Efficiency Implications of Knowledge Generation: Private versus Public Firms

Preliminary and incomplete. Do not circulate without permission.

Itay Goldstein Stefan Hirth Matthias Lassak*

August 27, 2024

Abstract

We provide an equilibrium analysis investigating efficiency differences between private and public firms' information generation strategies, emphasizing public firms' unique ability to learn additional information from financial markets through the feedback effect. The public firm generates a higher expected value than the private firm, as, in addition to the information generated internally, it has access to information reflected in market prices. However, the public firm features two mutually reinforcing sources of inefficiency. First, the public firm relies too much on market prices, as it does not incorporate information acquisition costs borne by market participants. Second, investors' incentives to acquire information are too strong, as they maximize private trading profits as opposed to real efficiency. As the private firm does not face these distorted information acquisition incentives in our model, it is associated with higher real efficiency.

Keywords: Private Firms, Public Firms, Efficiency, R&D, Market Feedback

*Goldstein: Wharton School, University of Pennsylvania; 3620 Locust Walk, Philadelphia, PA 19104, USA. Email: itayg@wharton.upenn.edu. Hirth and Lassak: Aarhus University and Danish Finance Institute (DFI); Fuglesangs Allé 4, 8210 Aarhus, Denmark. Emails: shirth@econ.au.dk, lassak@econ.au.dk. Hirth and Lassak gratefully acknowledge financial support from the DFI and the Independent Research Fund Denmark, Grant 0133-00087B.

1 Introduction

Since the mid-1990s, the U.S. has experienced a substantial decline in the number and share of publicly listed firms (Doidge et al. [2017]). Commentators have expressed concerns that this reduction inhibits the innovation ability and growth prospects of the U.S. economy (Weild and Kim [2009]). A large theoretical (e.g,. Bhattacharya and Ritter [1983]) and empirical (e.g., Bernstein [2015]) literature investigates differences between the innovation strategies and outcomes of private and public firms. However, one important difference has received little attention. Namely, only public firms can draw information from their stock prices. Thus, private and public firms operate in different information environments, which creates heterogeneous incentives for innovation and knowledge generation.

By aggregating information from different market participants, financial asset prices can provide useful information to the public firm. Indeed, the so-called feedback literature documents that managers use price signals when making corporate decisions (e.g., Bond et al. [2012], Goldstein [2023]). In particular, Blanco and Wehrheim [2017] show that the information generated by financial markets affects a public firm's innovation strategy.

In this paper, we provide an equilibrium analysis investigating efficiency differences between private and public firms' information generation strategies, emphasizing public firms' unique ability to learn additional information from financial markets through the feedback effect. As a public firm may simply ignore the information contained in prices, one might expect that public firms' innovation strategies are at least as efficient as those of their private counterparts. The main insight of our paper is that this prediction is generally inaccurate.

A public firm's value-maximizing manager considers the interdependence between her investment in internal information acquisition and the information generated by the market. In particular, we show that in an effort to stimulate information acquisition by market participants, a public firm invests less (more) in internal information acquisition relative to an otherwise identical private firm, if the firm's and traders' information are substitutes (complements). Doing so increases the informativeness of the stock price, helping the public firm manager to make better investment decisions. Consequently, the public firm generates a higher expected value than the private firm, irrespective of the information structure.[1]

However, when comparing economy-wide efficiency implications of public and private firms' choices, it is insufficient to measure efficiency by the public firm's better investment-making brought about by the feedback effect. Indeed, more informative prices are a consequence of more information acquisition by market participants, generated with higher information acquisition costs. Thus, we define real efficiency as the expected firm value net of all information acquisition costs, including the firm's internal cost of knowledge generation (e.g., R&D), and the financial market participants' cost of information acquisition.

A private firm's information acquisition choice is efficient as it maximizes its firm value, net of its privately borne information costs. In contrast, we identify two sources of inefficiency in the public firm's information acquisition. First, the public firm adjusts its information acquisition activities in anticipation of its ability to learn from the market. However, the public firm does not consider the speculators' information acquisition costs, resulting in too much reliance on market prices. Second, the financial market participants are not maximizing real efficiency, but rather their own trading profits when deciding upon how much information to acquire. Because the market price reveals the traders' information only with noise, the traders' incentives to acquire information are too strong from an efficiency perspective.

Importantly, these two sources of inefficiency are mutually reinforcing. Suppose the firm's and traders' information are substitutes. As the traders want to maximize their trading profits as opposed to efficiency, they acquire too much information. Anticipating a highly informative stock price, the firm faces weaker incentives to acquire information itself, as the market's information substitutes for the information generated internally. Less information acquisition by the firm increases the firm's value uncertainty, in turn, incentivizing the traders to acquire more information. This mutual reinforcement leads to too much information acquisition by the traders and too little by the firm, generating lower efficiency for the

[1] We abstract away from any other differences between private and public firms affecting their information generation incentives.

public firm relative to the private firm in equilibrium.

The finding that efficiency is higher in the case of a private firm also holds under a complement information structure. Again, the traders are maximizing their personal trading profits as opposed to economic efficiency, leading to too strong information acquisition incentives. A highly informative stock price increases the firm's information acquisition incentives due to the complement nature of information. In turn, more information acquisition by the firm increases the marginal value of information for the traders, implying more effort on their side and generating even more informative prices. In contrast to a substitute information structure, the firm is investing too much in internal information acquisition under a complement information structure. Nevertheless, the mutual reinforcement of the public firm's and traders' inefficient information acquisition choices also results in lower efficiency for the public firm relative to the private firm.

To fix ideas, consider a manager's decision whether to expand the firm's operations into a new geographical market. Prior to making these considerations public, the manager estimates the new market's potential through internal market research. The announcement of the firm's expansion plan reflects some of the manager's information and triggers a financial market reaction, which the manager can incorporate when choosing the scale of expansion. In this example, both the manager and market participants try to acquire similar information about the attractiveness of the new market, the local competition to be faced, and potential regulatory hurdles, reflecting a *substitute* information structure.

Another example is a pharmaceutical firm's application for approval of a new drug to the regulator, such as the FDA or the EMA. In addition to the approval itself, the regulatory bodies' disclosures feature a summary of the drug's efficacy, potential side effects, working mechanisms, and other crucial information about the health potential of the drug.[2] While the R&D efforts by the firm are clearly not realizable by the market, investors may acquire

[2]Take for instance the FDA's approval of WAINUA, a familial amyloid polyneuropathy drug, developed by Ionis Pharmaceuticals, Inc. The FDA's approval on December 21, 2023 featured a lengthy "drug trial snapshop", providing detailed information about the procedure and outcome of clinical trials (FDA [2024]).

information that may *complement* the pharmaceutical's knowledge generation. In particular, investor's may generate information about the market size, optimal pricing strategy, and competitive landscape of the drug. Ultimately, more precise information generated by the firm about the drug itself may increase the value of market participants' information. Depending on the financial market reaction upon the drug's approval, the manager can then choose the scale of the drug's production facilities.

Importantly, those two examples highlight the endogenous nature and interdependence of the firm's and investors' information acquisition efforts. Whenever the information is substitutive (complementary) in nature, less (more) knowledge generation by the firm crowds in information acquisition by traders, and, in turn, more informative market prices discourage (incentivize) the firm's information acquisition efforts.

The two examples are captured more generally by our model, in which we consider a firm that has to make two subsequent decisions. First, how much to spend on internal information acquisition (e.g., R&D) to learn about the quality of an investment project. Second, with which scale to conduct the actual investment project. While the public firm has access to the very same *internal* information technology as the private one, its shares are traded in a financial market that may serve as an additional source of information regarding the optimal scale decision. Specifically, the financial market includes a sophisticated trader who may choose to acquire costly information about the returns of the firm's investment project. The trading activity of the (potentially informed) sophisticated trader may render the stock price informative, which helps the manager in making a better investment scale decision.

