debt as a response to the climate risk exposure. We can disentangle the direct and indirect effects of global warming in all our credit metrics. It appears that the scenarios put forward by the IPCC may affect corporate decisions and associated capital structures quite differently. The modeling approach developed in this paper may have a number of limitations for some readers. Firstly, the capital structure analyzed involves a single perpetual debt, which allows for numerous potential extensions. For instance, future research could explore the scenario of a firm with finite maturity debt, considering whether the time horizon of stranded assets precedes or follows the average maturity of existing debt. Another interesting topic would be the existence of senior and subordinated debtors... It is clear that lenders here should not have the same view of climate credit risk.

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Appendix

A.1. IPCC scenarios

The 6th assessment report of the IPCC has chosen to assess the climate response to five socio-economic scenarios that cover the range of possible future developments of anthropogenic factors of climate change found in the literature. The five scenarios selected by the IPCC are as follows: Two scenarios with high and very high greenhouse gas emissions: SSP3-7.0 and SSP5-8.5; one scenario with intermediate greenhouse gas emissions: SSP2-4.5 and two scenarios with very low and low greenhouse gas emissions: SSP1-1.9 and SSP1-2.6.

- **Scenario 1 - Most optimistic:** $\theta^{IPCC} = 1.5°C$ **by 2050**
  SSP1-1.9:
  This first scenario is the only one that meets the Paris Agreement's goal of keeping global warming to around $1.5°C$ above the pre-industrial temperature level, with warming hitting $1.5°C$ but then dipping back down and stabilizing around 1.4C by the end of the century. This scenario is also call the **"Net Zero scenario**". if this scenario is achieved, the IPCC projects that it would likely limit global warming to $1.5°C$ above pre-industrial levels and significantly reduce the risks of extreme weather events, sea level rise, and other impacts associated with climate change.

- **Scenario 2 – Next Best:** $\theta^{IPCC} = 1.8°C$ **by 2100**
  SSP1-2.6:
  In the next-best scenario, global CO2 emissions are cut severely, but not as fast, reaching net-zero after 2050. It imagines the same socioeconomic shifts towards sustainability as Net Zero Scenario. But temperatures stabilize around $1.8°C$ higher by the end of the century. This scenario is also know as the **Sustainable Development Scenario** (SDS) developed by the International Energy Agency (IAE) that outlines a pathway towards meeting the goals of the Paris Agreement, while also achieving universal energy access and other sustainable development objective. The SDS includes a range of measures to reduce greenhouse gas emissions, increase energy efficiency, and promote the use of renewable energy sources. These measures are designed to limit global temperature increase to well below 2°C, and preferably to 1.5°C above pre-industrial levels, as specified in the Paris Agreement.

- **Scenario 3 – Middle of the road:**$\theta^{IPCC} = 2.7°C$ **by 2100**
  SSP2-4.5:
  This is a "middle of the road" Scenario also know as **Stated Policy Scenario** (STEPS). CO2 emissions hover around current levels before starting to fall mid-century, but do not reach net-zero by 2100. In this scenario, temperatures rise $2.7°C$ by the end of the century.

- **Scenario 4 – Dangerous:** $\theta^{IPCC} = 3.7°C$ **by 2100**
  SSP3-7.0:
  This is a scenario that assumes high levels of inequality and a focus on economic growth, resulting in a continuation of current trends in emissions and energy use. Under this scenario, global temperatures are likely to increase by around $3.7°C$ above pre-industrial levels by the end of the century.

- **Scenario 5 – Avoid at all costs:** $\theta^{IPCC} = 4.4°C$ **by 2100**
  SSP5-8.5:
  This is a scenario that assumes continued economic and population growth, with no significant efforts to reduce greenhouse gas emissions. Under this scenario, global temperatures are likely to increase by around $4.4°C$ above pre-industrial levels by the end of the century.