We assume that once the public manager has learned the outcome of the internal information acquisition, she informs the market about her *intended* investment scale. Jayaraman and Wu [2020] show that managers incorporate the market reaction upon CAPEX (capital expenditure) forecasts in their *actual* investment decisions. Our modeling assumption closely follows this evidence in the sense that the public manager announces the investment scale that she implements if the stock price does *not* provide her with new information. However, whenever the market reveals new positive (negative) information to her, she chooses a larger (smaller) scale than previously announced. Importantly, the announcement of the *intended* investment scale reveals the public manager's internally generated information to the market.

We analyze the model considering two information structures. First, we focus on a substitute information structure, where the manager and the trader have access to the same fundamental being informative of the project's profitability. Second, we study the case of complement information, where the manager and the trader may acquire information about different fundamentals jointly reflecting the project's profitability.

In the model, the result on the public firm's inefficient information acquisition choice hinges on the interplay of three links: First, the firm adjusts its scale investment based on the information revealed through market prices. The dependence of a firm's investments on the information reflected in market prices is documented by a variety of studies (e.g., Chen et al. [2007], Bakke and Whited [2010], Foucault and Frésard [2012], and Edmans et al. [2017]), building the cornerstone of the feedback literature. Second, the extent of informed trading is stimulated by the firm's revealed information through the intended investment announcement. Jayaraman and Wu [2020] and Fox et al. [2021] provide direct evidence that managers use CAPEX guidance as a way to stimulate market feedback.[3] Third, the firm adjusts its internal information acquisition to stimulate informative stock prices. Blanco and Wehrheim [2017] show that firms with more options trading activity are more innovative. However, their findings may be driven by firms changing their innovation strategies in anticipation of, or in reaction to informative stock prices. More directly, Friberg et al. [2024] show that in anticipation of less informative stock prices brought about by an exogenous change in the composition of equity ownership, a firm adjusts its corporate policies, including its R&D spending. To sum up, we perceive our chain of theoretical links to be largely consistent with the empirical literature, highlighting the relevance of our theoretical study

[3]In addition, Jayaraman and Wu [2019], Bird et al. [2021], and Goldstein et al. [2023] show that other public disclosures affect the degree of informed trading and the feedback effect.

and its implications.

Our paper contributes to the feedback literature by investigating the interdependence between the firm's internal information acquisition and information generated by market participants.[4] In contrast to the extant literature which we review below, our definition of real efficiency includes the information acquisition costs incurred by financial market participants. To judge whether the public firm dominates the private firm and whether the "feedback effect" enhances economic efficiency, it is insufficient to focus on its marginal benefits in terms of better investment-making by the public firm. Indeed, a stronger feedback effect arises due to more information acquisition by market participants, associated with higher information acquisition costs.

Summarizing, our analysis identifies two sources of inefficiency brought about by the feedback effect, resulting in lower real efficiency in an economy with public as opposed to private firms. While the literature documents that public markets have become more informative (Bai et al. [2016]) and improve public firms' investment-making (e.g., Bennett et al. [2020]), we caution that these findings do not imply higher efficiency when considering the information acquisition costs borne by market participants.[5]

The insights on public firms' information generation efficiency shed a new light on financial market regulation. A common theme in regulatory efforts is to increase price efficiency to enhance the market's role in resource allocation. In contrast, we show that even though more informative prices generate value for firms through market feedback, they may be detrimental for real efficiency by distorting the allocation of information acquisition efforts

[4]Similar as Goldstein et al. [2020], we study the information acquisition choices of a firm and financial market participants about the firm's profitability drivers A and B. However, while Goldstein et al. [2020] study the incentives of the firm to learn about the same or a different profitability driver than the market, we focus on the intensity of learning, holding constant about which fundamental the firm can learn. The two studies are complementary as Goldstein et al. [2020] focus on an exogenous intensity and endogenous fundamental choice, i.e., choosing whether to learn about A or B, while our focus is on an endogenous intensity, but exogenous fundamental choice, i.e., choosing to learn more or less about A (or B).

[5]Using the estimates from French [2008], spending on price discovery has risen from 0.3% to 1% of gross domestic product since 1980, a substantial cost for society. We are unaware of empirical studies investigating whether these firm-level benefits of increased price discovery outweigh the associated information acquisition costs of financial market participants.

between firms and the financial market.

Our model abstracts away from many crucial differences between private and public firms.[6] Nevertheless, it provides a new perspective on the mixed evidence on the knowledge generation intensity and success of private versus public firms.[7] Our theory predicts that a public firm invests more (less) in internal information generation relative to an identical private firm, if the information it generates complements (substitutes) the information produced by market participants. Therefore, it is crucial whether the information that the firm generates through its internal knowledge generation such as R&D activities has a substitute or complement nature relative to what it can learn from market feedback.

2 Related theoretical literature

While the manager is certainly the best-informed agent in the economy about the firm's prospects, it is intuitive that market participants may possess incremental value-relevant information. By aggregating the different pieces of information from many traders, the market price represents an additional piece of information a rational manager should consider when making decisions. Indeed, a large empirical literature shows that the information contained in market prices affects corporate decision-making.[8] Consequently, there is a two-way flow of information from the firm to the market and from the market to the firm. A growing literature studies how firm decisions affect market participants' trading behavior and ultimately the usefulness of the market price as a source of information for the manager. Examples include financing choices (Machado and Pereira [2021], Chemla and Tinn [2020]), oligopolistic competition (Xiong and Yang [2021]), compensation (Lin et al. [2019], Machado and Pereira [2020]), and voluntary disclosure (Schneemeier [2023]). Bond et al. [2012], Goldstein and

[6] For instance, public and private firms differ in their access to capital (markets), their disclosure mandates, and the prevalence of agency problems.

[7] For instance, Lerner et al. [2011] and Bernstein [2015] find that private firms are more innovative, while Feldman et al. [2021] document that public firms are more R&D intensive.

[8] See, among others, Luo [2005], Chen et al. [2007], Foucault and Frésard [2012], Foucault and Frésard [2014], and Jayaraman and Wu [2020].

Yang [2017], and Goldstein [2023]) provide excellent surveys about the so-called feedback effect literature.

Specifically, our paper contributes to the part of the financial market feedback literature that considers several dimensions of information and different agents (e.g., the firm's manager and financial market participants) having heterogeneous abilities or interests to learn about those dimensions. Boot and Thakor [2001] find that complementary (substitute) information disclosure strengthens (weakens) investors' incentives to acquire and trade on information. While Boot and Thakor [2001] already provide some of the basic ingredients of our analysis, they do not consider the firm's learning from market prices.[9]

In models featuring the feedback effect, Bond et al. [2010], Bond and Goldstein [2015], Goldstein and Yang [2019], and Goldstein et al. [2024] consider two fundamentals the traders and the decision-maker may be informed about. We follow Goldstein and Yang [2019] by assuming a multiplicative impact of the invested capital and the two fundamental factors on the firm's value. However, our focus is the feedback effects' impact on the equilibrium knowledge generation incentives. Specifically, we study the interdependence between the manager's and the speculator's *endogenous and interdependent* information acquisition choices, while Bond et al. [2010], Bond and Goldstein [2015], and Goldstein et al. [2024] treat the information endowment exogenous, and Goldstein and Yang [2019] only consider endogenous information acquisition by traders. Thus, the distortions in information acquisition incentives do not arise in their settings.

Goldstein et al. [2020] also consider two dimensions of uncertainty and consider the traders' and manager's incentives to be informed about either of the fundamentals. However, their timeline is such that traders and the manager acquire private information simultaneously. In contrast, we focus on subsequent information choices where the trader conditions on the outcome of the firm's learning. Moreover, as part of their paper's title "who learns

[9]Goldstein and Yang [2015] model a setting in which different traders are informed of different fundamentals. They show how trading intensities depend on whether information on one fundamental is a complement or substitute for the information on the other fundamental.

what" suggests, they study the manager's and speculators' information choices of becoming informed about either of the two fundamentals A or B. Thus, their main insight is a fundamental mismatch where the traders want to collect the same information as the firm to maximize trading profits (e.g., both parties learning about A), while the firm prefers to diversify its information sources and acquires orthogonal information (e.g., learning about A, while traders learn about B). While Goldstein et al. [2020] focus on the agents' choices of becoming informed about fundamental A or B, they assume that they can do so free of charge with exogenous precisions of the associated signals. In contrast, we focus on the agents' decisions to costly acquire more precise signals, while fixing whether the agents can learn about A or B. Our insight that the feedback effect distorts the equilibrium information acquisition *intensities* is therefore unique to our model.

A key insight from the feedback literature is a more nuanced perspective on efficiency. What matters most for real efficiency is whether market prices reveal new information to decision-makers, helping them to make value-increasing decisions.[10] Bond et al. [2012] coin this efficiency definition as "revelatory price efficiency" (RPE). From the RPE perspective, the public firm in our model is efficient as it maximizes its firm value net of information acquisition costs, rationally taking into account all available information sources, including the information in market prices. However, as the firm's choices affect the speculator's incentives to acquire information himself, one crucial aspect is missing in the literature and the RPE definition: The information in prices is generally not costless. As the public firm is able to generate additional value by learning from the market, a fair comparison between the public and the private firm considers all associated information acquisition costs, including those incurred by market participants. When considering the information acquisition costs borne by market participants, our paper shows that the public firm's information acquisition choices are distorted, resulting in lower efficiency of the public firm.

Several recent papers share our insight that the focus should go beyond RPE. For in-

[10] In contrast, the traditional view of "market efficiency" or forecasting price efficiency focuses on whether prices can accurately predict cash flows.

stance, Dow and Rahi [2003] and Banerjee et al. [2023] study how informative prices and the feedback effect influence hedging opportunities and thus investor welfare. However, both papers assume exogenous and costless information. In contrast, Hapnes [2021] and Gervais and Strobl [2023] incorporate traders' information acquisition costs in their definitions of real efficiency or welfare. However, as they do not endogenize both the manager's and the traders' information acquisition choices, the inefficiency in the allocation of information acquisition is unique to our paper.

Davis and Gondhi [2024] and Banerjee et al. [2021] share our insight that informative prices may distort firm decisions, however, through different channels. They focus on the impact of price information on agency conflicts between debt and equity holders (Davis and Gondhi [2024]) and in a moral hazard principal-agent context (Banerjee et al. [2021]). In contrast, our channel works in the absence of agency conflicts and operates purely through the interdependence between the manager's (or firm owner's) and speculators' information acquisition incentives.

3 Model

3.1 Setup

We consider an economy that consists of a single firm and a manager ("she") who runs its operations. The firm may be either private or public which is common knowledge. We refer to the public (private) firm's manager as the public (private) manager for brevity. The financial market is populated by a sophisticated investor ("he"), uninformed noise traders, and a competitive market maker. All agents are risk-neutral and the manager's role in this model is twofold: First, she chooses the *R&D intensity*, γ, that is, how much to spend on internal information acquisition. Second, she determines the *investment scale* $K \geq 0$ of a growth option. We abstract away from any agency issues and assume that the manager maximizes the expected firm value, net of information acquisition costs, with her choices of

γ and K.

We assume that the profitability of the growth option and ultimately the firm value V depends on two random variables A and B. The fundamentals are binary and independently distributed, with $i \in \{L = \theta - \sigma, H = \theta + \sigma\}$, and a prior probability of $Pr(i = H) = \frac{1}{2}$ for $i \in \{A, B\}$. In addition, we require $\sigma, \theta > 0$ and $\sigma < \theta$, implying strictly positive fundamental values.

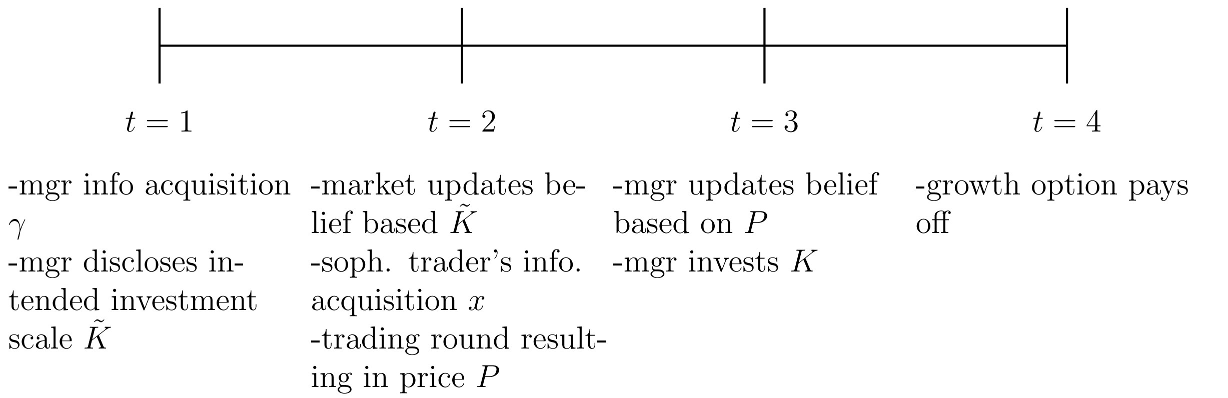

Figure 1: Timeline

The timing of the model is as follows. At $t = 1$, the manager conducts internal information acquisition about the profitability of the growth option and announces the outcome publicly. At $t = 2$, the sophisticated investor can himself at a cost acquire information about the fundamental A. Thereafter, trading takes place in the financial market. As will be clear shortly, the resulting stock price partially reveals the sophisticated trader's information, making it a useful piece of information for the manager. Thus, at $t = 3$, the manager makes her investment scale decision K conditional on the information learned internally and revealed through the stock price. At $t = 4$, the fundamentals A and B realize and all payments are made. The private firm's shares are not publicly traded, making the trading stage at $t = 2$ superfluous.

We consider two alternative information structures: First, we focus on the manager learning about the same fundamental as the sophisticated investor, A, leading to a *substitute information structure*. Second, we consider a *complement information structure*, where the manager learns about the other fundamental B. Sections 4 and 5 contain the analysis of the substitute and complement information case, respectively.

For both information structures, the model proceeds as follows:

$t = 1$: Internal Learning Stage

The manager chooses the intensity of costly internal information acquisition, which we interpret as expenses for R&D. Specifically, the manager chooses the probability $\gamma \in [0, 1]$ for which the signal a (b) in the substitute (complement) information case perfectly reveals the fundamental A (B). With probability $1 - \gamma$, the signal is uninformative, implying that her belief about the respective fundamental stays at its prior, $Pr(A = H|a = \emptyset) = Pr(B = H|b = \emptyset) = \frac{1}{2}$. Thus, the signals take the values $a, b \in \{L, H, \emptyset\}$. Information acquisition is costly, captured by the convex cost function $\frac{c}{2}\gamma^2$, with $c > 0$. The choice of γ is publicly observable.