A.2: Demonstrations and additional analytical results

Demonstration of Proposition (3.1)

This is a proof by *reductio ad absurdum*. When the bankruptcy cost α is constant, the Leland's analysis applies and one knows it is optimal for the equityholder to select a constant threshold V_B. Since, in our setting, the bankruptcy cost is deterministic and time-dependent, let's assume for a while that $V_B = V(\tau_B)$ is time-dependent (and hence that the equity value effectively is impacted by the global warming). The time-dependent default threshold may thus be written $V_B(t)$ as a consequence of the time-dependency of $\alpha(t)$. Any other parameters are assumed to be given. Then, the financial value of the firm and the debt value are respectively given by

$$
\begin{aligned}
v(V) &= V + TB - BC \\
&= V + E^{\mathbb{Q}}\left[\eta \frac{c}{r}\left(1 - e^{-r\tau_B}\right)\right] - E^{\mathbb{Q}}\left[\alpha\left(T^{\circ}(\tau_B)\right) V_B(\tau_B) e^{-r\tau_B}\right] \\
&= V + \eta \frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right) - E^{\mathbb{Q}}\left[\alpha(\tau_B) V(\tau_B) e^{-r\tau_B}\right] \\
D(V) &= E^{\mathbb{Q}}\left[\frac{c}{r}\left(1 - e^{-r\tau_B}\right) + \left(1 - \alpha\left(T^{\circ}(\tau_B)\right)\right) V_B(\tau_B) e^{-r\tau_B}\right] \\
&= \frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right) + E^{\mathbb{Q}}\left[V_B(\tau_B) e^{-r\tau_B}\right] - E^{\mathbb{Q}}\left[\alpha(\tau_B) V(\tau_B) e^{-r\tau_B}\right]
\end{aligned}
$$

where $V_B(\tau_B)$ stands for the value of the time-dependent default threshold at default time. And the equity value then satisfies

$$
\begin{aligned}
Eq(V) &= v(V) - D(V) \\
&= \left\{V - (1-\eta)\frac{c}{r}\right\} + (1-\eta)\frac{c}{r} E^{\mathbb{Q}}\left[e^{-r\tau_B}\right] \\
&- E^{\mathbb{Q}}\left[V_B(\tau_B) e^{-r\tau_B}\right]
\end{aligned}
$$

and this last expression shows that the equity value does not depend on **the time-dependent bankruptcy cost**. Consequently, there is no argument to derogate from the Leland's policy because there is no reason to make the default decision time-dependent. The default threshold is a constant boundary. The closer the value of the firm to $V(\tau_B)$, the closer the value of the equity to zero. Whatever the date at which the asset value V reaches $V(\tau_B)$, the equityholder receives nothing 0$. So $V(\tau_B)$ is time-homogeneous. Put differently, the price of any corporate security $F(V,t)$ paying a continuous income flow d satisfies the following PDE:

$$
\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F(V,t)}{\partial V^2} + \mu V \frac{\partial F(V,t)}{\partial V} + \frac{\partial F(V,t)}{\partial t} = rF(V,t) - d.
$$

And, when the security admits no time dependence, the term $\frac{\partial F(V,t)}{\partial t} = 0$ and the above equation becomes an O.D.E. This is exactly the case for the equity value written by $Eq(V)$. One finds:

$$
\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 Eq(V)}{\partial V^2} + \mu V \frac{\partial Eq(V)}{\partial V} = rEq(V)
$$

with boundary conditions :

$$
\begin{aligned}
&\text{at } V = V_B, \qquad Eq(V_B) = 0 \\
&\text{as } V \to \infty, \qquad Eq(V_B) = V - (1-\eta)\frac{c}{r}
\end{aligned}
$$

This general solution of the PDE is $Eq(V) = A_0 + A_1 V + A_2 V^{-x}$whose constants A_0,A_1 and A_2can be determined by using the boundary conditions. This in turn provides us an alternative derivation of the pricing formula provided in the core text. ■

Demonstration of Proposition (3.2)