Learning about the fundamentals is of value for the firm, as it allows the manager to choose the optimal investment scale K for the growth option (see description about $t = 3$ below). We assume that the firm has no assets in place and its value is determined solely by the growth option:[11]:

$$V = ABK - \frac{1}{2}K^2 - \frac{1}{2}c\gamma^2. \tag{1}$$

We abstract from standard agency conflicts which implies that the manager chooses γ to maximize the expected firm value (1), net of information acquisition costs.

After having received her internal signal a or b, the manager announces her intended investment scale publicly and truthfully, which may be interpreted as a CAPEX forecast (Jayaraman and Wu [2020]). Specifically, the manager announces $\tilde{K}(a) = K^*(a)$ ($\tilde{K}(b) = K^*(b)$), capturing her *intended* equilibrium investment scale K^* in the absence of other information than her signal a (b) in the substitute (complement) information case. While reflecting her honest intentions, the public manager may choose a different investment scale after having observed the financial market outcome.

[11] We assume a similar investment technology as Goldstein and Yang [2019] and Petrov and Schantl [2023].

As will be clear shortly, the announcement reveals the manager's information to the market, resulting in the common belief of $\mu_A = Pr(A = H|a)$ ($\mu_B = Pr(B = H|b)$) for the fundamental A (B).

<u>$t = 2$: Trading Stage</u>

After updating his belief to μ_i from the manager's announcement, the sophisticated trader makes an optimal information acquisition choice.[12] In particular, he chooses the probability $x \in [0, 1]$ for which the signal α perfectly reveals the fundamental A. With probability $1-x$, the signal is uninformative, implying that $\alpha \in \{L, H, \emptyset\}$. The trader's cost of information acquisition is $\frac{\kappa}{2}x^2$, with $\kappa > 0$.

To avoid discussing various corner solutions in the text, we restrict our attention to interior values of the information acquisition choices by the firm ($\gamma \in (0, 1)$) and the sophisticated trader ($x \in (0, 1)$), by assuming sufficiently high marginal costs of information acquisition c and κ. We specify these assumptions when solving for the equilibrium information choices.

After potentially learning the fundamental A, the speculator trades w units of the asset. Without loss of generality, we assume that the speculator does not trade ($w^* = 0$) when indifferent. In addition to the speculator, the financial market is populated by uninformed liquidity traders. The aggregate liquidity trades result in random order flows of $y \in \{-1, 1\}$ with equal probability. As standard in markets similar to Kyle [1985], the speculator trades $w \in \{-1, 1\}$ in equilibrium to mimic the liquidity trades. Finally, the market maker sets the stock price P as the expected value of the firm conditional on the total order flow $z = w + y$ and all other public information.

<u>$t = 3$: Scale Investment Stage</u>

The resulting stock price in $t = 2$ partially reveals the trader's information. Thus, in addition to her belief μ_i, the public manager takes into account the market price when making her investment scale decision K. The manager chooses K to maximize the final firm value specified in (1).

[12] We model the financial market similar as in Gao and Liang [2013], Edmans et al. [2015], and Lassak [2023].

$t = 4$: Payoffs

Finally, at $t = 4$, A and B as well as the resulting payoffs realize, and the game ends.

As an equilibrium solution concept, we use Perfect Bayesian Nash Equilibrium which, hereafter, we refer to simply as equilibrium. The equilibrium is characterized by: 1) the manager's optimal investment scale decision K^*, maximizing expected firm value (1); 2) a pricing rule, $P^*(z)$, such that the market maker breaks even on average; 3) the strategic investor's profit-maximizing trade w^*; 4) the strategic trader's choice of the intensity of information acquisition, x^*, which maximizes his expected trading profit net of information acquisition costs; 5) the manager's choice of R&D intensity γ^*, maximizing the expected firm value net of information costs; 6) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium. All proofs are contained in Appendix A. We solve the model by backward induction and start with the substitute information structure.

4 Substitute Information

4.1 Equilibrium

4.1.1 Scale Investment Subgame

The information set of the manager at this stage, which we denote by $\mathcal{T}$, includes her private signal a and, if it is a public firm, the stock price P. As we will specify shortly, both a and P will affect the manager's belief about A in the substitute case, whereas the belief about B is not updated and stays at the ex-ante expected value of θ.

Conditional on her information set, the manager chooses the investment scale K^* to maximize the expected firm value after investment

$$K^* = \arg\max_K \ \mathbb{E}\left[AB|\mathcal{T}\right] K - \frac{1}{2}K^2 - \frac{1}{2}c\gamma^2 = \mathbb{E}\left[A|\mathcal{T}\right]\theta, \tag{2}$$

generating an expected firm value after investment of

$$\mathbb{E}\left[V|\mathcal{T},K^*\right]=\frac{1}{2}(\mathbb{E}\left[AB|\mathcal{T}\right])^2-\frac{1}{2}c\gamma^2=\frac{1}{2}\theta^2(\mathbb{E}\left[A|\mathcal{T}\right])^2-\frac{1}{2}c\gamma^2. \tag{3}$$

The growth option's profitability per unit of K depends on the fundamentals A and B. Thus, the higher the manager's belief μ_i about fundamental $i \in \{A, B\}$, the higher is her scale investment K^* ultimately leading to a more elevated expected firm value. Given that the profit on the growth option is convex in the manager's belief, μ_i, the manager wants her information about A and B to be as precise as possible.

4.1.2 Financial Market Subgame

Let's consider the financial market the public firm's shares are traded in. As the private firm's shares are not traded publicly, this stage is superfluous for the private firm. In the substitute information case, both the manager and the strategic trader can generate information about the same fundamental, A, while the belief about B remains unchanged. Our information and signal structure implies that the manager either learns A perfectly through internal R&D ($a \in \{L, H\}$) or receives a completely uninformative signal ($a = \emptyset$) . In the first case, there will be no incentives for the strategic trader to trade on his information about A, as the manager's information becomes public knowledge once she announces the intended investment scale $\tilde{K}(a)$. Indeed, the perfectly revealing signal eliminates all public uncertainty and thus the potential of generating trading profits for the speculator with private information.

In contrast, if the manager's signal is not informative, $a = \emptyset$, the strategic trader has the potential to generate trading gains with her private information. The speculator's trading activities injects information about A into the price, making it an important source of information for the manager of the public firm. Thus, the next lemma describes the equilibrium ("EQ") in the trading stage, taking into account the resulting investment behavior of the manager.

Lemma 1 (Trading & Investment EQ – Substitute Information) *For a public belief of $\mu_A \in \{0, \frac{1}{2}, 1\}$ prior to trading, the unique pure strategy equilibrium in the trading stage is as follows:*

1) Strategic trader's demand: For a public belief of $\mu_A \in \{0, 1\}$ and any signal α, the speculator is indifferent and chooses not to trade ($w^* = 0$). For a public belief of $\mu_A = \frac{1}{2}$, the speculator trades $w^* = 1$ ($w^* = -1$) after observing the signal $\alpha = H = \theta + \sigma$ ($\alpha = L = \theta - \sigma$), and chooses $w^* = 0$ for a signal of $\alpha = \emptyset$.

2) Price setting: The market maker sets prices as a function of observed order flow z and the belief μ_A

$$P^*(z, \mu_A) = \begin{cases} P_H \equiv \frac{1}{2}(\theta + \sigma)^2\theta^2 - \frac{1}{2}c\gamma^2, & \textit{for } z = 2 \textit{ or } \mu_A = 1, \\ P_\emptyset \equiv \frac{1}{2}\theta^4 - \frac{1}{2}c\gamma^2, & \textit{for } z \in \{-1, 0, 1\} \textit{ and } \mu_A = \frac{1}{2}, \\ P_L \equiv \frac{1}{2}(\theta - \sigma)^2\theta^2 - \frac{1}{2}c\gamma^2, & \textit{for } z = -2 \textit{ or } \mu_A = 0. \end{cases}$$

3) Investment scale: The manager chooses the investment scale as a function of the observed stock price P and the belief μ_A

$$K^*(P, \mu_A) = \begin{cases} (\theta + \sigma)\theta, & \textit{for } P^* = P_H \textit{ or } \mu_A = 1, \\ \theta^2, & \textit{for } P^* = P_\emptyset \textit{ and } \mu_A = \frac{1}{2}, \\ (\theta - \sigma)\theta, & \textit{for } P^* = P_L \textit{ or } \mu_A = 0. \end{cases}$$

Naturally, the speculator can only monetize on his private information if he trades in the direction of his private signal and the fundamental A has not been revealed.[13] Thus, he buys if $\alpha = H$ and sells if $\alpha = L$. This implies that the order flows of $z = 2$ or $z = -2$ reveal the true A to the market maker, who sets the price anticipating that the

[13] The properties of the investment game rule out manipulative selling as highlighted by Goldstein and Guembel [2008]. Whenever the price indicates that the trader is selling, the firm chooses $K^* = (\theta - \sigma)\theta$, resulting in the firm value being *insensitive* to the traders information. This rules out that the trader can benefit from uninformed shorting.

manager will invest accordingly. In line with the revealed realization of A, the manager invests $K^*(z=2) = (\theta+\sigma)\theta$ or $K^*(z=-2) = (\theta-\sigma)\theta$, respectively. On the contrary, an order flow of $z \in \{-1, 0, 1\}$ is not informative. In particular, whenever the order flow is $z \in \{-1, 1\}$, the market maker realizes that the speculator is uninformed and not active in the financial market. For $z = 0$, the market maker does not know whether the speculator has entered a long or short position, leaving her belief unchanged at $\mu_A = \frac{1}{2}$. Consequently, the manager bases the investment decision on the prior belief μ_A.

Before trading, the speculator takes into account his potential trading gains when being informed and chooses the optimal level of information acquisition x^* as the next lemma describes.

Lemma 2 (Speculator's Information Acquisition EQ – Substitute Information) *For a public belief of $\mu_A \in \{0, \frac{1}{2}, 1\}$ prior to trading, the trader's unique information acquisition choice satisfies*

$$x^* = \begin{cases} 0 & \text{if } \mu_A \in \{0,1\}, \\ x^*_\emptyset \equiv \frac{\theta^3\sigma}{2\kappa} & \text{if } \mu_A = \frac{1}{2}. \end{cases}$$

Assumption 1 ensures $x^* \in (0, 1]$.

Assumption 1 *We assume the trader's marginal information acquisition cost to be $\kappa \in [\underline{\kappa}^{sub}, \infty)$, with $\underline{\kappa}^{sub} = \frac{\theta^3\sigma}{2}$.*

Following the intuition from the trading stage, the trader only acquires information whenever the fundamental has not been revealed, that is, following a public belief of $\mu_A = \frac{1}{2}$. In this case, her information acquisition intensity is decreasing in the marginal cost factor κ, and increasing in the profitability uncertainty σ, and the average size of the growth fundamentals θ.

4.1.3 Firm's Information Acquisition Subgame

As highlighted by Lemma 1, the manager's equilibrium scale investment K^* depends on her belief about the fundamental A after having observed her internal signal a and the stock price. In particular, the manager chooses a high or low investment scale if the fundamental A has been revealed to be H or L, respectively, or a medium investment scale, if the manager did not receive further information about A. Based on these beliefs and investment scale decisions, the expected firm values are P_H, P_L, and $P_\emptyset$, respectively.

Anticipating the potential outcomes of her internal information acquisition and the information revealed by the financial market, the public manager's ex-ante firm value expectation is

$$\begin{aligned}\mathbb{E}\left[V\right] =&\gamma\left(\frac{P_H+P_L}{2}\right)+(1-\gamma)\left(\frac{x_\emptyset^*}{2}\frac{P_H+P_L}{2}+\left(1-\frac{x_\emptyset^*}{2}\right)P_\emptyset\right)\\ =&\underbrace{\frac{1}{2}\theta^4}_{\text{prior value}}+\underbrace{\frac{1}{2}\theta^2\sigma^2}_{\text{benefit of learning}}\underbrace{\left(\gamma+(1-\gamma)\frac{x_\emptyset^*}{2}\right)}_{\text{probability of learning}}-\frac{1}{2}c\gamma^2. \end{aligned} \tag{4}$$

If the manager neither learns additional information from her internal information acquisition nor from the financial market, she has to base her investment decision on her prior belief of $\mu_A=\frac{1}{2}$, generating an expected firm value of $\frac{1}{2}\theta^4-\frac{1}{2}c\gamma^2$. In contrast, if the manager acquires information internally, she either learns $a=H$ or $a=L$ with probability $\frac{\gamma}{2}$ each, allowing her to adjust K^* to A's value. With probability $1-\gamma$, her internal information acquisition turns out to be unsuccessful and generates an uninformative signal $a=\emptyset$. In this case, however, she may learn from the financial market. The speculator gets informed with probability $x_\emptyset$, and with probability $\frac{1}{2}$, this information is revealed through the stock price. Thus, combining the possibility of learning from internal sources as well as from the financial market, the manager learns the fundamental perfectly with probability $\gamma+(1-\gamma)\frac{x_\emptyset^*}{2}$. Knowing whether the fundamental is H or L helps the manager to create additional value of $\frac{1}{2}\theta^2\sigma^2$ by choosing a more appropriate investment scale.

The public manager's optimal information acquisition maximizes the expected firm value net of information acquisition costs characterized by (4). In addition, it is straightforward to derive the private manager's γ choice by setting $x^*_\emptyset = 0$, as he naturally does not observe the price of his firm's shares and thus cannot learn from the market. Lemma 3 describes the information acquisition choices by the public and the private firm.

Lemma 3 (Firm's Information Acquisition EQ – Substitute Information)) *The private firm chooses $\gamma^*_{pri} = \frac{\theta^2\sigma^2}{2c}$ as their information acquisition intensity. The public firm chooses*

$$\gamma^*_{pub} = \gamma^*_{pri}\left(1 - \frac{x^*_\emptyset}{2}\right) = \gamma^*_{pri}\left(1 - \frac{\theta^3\sigma}{4\kappa}\right). \tag{5}$$

Assumption 2 ensures $\gamma^*_{pri}, \gamma^*_{pub} \in (0, 1]$.

Assumption 2 *We assume the firm's marginal information acquisition cost to be $c \in [\underline{c}^{sub}, \infty)$, with $\underline{c}^{sub} = \frac{\theta^2\sigma^2}{2}$.*

Having solved for the equilibrium, the next section analyzes its implications.

4.2 Analysis

4.2.1 Comparative Statics

The next corollary summarizes how the private and public firm's equilibrium information acquisition intensities vary with model parameters:

Corollary 1 (Comparative Statics – Substitute Information) *The private and public firm's information acquisition equilibrium intensities, $\gamma^*_{pri}, \gamma^*_{pub}$, are increasing in σ and decreasing in c. γ^*_{pri} is increasing in θ, while γ^*_{pub} is increasing (decreasing) in θ for sufficiently large (small) κ.*[14] *γ^*_{pub} is decreasing in κ.*

[14]The precise threshold for κ is $\frac{5}{4}\underline{\kappa}^{sub}$.

Naturally, the private and public firms' information acquisition is decreasing in the marginal cost parameter c and increasing in the profitability uncertainty σ. Similarly, the *private* firm's γ^*_{pri} is increasing in the average fundamental size θ, highlighting a higher average value upon investment.