The formula for the coupon flow can be demonstrated by many different ways.
First,

$$\begin{aligned} E^{\mathbb{Q}}\left[\int_0^{\tau_B} ce^{-rs}ds\right] &= E^{\mathbb{Q}}\left[\int_0^{\infty} ce^{-rs}ds - \int_{\tau_B}^{\infty} ce^{-rs}ds\right] \\ &= \frac{c}{r} - E^{\mathbb{Q}}\left[e^{-r\tau_B}\int_{\tau_B}^{\infty} ce^{-r(s-\tau_B)}ds\right] \end{aligned}$$

and a change of variable $u = s - \tau_B$ in the second integral gives the result.
Second,

$$\begin{aligned} E^{\mathbb{Q}}\left[\int_0^{\tau_B} ce^{-rs}ds\right] &= E^{\mathbb{Q}}\left[\int_0^{\infty} ce^{-rs}1_{\{s<\tau_B\}}ds\right] \\ &= \int_0^{\infty} ce^{-rs}\mathbb{Q}\left[s<\tau_B\right]ds \\ &= \frac{c}{r} - \int_0^{\infty} ce^{-rs}\mathbb{Q}\left[\tau_B<s\right]ds \\ \textit{int. by part} &= \frac{c}{r} - \left\{\underbrace{\left[c\frac{e^{-rs}}{-r}\mathbb{Q}\left[\tau_B<s\right]\right]_0^{+\infty}}_{=0} - \underbrace{\int_0^{\infty} c\frac{e^{-rs}}{-r}f_{\tau_B}(s)\,ds}_{-\frac{c}{r}E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]}\right\} \end{aligned}$$

Consider $(V_t)_t$ a geometric Brownian motion starting at V_0 (at time 0) and a default threshold V_B (lower than V_0). The drift is μ and the volatility σ so that

$$d\ln V_t = mdt + \sigma dW_t$$

with $m = \mu - \frac{1}{2}\sigma^2$. Denote by $\tau_B = \inf\{t \geq 0 \,:\, V_t \leq V_B\}$ the first time the GBM hits the threshold and set $a = \ln(V_B/V_0) < 0$. Then, the pdf of the first hitting time is

$$f_{\tau_B}(t; m, \sigma) = \frac{|a|}{2\sigma\sqrt{2\pi t^3}}\exp\left(-\frac{(a-mt)^2}{2\sigma^2 t}\right)$$

and its cdf is

$$F_{\tau_B}(T; m, \sigma) = N\left[-d_2\left(V_0, V_B; m, T\right)\right] + \left(\frac{V_0}{V_B}\right)^{-2m/\sigma^2} N\left[d_2\left(V_B, V_0; m, T\right)\right]$$

With N the Gaussian cdf and $d_2(V_0, V_B; m, t) = \frac{\ln(V_0/V_B)+mt}{\sigma\sqrt{t}}$. The Laplace transform of the first-hitting time is

$$\mathcal{L}_f^r \equiv E\left[e^{-r\tau_B}\right] = \int_0^\infty e^{-rt} f_{\tau_B}(t; m, \sigma)\, dt = e^{\frac{m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}a} = \left(\frac{V_0}{V_B}\right)^{-\frac{m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}}$$

In his setting, Leland (1994) considers a risk-neutral setting without any pay-out rate, so $m = r - \frac{1}{2}\sigma^2$ and $\sqrt{m^2+2r\sigma^2} = \sqrt{\left(r+\frac{1}{2}\sigma^2\right)^2} = r + \frac{1}{2}\sigma^2$ and the above Laplace transform simplifies to $\left(\frac{V_0}{V_B}\right)^{-\frac{2r}{\sigma^2}}$, the Leland's core building block. The *incomplete* Laplace transform of the first-hitting time is

$$\begin{aligned}\mathcal{L}_f^r(T) \equiv E\left[e^{-r\tau_B}1_{\{\tau_B\le T\}}\right] = G_{\tau_B}^r(T) &= \int_0^T e^{-rt} f_{\tau_B}(t; m, \sigma)\, dt \\ &\overset{\star}{=} \left(\frac{V_0}{V_B}\right)^{\frac{-m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}} F_{\tau_B}\left(t; \sqrt{m^2+2r\sigma^2}, \sigma\right)\end{aligned}$$

where "$\overset{\star}{=}$" is obtained by completing the square. This formula yields to

$$\begin{aligned}G_{\tau_B}^r(T) &= \left(\frac{V_0}{V_B}\right)^{\frac{-m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}} \left\{ \begin{array}{c} N\left[-d_2\left(V_0, V_B; \sqrt{m^2+2r\sigma^2}, T\right)\right] \\ + \left(\frac{V_0}{V_B}\right)^{-2\sqrt{m^2+2r\sigma^2}/\sigma^2} N\left[d_2\left(V_B, V_0; \sqrt{m^2+2r\sigma^2}, T\right)\right] \end{array} \right\} \\ &= \left(\frac{V_0}{V_B}\right)^{\frac{-m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}} N\left[-d_2\left(V_0, V_B; \sqrt{m^2+2r\sigma^2}, T\right)\right] \\ &\quad + \left(\frac{V_0}{V_B}\right)^{\frac{-m-\sqrt{m^2+2r\sigma^2}}{\sigma^2}} N\left[d_2\left(V_B, V_0; \sqrt{m^2+2r\sigma^2}, T\right)\right]\end{aligned}$$

Now, with $z_r = \sqrt{m^2+2r\sigma^2}$, $\tau_{\min} = \inf\{t > 0 \,:\, \delta T^\circ(t) = T_{\min}\}$ and $\tau_{\max} = \inf\{t > 0 \,:\, \delta T^\circ(t) = T_{\max}\}$, one finds:

- For short horizons

$$\begin{aligned}E\left[e^{-r\tau_B}1_{\{\tau_B\le\tau_{\min}\}}\right] = \mathcal{L}_f^r(\tau_{\min}) &= G_{\tau_B}^r(\tau_{\min}) \\ &= \left(\frac{V_0}{V_B}\right)^{\frac{-m+z_r}{\sigma^2}} N\left[-d_2(V_0, V_B; z_r, \tau_{\min})\right] \\ &\quad + \left(\frac{V_0}{V_B}\right)^{\frac{-m-z_r}{\sigma^2}} N\left[d_2(V_B, V_0; z_r, \tau_{\min})\right]\end{aligned}$$

- For long horizons

$$
\begin{aligned}
E\left[e^{-r\tau_B}1_{\{\tau_{max}<\tau_B\}}\right] &= E\left[e^{-r\tau_B}\right] - E\left[e^{-r\tau_B}1_{\{\tau_B\leq\tau_{max}\}}\right] \\
&= \mathcal{L}_f^r - \mathcal{L}_f^r\left(\tau_{\max}\right) \\
&= \left(\frac{V_0}{V_B}\right)^{-\frac{m+z_r}{\sigma^2}} - G_{\tau_B}^r\left(\tau_{\max}\right) \\
&= \left(\frac{V_0}{V_B}\right)^{\frac{-m-z_r}{\sigma^2}} N\left[-d_2\left(V_0, V_B; z_r, \tau_{\max}\right)\right] \\
&\quad - \left(\frac{V_0}{V_B}\right)^{\frac{-m+z_r}{\sigma^2}} N\left[d_2\left(V_B, V_0; z_r, \tau_{\max}\right)\right]
\end{aligned}
$$

- For intermediate horizons (such that $\{\tau_{\min} < \tau_B \leq \tau_{\max}\}$, the bankruptcy costs amount to

$$
\begin{aligned}
\alpha\left(\tau_B\right) &= \alpha_0 + \beta\left(\left(\theta^\circ - \delta T_{\min}\right) - \left(\theta^\circ - \delta T^\circ\left(0\right)\right)e^{-\kappa\tau_B}\right) \\
&= \left(\alpha_0 + \beta\left(\theta^\circ - \delta T_{\min}\right)\right) - \beta\left(\theta^\circ - \delta T^\circ\left(0\right)\right)e^{-\kappa\tau_B} \\
&= \gamma_1 - \gamma_2 e^{-\kappa\tau_B}
\end{aligned}
$$

with appropriate γ_1 and γ_2, one therefore has

$$
\begin{aligned}
E\left[\alpha\left(\tau_B\right)e^{-r\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right] &= \gamma_1 E\left[e^{-r\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right] \\
&\quad - \gamma_2 E\left[e^{-(r+\kappa)\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right].
\end{aligned}
$$

where the last two terms can be computed by considering λ (equal to either r or $r+\kappa$ and