The public firm's γ^*_{pub} is decreasing in the speculator's information acquisition intensity $x_\emptyset$. As the firm's and the trader's information are substitutes, the public firm receives the same information when learning internally or from the market. An informative stock price captured by $x^*_\emptyset > 0$, therefore, decreases the public firm's information acquisition incentives as it hopes to learn the fundamental A from market prices without incurring the costs of internal information acquisition.

As the trader is acquiring less information when it is costlier, γ^*_{pub} is increasing in κ. As $x^*_\emptyset$ is increasing in θ, the *public* firm's information acquisition intensity changes ambiguously in θ. First, a higher average fundamental θ increases the value of information, just as for the private firm, increasing information acquisition incentives for the public firm. Second, however, it strengthens the trader's information acquisition intensity, making the stock price more informative, ultimately decreasing the firm's incentive to acquire information internally. Whenever κ is sufficiently low and the stock price is sufficiently informative, the latter effect dominates, resulting in less information acquisition by the public firm for a higher average fundamental θ.[15]

4.2.2 Gross Firm Values

While the public firm learns less from internal information acquisition it has access to an additional source of information, the stock price. Therefore, we analyze the respective expected firm values in equilibrium.

As a first step, ignoring the firms' information acquisition costs, the ex-ante gross ex-

[15] One may expect a similar result for the comparative static with respect to σ. This is not the case, however, as the firm's strengthened information acquisition incentive always dominates the increase in market feedback.

pected value are $\mathbb{E}^{gross}[V] = \frac{1}{2}\theta^4 + (\gamma^*_{pub} + (1-\gamma^*_{pub})\frac{x_\emptyset}{2})\frac{1}{2}\theta^2\sigma^2$ and $\mathbb{E}^{gross}_{pri}[V] = \frac{1}{2}\theta^4 + \gamma^*_{pri}\frac{1}{2}\theta^2\sigma^2$ according to (4), for the public and private firm, respectively. As the firms' and trader's information are substitutes, the difference in expected gross firm values boils down to the differential likelihood of learning the fundamental A. Indeed, the private firm's gross value is higher if $\gamma^*_{pub} + (1-\gamma^*_{pub})\frac{x_\emptyset}{2} < \gamma^*_{pri}$. While the private firm learns the fundamental with probability γ^*_{pri}, the public firm also has the opportunity to learn from the market, in case its private information acquisition turns out to be unsuccessful.

The next lemma highlights which firm type's expected gross firm value is higher in equilibrium.

Lemma 4 (Gross Firm Value Difference – Substitute Information) *Whenever the firm's cost of information acquisition c is sufficiently large (small), the public (private) firm has a higher expected gross value in equilibrium.*[16]

If the firms' information acquisition costs are high, both manager types choose low R&D efforts. The firms are unlikely to learn the fundamental from internal sources, providing strong information acquisition incentives for the speculator of the public firm. In turn, the public firm benefits from the information revealed through the stock price. Whenever c is sufficiently high and thus the cost functions sufficiently convex, the additional information provided by the market dominates the public firm's lower internal information acquisition, leading to the public firm being more likely to be informed about A than the private firm. As a consequence, the public firm's expected gross value is higher.

In contrast, for sufficiently low information acquisition costs c, the private firm acquires substantially more information than the public firm. An informative stock price narrows the information gap. However, it is not able to overturn the private firm's information advantage, resulting in a higher expected gross value for the private firm.

[16]The precise condition on the firm's cost of information acquisition is $c > \underline{c}^{sub}\left(2 - \frac{\theta^3\sigma}{4\kappa}\right)$.

4.2.3 Net Firm Values

The comparison in Section 4.2.2 only considers the gross expected firm values and ignores the cost of the internal R&D of $\frac{1}{2}c\gamma^2$. Next, we compare the expected firm values, net of information acquisition costs. Inserting the equilibrium information acquisition intensities outlined in Lemma 3, the private and public firm's net expected firm values are:

$$\mathbb{E}_{pri}\left[V\right] = \mathbb{E}_{pri}^{gross}\left[V\right] - \frac{1}{2}c\left(\gamma_{pri}^{*}\right)^{2} = \frac{1}{2}\theta^{4} + \frac{1}{2}(\gamma_{pri}^{*})^{2}c \tag{6}$$

$$\mathbb{E}\left[V\right] = \mathbb{E}^{gross}\left[V\right] - \frac{1}{2}c\left(\gamma_{pub}^{*}\right)^{2} = \frac{1}{2}\theta^{4} + \frac{x_{\emptyset}}{4}\theta^{2}\sigma^{2} + \frac{1}{2}(\gamma_{pub}^{*})^{2}c \tag{7}$$

Lemma 5 establishes that net of the internal R&D costs, the public firm is always at least as valuable as the private firm.

Lemma 5 (Net Firm Value Difference – Substitute Information) *Net of internal information costs, the public firm has a (weakly) higher expected value in equilibrium.*

Intuitively, the public manager could simply choose the private manager's information acquisition intensity, γ_{pri}^{*}, and additionally observe the informative stock price at no costs, already resulting in a higher expected net firm value. If the profit-maximizing manager chooses any other γ in equilibrium, it has to be associated with an even higher expected net firm value.

4.3 Real Efficiency

While the previous analysis shows that the public firm has a higher value net of information acquisition costs, it is unclear whether its practices are socially desirable. In particular, the public firm chooses to engage less in internal R&D in an effort to "outsource" this costly information acquisition activity to financial market participants. Therefore, while optimal for the public firm, it is unclear whether reducing internal R&D in anticipation of market feedback leads to an efficient allocation of information acquisition activities. To investigate the efficiency consequences of the private and public firm's information acquisition choices, we define real efficiency $\mathcal{E}$ as the ex-ante expected firm value, net of all participants' expected information acquisition costs in the economy[17]:

$$\mathcal{E}_{pri} = \mathbb{E}_{pri}[V] \tag{8}$$

$$\mathcal{E}_{pub} = \mathbb{E}_{pub}\left[V - \frac{1}{2}\kappa x^2\right] = \mathbb{E}_{pub}[V] - \frac{1}{2}(1-\gamma)\kappa\left(x_\emptyset\right)^2 \tag{9}$$

Real efficiency for the private firm simply equals the firm's expected value, net of R&D costs. For the public firm, in contrast, we need to consider also the expected information acquisition costs incurred by the speculator. As highlighted by Lemma 1, the trader acquires information only in the case the fundamental has not been revealed by the firm, which occurs with probability $1-\gamma$. Note that the speculator's trading profits equal the liquidity traders' losses and therefore cancel out in the aggregate.

Before moving to the efficiency implications of the equilibrium, consider the benchmark choices by a social planner. The social planner maximizes real efficiency in an economy with a private firm (8) and public firm (9) with the choices γ^{SP} and, if applicable, $x_\emptyset^{SP}$.[18] The next lemma highlights these choices:

Lemma 6 (Social Planner's Choices – Substitute Information) *A social planner would choose*

$$\gamma_{pri}^{SP} = \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2\right) = \gamma_{pri}^*,$$
$$\gamma_{pub}^{SP} = \frac{1}{c}\left(\left(1-\frac{x_\emptyset^{SP}}{2}\right)\frac{1}{2}\sigma^2\theta^2 + \frac{\kappa}{2}\left(x_\emptyset^{SP}\right)^2\right) = \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 - \frac{\theta^4\sigma^4}{32\kappa}\right) > \gamma_{pub}^*,$$
$$x_\emptyset^{SP} = \frac{\theta^2\sigma^2}{4\kappa} < x_\emptyset^*.$$

The private firm's real efficiency coincides with its expected net firm value, implying that

[17] A similar efficiency criterion is used by Hapnes [2021].