$$
\begin{aligned}
E\left[e^{-\lambda\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right] &= E\left[e^{-\lambda\tau_B}1_{\{\tau_B\leq\tau_{\max}\}}\right] - E\left[e^{-\lambda\tau_B}1_{\{\tau_B\leq\tau_{\min}\}}\right] \\
&= \mathcal{L}_f^\lambda\left(\tau_{\max}\right) - \mathcal{L}_f^\lambda\left(\tau_{\min}\right) \\
&= G_{\tau_B}^\lambda\left(\tau_{\max}\right) - G_{\tau_B}^\lambda\left(\tau_{\min}\right) \\
&= \left(\frac{V_0}{V_B}\right)^{\frac{-m+z_\lambda}{\sigma^2}} \left\{N\left[-d_2\left(V_0, V_B; z_\lambda, \tau_{\max}\right)\right] - N\left[-d_2\left(V_0, V_B; z_\lambda, \tau_{\min}\right)\right]\right\} \\
&\quad + \left(\frac{V_0}{V_B}\right)^{\frac{-m-z_\lambda}{\sigma^2}} \left\{N\left[d_2\left(V_B, V_0; z_\lambda, \tau_{\max}\right)\right] - N\left[d_2\left(V_B, V_0; z_\lambda, \tau_{\min}\right)\right]\right\}
\end{aligned}
$$

We can now price all corporate securities and firm value.

The debt value is

$$
\begin{aligned}
D\left(V,t\right) &= \frac{c}{r} + \left(V_B - \frac{c}{r}\right)E^{\mathbb{Q}}\left[e^{-r\tau_B}\right] - V_B E^{\mathbb{Q}}\left[\alpha\left(\tau_B\right)e^{-r\tau_B}\right] \\
&= \frac{c}{r} + \left(V_B - \frac{c}{r}\right)\left(\frac{V_0}{V_B}\right)^{-X} - \alpha_0 V_B E^{\mathbb{Q}}\left[e^{-r\tau_B}1_{\{\tau_B\leq\tau_{\min}\}}\right] \\
&\quad - V_B\left(\gamma_1 E\left[e^{-r\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right] - \gamma_2 E\left[e^{-(r+\kappa)\tau_B}1_{\{\tau_{\min}<\tau_B\leq\tau_{\max}\}}\right]\right) \\
&\quad - 100\% V_B E^{\mathbb{Q}}\left[e^{-r\tau_B}1_{\{\tau_{\max}<\tau_B\}}\right].
\end{aligned}
$$

Using the above expression and the building blocks, it is a simple exercise to derive the pricing formula provided in Proposition (3.2). It is worth noting that this expression is also the solution of the following PDE:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 D(V,t)}{\partial V^2} + \mu V \frac{\partial D(V,t)}{\partial V} + \frac{\partial D(V,t)}{\partial t} = rD(V,t) - c.$$

with boundary conditions:

$$\begin{aligned} &\text{at } V = V_B, \qquad D(V_B, \tau_B) = \alpha(\tau_B) V_B \\ &\text{as } V \to \infty, \qquad D(V_B, \tau_B) = \frac{c}{r} \end{aligned}$$

and that this is challenging to solve this way because the first boundary condition is random.

The firm value is given by

$$\begin{aligned} \upsilon(V) &= V + \eta \frac{c}{r}\left(1 - E^{\mathbb{Q}}\left[e^{-r\tau_B}\right]\right) - V_B E^{\mathbb{Q}}\left[\alpha(\tau_B) e^{-r\tau_B}\right] \\ &= V + \eta \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) - \alpha_0 V_B E^{\mathbb{Q}}\left[e^{-r\tau_B} 1_{\{\tau_B \leq \tau_{\min}\}}\right] \\ &- V_B\left(\gamma_1 E\left[e^{-r\tau_B} 1_{\{\tau_{\min} < \tau_B \leq \tau_{\max}\}}\right] - \gamma_2 E\left[e^{-(r+\kappa)\tau_B} 1_{\{\tau_{\min} < \tau_B \leq \tau_{\max}\}}\right]\right) \\ &- 100\% V_B E^{\mathbb{Q}}\left[e^{-r\tau_B} 1_{\{\tau_{\max} < \tau_B\}}\right] \end{aligned}$$

Here again, one may derive the pricing formula provided in Proposition (3.2) with the above expression and the building blocks. Finally, the value of equity is found by computing the difference, namely $Eq(V) = \upsilon(V) - D(V)$ ■