[18] If the firm learns the fundamental internally ($a \in \{H, L\}$), there is no more uncertainty and the social planner would choose $x = 0$.

the social planner and the private manager face the same incentives. Therefore, the social planner's information acquisition coincides with the choice of the private manager.

In contrast, the public firm invests too little in information acquisition from the planner's perspective. Indeed, the public firm's marginal value of an increase in γ is $\left(1-\frac{x_\emptyset}{2}\right)\frac{1}{2}\sigma^2\theta^2$, the marginal increase in firm value from learning the fundamental. The social planner, however, also considers the consequence of this decision on the speculator. If the firm invests more in information acquisition, it is more likely it learns the fundamental and announces it publicly. Thus, the trader will less likely acquire information himself, creating lower information acquisition costs in expectation. While the social planner takes this cost saving aspect into account, the public firm does not, and chooses a too low internal information acquisition intensity $\gamma_{pub}^{SP} > \gamma_{pub}^*$.

In addition, the speculator acquires too much information from the social planner's perspective. The marginal social value from informed trading is purely captured by the expected increase in firm value informative prices bring about. In particular, the stock price is informative with probability $\frac{1}{2}$, and increases expected firm value by $\frac{1}{2}\sigma^2\theta^2$. In contrast, the speculator considers only his expected trading gains when making his information acquisition choice. The expected trading gains are $\frac{1}{2}\sigma\theta^3$ and thus provide stronger incentives of information acquisition than the pure increase in expected firm value, leading to an over-investment in $x_\emptyset^*$ by the speculator.

The next proposition highlights the efficiency differences between the public and private firm in equilibrium.

Proposition 1 (Real Efficiency – Substitute Information) *In equilibrium, real efficiency is (weakly) higher if the firm is private:* $\mathcal{E}_{pri}^* \geq \mathcal{E}_{pub}^*$.

Even though the public firm has access to an additional information source, the stock price, it generates lower efficiency than the private firm. In particular, the inefficiency is driven by two channels. First, as highlighted by Lemma 3, the public firm outsources part of its information acquisition activities to the financial market in an effort to save costs. By doing so, however, the firm does not take into consideration the resulting information acquisition costs of the speculator, resulting in too little information acquisition by the firm from an efficiency perspective ($\gamma_{pub}^{SP} > \gamma_{pub}^{*}$). Second, the information acquisition by the speculator is inefficient in itself. As the speculator tries to maximize trading profits, his incentives are not perfectly aligned with real efficiency, resulting in too much information acquisition ($x_{\emptyset}^{SP} < x_{\emptyset}^{*}$).

Importantly, these two channels are mutually reinforcing. Indeed, the firm's ignorance of the speculator's information acquisition cost result in too little internal information acquisition, incentivizing the speculator to generate even more information. In turn, the trader's skewed information acquisition incentives result in an even higher willingness of the public firm to outsource information acquisition to the speculator.

As a consequence, the public firm is associated with lower real efficiency than the private firm as the two distortions faced by the public firm are absent for the private firm.

5 Complement Information

In this section, we solve and analyze our model assuming a complement information structure where the manager and trader can acquire information about different fundamentals. In particular, we assume that the manager has access to the signal b, providing her with information about the fundamental B. In contrast, the trader can acquire information about the fundamental A as before.

All other parts of the model are unchanged and we solve the model by backward induction.

5.1 Equilibrium

5.1.1 Scale Investment Subgame

The investment scale subgame equilibrium follows the same considerations as outlined in Section 4.1.1. The public manager's information set $\mathcal{T}$ includes her signal b as well as the stock price, which provides her with information about A. Thus, the manager's scale investment features

$$K^* = \arg\max_K \ \mathbb{E}\left[AB|\mathcal{T}\right] K - \frac{1}{2}K^2 - \frac{1}{2}c\gamma^2 = \mathbb{E}\left[AB|\mathcal{T}\right], \tag{10}$$

generating an expected firm value after investment of

$$\mathbb{E}\left[V|\mathcal{T}, K^*\right] = \frac{1}{2}(\mathbb{E}\left[AB|\mathcal{T}\right])^2 - \frac{1}{2}c\gamma^2 = \frac{1}{2}(\mathbb{E}\left[AB|\mathcal{T}\right])^2 - \frac{1}{2}c\gamma^2. \tag{11}$$

5.1.2 Financial Market Subgame

In contrast to the substitute information case, the speculator always has incentives to trade on his information, as he is the only agent informed about A. Indeed, the firm's investment scale forecast $\tilde{K}(b)$ captures information about the fundamental B and drives the belief μ_B, but does not affect the public belief about A. The next lemma describes the equilibrium in the trading stage, taking into account the resulting investment behavior of the manager.

Lemma 7 (Trading & Investment EQ – Complement Information) *1) Strategic trader's demand: For any public belief μ_B, the speculator trades $w^* = 1$ ($w^* = -1$) after observing the signal $\alpha = H = \theta + \sigma$ ($\alpha = L = \theta - \sigma$), and chooses $w^* = 0$ for a signal of $\alpha = \emptyset$.*

2) Price setting: The market maker sets prices as a function of observed order flow z and *the belief* μ_B

$$P^*(z,\mu_B) = \begin{cases} P_{HH} \equiv \frac{1}{2}H^4 - \frac{1}{2}c\gamma^2, & \textit{for } z=2 \textit{ and } \mu_B = 1, \\ P_H \equiv \frac{1}{2}H^2\theta^2 - \frac{1}{2}c\gamma^2, & \textit{for } z=2 \textit{ and } \mu_B = \frac{1}{2}, \textit{ or } z \in \{-1,0,1\} \textit{ and } \mu_B = 1, \\ P_{HL} \equiv \frac{1}{2}H^2L^2 - \frac{1}{2}c\gamma^2, & \textit{for } z=2 \textit{ and } \mu_B = 0, \textit{ or } z=-2 \textit{ and } \mu_B = 1, \\ P_\emptyset \equiv \frac{1}{2}\theta^4 - \frac{1}{2}c\gamma^2, & \textit{for } z \in \{-1,0,1\} \textit{ and } \mu_B = \frac{1}{2}, \\ P_L \equiv \frac{1}{2}L^2\theta^2 - \frac{1}{2}c\gamma^2, & \textit{for } z=-2 \textit{ and } \mu_B = \frac{1}{2}, \textit{ or } z \in \{-1,0,1\} \textit{ and } \mu_B = 0, \\ P_{LL} \equiv \frac{1}{2}L^4 - \frac{1}{2}c\gamma^2, & \textit{for } z=-2 \textit{ and } \mu_B = 0, \end{cases}$$

3) Investment scale: The manager chooses the investment scale as a function of the observed stock price P *and the belief* μ_B

$$K^*(P,\mu_B) = \begin{cases} H^2, & \textit{for } P^* = P_{HH} \textit{ and } \mu_B = 1, \\ H\theta, & \textit{for } P^* = P_H \textit{ and } \mu_B = \frac{1}{2}, \textit{ or } P^* = P_H \textit{ and } \mu_B = 1, \\ HL, & \textit{for } P^* = P_{HL} \textit{ and } \mu_B = 1, \textit{ or } P^* = P_{HL} \textit{ and } \mu_B = 0, \\ \theta^2, & \textit{for } P^* = P_\emptyset \textit{ and } \mu_B = \frac{1}{2}, \\ L\theta, & \textit{for } P^* = P_L \textit{ and } \mu_B = \frac{1}{2}, \textit{ or } P^* = P_L \textit{ and } \mu_B = 0, \\ L^2, & \textit{for } P^* = P_{LL} \textit{ and } \mu_B = 0. \end{cases}$$

The market maker's pricing reflects the information about B contained in the firm's announcement of the intended investment scale $\tilde{K}(b)$, as well as the information about A contained in the order flow, taking into account the manager's resulting actual investment behavior. For instance, whenever the manager's announcement revealed that her signal was uninformative $b = \emptyset$ and the orderflow is $z \in \{-1, 0, 1\}$, the market maker does not learn any new information. In this case, the manager invests $K^* = \theta^2$ and the expected firm value is $P_\emptyset$. In contrast, if the belief about B is $\mu_B = 1$ prior to trading, and the market maker observes $z = -2$, he learns that $B = H$ and $A = L$, and sets the price to P_{HL}. Consequently, the manager invests $K^* = HL$ based on good information about B and bad information about A.