Supplementary materials on the valuation of corporate securities

Proposition 3.2 provides some valuation formulae that use Leland (1994) as a benchmark. For readers' convenience, it is worth recalling Leland's formulae that are

$$D_L(V; c, \alpha_0) = \frac{c}{r} + \left((1-\alpha_0)V_B - \frac{c}{r}\right)\left(\frac{V_0}{V_B}\right)^{-X} \tag{14}$$

$$\upsilon_L(V; c, \alpha_0) = V + \eta \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) - \alpha_0 V_B \left(\frac{V_0}{V_B}\right)^{-X} \tag{15}$$

$$Eq_L(V; c) = V - (1-\eta)\frac{c}{r} + \left((1-\eta)\frac{c}{r} - V_B\right)\left(\frac{V_0}{V_B}\right)^{-X}. \tag{16}$$

These equations may be rewritten

$$D_L(V; c, \alpha_0) = \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + (1-\alpha_0) V_B \left(\frac{V_0}{V_B}\right)^{-X} \tag{17}$$

$$\upsilon_L(V; c, \alpha_0) = V + \eta \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + (1-\alpha_0) V_B \left(\frac{V_0}{V_B}\right)^{-X} - V_B \left(\frac{V_0}{V_B}\right)^{-X}. \tag{18}$$

Actually, there are many alternative expressions for the values of corporate securities. For instance, the expressions of Proposition 3.2 may be be rewritten

$$
\begin{aligned}
D(V) = \frac{c}{r} + \left(V_B - \frac{c}{r}\right)\left(\frac{V_0}{V_B}\right)^{-X} - \alpha_0 V_B G^{r}_{\tau_B}(\tau_{\min}) \\
- \left(\alpha_0 + \beta\left(\theta^{\circ} - \delta T_{\min}\right)\right) V_B \left(G^{r}_{\tau_B}(\tau_{\max}) - G^{r}_{\tau_B}(\tau_{\min})\right) \\
+ \beta\left(\theta^{\circ} - \delta T^{\circ}(0)\right) V_B \left(G^{r+\kappa}_{\tau_B}(\tau_{\max}) - G^{r+\kappa}_{\tau_B}(\tau_{\min})\right) \\
- V_B\left[\left(\frac{V_0}{V_B}\right)^{-X} - G^{r}_{\tau_B}(\tau_{\max})\right]
\end{aligned}
$$

and

$$
\begin{aligned}
v(V) = V + \eta\frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) - \alpha_0 V_B G^{r}_{\tau_B}(\tau_{\min}) \\
- \left(\alpha_0 + \beta\left(\theta^{\circ} - \delta T_{\min}\right)\right) V_B \left(G^{r}_{\tau_B}(\tau_{\max}) - G^{r}_{\tau_B}(\tau_{\min})\right) \\
+ \beta\left(\theta^{\circ} - \delta T^{\circ}(0)\right) V_B \left(G^{r+\kappa}_{\tau_B}(\tau_{\max}) - G^{r+\kappa}_{\tau_B}(\tau_{\min})\right) \\
- V_B\left[\left(\frac{V_0}{V_B}\right)^{-X} - G^{r}_{\tau_B}(\tau_{\max})\right]
\end{aligned}
$$

Note the similarity of the first line in these pricing formulae with Leland's one.

When $\delta T_{\min} = \delta T^{\circ}(0)$, the formulae simplify to

$$
\begin{aligned}
D(V) = \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + (1 - \alpha_0) V_B G^{r}_{\tau_B}(\tau_{\max}) \\
- \beta\left(\theta^{\circ} - \delta T^{\circ}(0)\right) V_B \left(G^{r}_{\tau_B}(\tau_{\max}) - G^{r+\kappa}_{\tau_B}(\tau_{\max})\right)
\end{aligned}
$$

and

$$
\begin{aligned}
v(V) = V + \eta\frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + (1 - \alpha_0) V_B G^{r}_{\tau_B}(\tau_{\max}) - V_B\left(\frac{V_0}{V_B}\right)^{-X} \\
- \beta\left(\theta^{\circ} - \delta T^{\circ}(0)\right) V_B \left(G^{r}_{\tau_B}(\tau_{\max}) - G^{r+\kappa}_{\tau_B}(\tau_{\max})\right)
\end{aligned}
$$

so that, of course, the value of equity aligns to the one derived by Leland (1994). Note that, when the global warming sensitivity, β tends to ∞ then the debt and firm values satisfy

$$
\lim_{\beta\to\infty} D(V,\beta) = \frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + (1 - \alpha_0) V_B G^{r}_{\tau_B}(\tau_{\min})
$$

and

$$
\lim_{\beta\to\infty} v(V,\beta) = V + \eta\frac{c}{r}\left(1 - \left(\frac{V_0}{V_B}\right)^{-X}\right) + V_B\left((1 - \alpha_0) G^{r}_{\tau_B}(\tau_{\min}) - \left(\frac{V_0}{V_B}\right)^{-X}\right)
$$

Once more, it is understood that if $\delta T^{\circ}(0) = \delta T_{\min}$, then $\tau_{\min} = 0$ and $G^{r}_{\tau_B}(\tau_{\min}) = 0$. Hence, these two limits coincide with a Leland-style situation without any recovery and differ from the standard Leland situation. These limits are a direct consequence of our choice of using a step function specification for modeling the global warming-related recovery. This mathematically can explain the jump one could observe in simulations.

A.3. Additional figures for the Net-Zero Scenario

Figure 3 bis: Climate exposure & existing capital structure

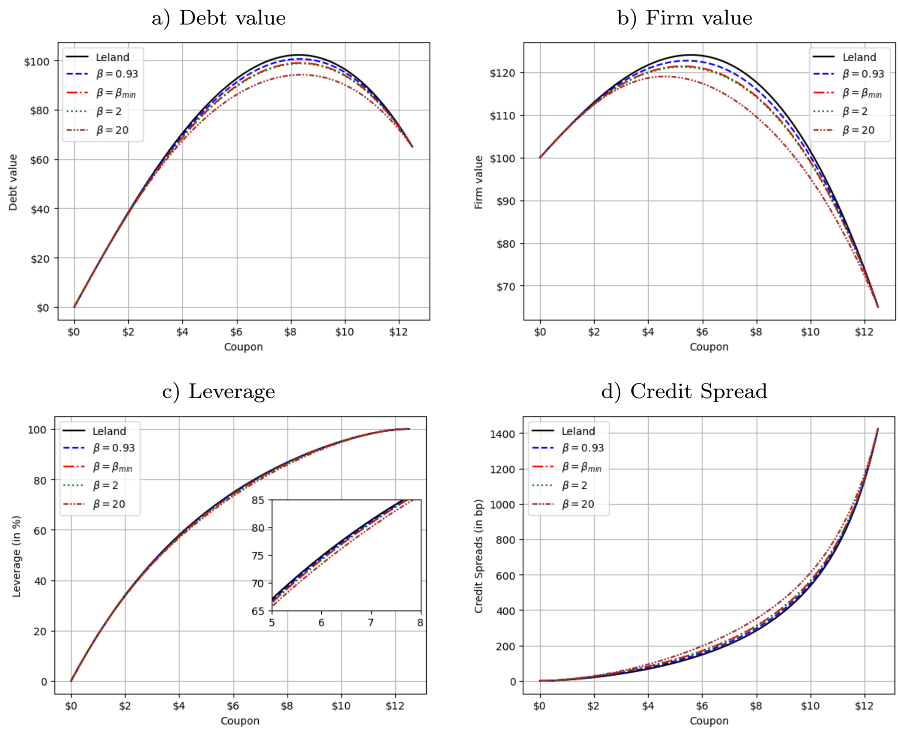


Figure 6 bis: Credit Spreads & Climate Credit Risk

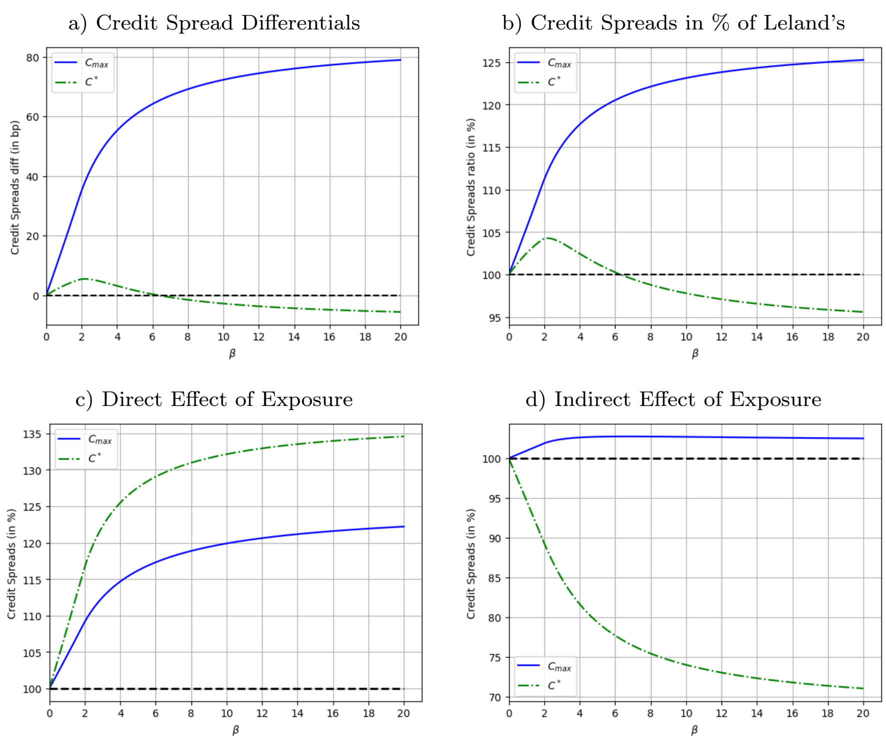
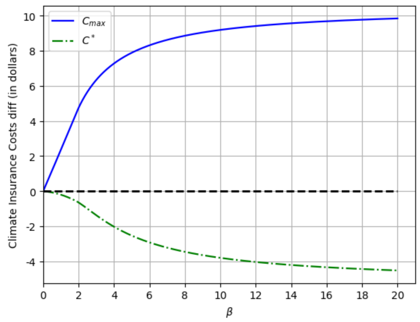
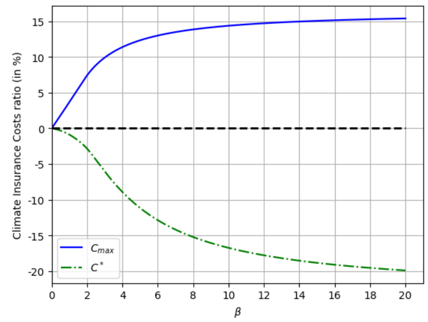


Figure 7 bis: Insurance Costs & Climate Credit Risk

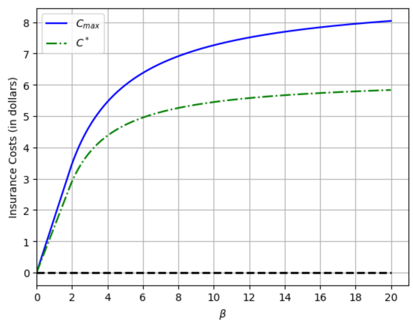
a) Insurance Costs Differentials



b) Insurance Costs in % of Leland's



c) Direct Effect of Exposure



d) Indirect Effect of Exposure

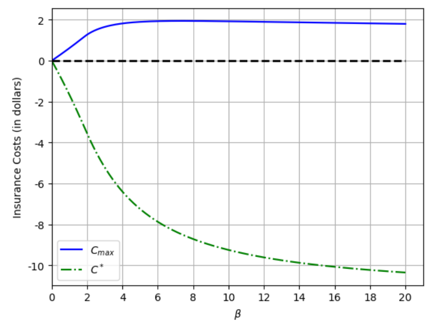
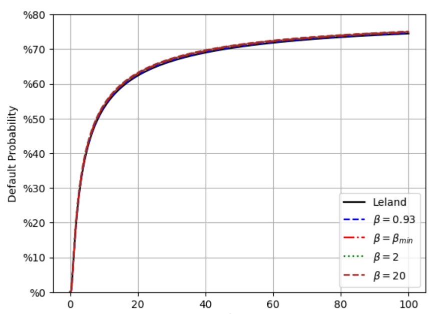


Figure 8 bis: Probability of Default: Term Structures

a) For $C = C_{\max}$



b) For $C = C^*$