In contrast to the substitute information case, the speculator is able to monetize on his superior information about fundamental A, whether or not the firm's announcement revealed the fundamental B. Before trading, the speculator takes into account his potential trading gains and chooses the optimal level of information acquisition x^* as the next lemma describes. Importantly, the profits from being informed will depend on the public belief μ_B about fundamental B prior to trading.

Lemma 8 (Speculator's Information Acquisition EQ – Complement Information) *For a public belief of $\mu_B \in \{0, \frac{1}{2}, 1\}$, the trader's unique information acquisition choice satisfies*

$$x^* = \begin{cases} x_L^* = \frac{\theta\sigma}{2\kappa} L^2 & \text{if } \mu_B = 0, \\ x_H^* = \frac{\theta\sigma}{2\kappa} H^2 & \text{if } \mu_B = 1, \\ x_\emptyset^* = \frac{\theta^3\sigma}{2\kappa} & \text{if } \mu_B = \frac{1}{2}. \end{cases}$$

Assumption 3 ensures $x^* \in [0, 1]$.

Assumption 3 *We assume the trader's marginal information acquisition cost to be $\kappa \in [\underline{\kappa}^{com}, \infty)$, with $\underline{\kappa}^{com} = \frac{\theta\sigma}{2} H^2$.*

In contrast to the substitute case, the trader's information advantage about A is not eliminated by the firm's revelation of B. Consequently, the speculator always acquires information about A. The firm's announced investment scale, however, changes the marginal value of information for the trader. When the firm's announcement reveals $B = H$ ($B = L$), it is common knowledge that the firm received better (worse) news about B relative to the prior of $\frac{1}{2}$. Thus, it will choose a larger (smaller) investment scale based on the information about B. As the firm operates on a large (small) scale, the marginal value of knowing the second fundamental, A, is higher (lower) for the speculator. Consequently, the trader invests more in information acquisition the better the public belief is about B, that is, $x^*_H > x^*_\emptyset > x^*_L$.

5.1.3 Firm's Information Acquisition Subgame

Anticipating the potential outcomes of her internal information acquisition and the information revealed by the financial market, the public manager's ex-ante firm value expectation is

$$\begin{aligned}\mathbb{E}[V] =& \gamma \left[\frac{1}{2}\left(P_H + \frac{x^*_H}{2}\left(\frac{P_{HH}+P_{HL}}{2} - P_H\right)\right) + \frac{1}{2}\left(P_L + \frac{x^*_L}{2}\left(\frac{P_{LL}+P_{HL}}{2} - P_L\right)\right)\right] \\ &+ (1-\gamma)\left[P_\emptyset + \frac{x^*_\emptyset}{2}\left(\frac{P_H+P_L}{2} - P_\emptyset\right)\right] - \frac{1}{2}c\gamma^2 \\ =& \underbrace{\frac{1}{2}\theta^4}_{\text{prior value}} + \underbrace{\frac{\gamma\sigma^2}{2}\left[\theta^2 + \frac{x^*_H}{4}H^2 + \frac{x^*_L}{4}L^2 - \frac{x^*_\emptyset}{2}\theta^2\right]}_{\text{marg. benefit of info. acq.}} + \frac{x^*_\emptyset}{4}\theta^2\sigma^2 - \frac{1}{2}c\gamma^2. \end{aligned} \tag{12}$$

As in the substitute information case, the manager benefits from learning the fundamentals as she is able to choose a more appropriate investment scale. For instance, if she acquires information internally, she will learn the fundamental B, allowing her to create additional value of $\frac{1}{2}\theta^2\sigma^2$. Importantly, due to the complement information structure, the marginal benefit of learning about fundamental B is affected by the information revealed about A trough informative stock prices. When she learns that $b = H$ ($b = L$), which occurs with a probability of $\frac{1}{2}$, the market will reveal the A fundamental with probability $\frac{x^*_H}{2}$ ($\frac{x^*_L}{2}$), generating additional value of $\frac{\sigma^2 H^2}{2}$ ($\frac{\sigma^2 L^2}{2}$). However, by choosing a higher γ, the manager foregoes the opportunity of learning from the market upon an uninformative internal signal $b = \emptyset$, which would create an additional firm value increase of $\frac{\sigma^2\theta^2}{2}$. The private firm's expected value follows from (12) by setting $x^*_H = x^*_L = x^*_\emptyset = 0$.

The next lemma describes the private and public firms' information acquisition for the case of complement information.

Lemma 9 (Firm's Information Acquisition EQ – Complement Information) *The* *private firm chooses* $\gamma^*_{pri} = \frac{\theta^2\sigma^2}{2c}$ *as their information acquisition intensity. The public firm chooses*

$$\gamma^*_{pub} = \gamma^*_{pri}\left(1 + \frac{\frac{1}{2}\left(x^*_H H^2 + x^*_L L^2\right) - x^*_{\emptyset}\theta^2}{2\theta^2}\right) = \gamma^*_{pri}\left(1 + \frac{\sigma^3\left(\sigma^2 + 6\theta^2\right)}{4\kappa\theta}\right). \tag{13}$$

Assumption 4 ensures $\gamma^*_{pri}, \gamma^*_{pub} \in (0, 1]$.

Assumption 4 *We assume the firm's marginal information acquisition cost to be* $c \in [\underline{c}^{com}, \infty)$, *with* $\underline{c}^{com} = \frac{\sigma^2\theta^2}{2} + \frac{\sigma^5\theta\left(\sigma^2+6\theta^2\right)}{8\kappa}$.

Naturally, the information structure only affects the distribution of information between the public firm and the speculator, leading to the same information acquisition choice for the private firm γ^*_{pri} as before.

However, in the complement case, the public firm acquires more information than the private firm. By acquiring more information herself, the manager knows that she will incentivize the speculator to acquire either x^*_H or x^*_L, depending on whether she learned $b = H$ or $B = L$, respectively. Importantly, the information acquisition intensity is higher on average after the manager learned the fundamental B than if she did not, that is, $\frac{1}{2}\left(x^*_H + x^*_L\right) > x^*_{\emptyset}$. Consequently, the manager's and trader's information acquisitions are strategic complements.

5.2 Analysis

5.2.1 Comparative Statics

The next corollary summarizes how the private and public firm's equilibrium information acquisition intensities vary with model parameters for the case of a complement information structure:

Corollary 2 (Comparative Statics – Complement Information) *The private and public firm's information acquisition equilibrium intensities,* $\gamma^*_{pri}, \gamma^*_{pub}$, *are increasing in* σ *and* θ, *and decreasing in* c. γ^*_{pub} *is increasing in* κ.

Intuitively, the private firm acquires more information if information costs are lower (c), and if the value of information is higher due to a higher degree of uncertainty (σ) or the a higher average size of investment (θ). The latter factors also increase the speculator's incentives to acquire information (see Lemma 8). Thus, due to the complement nature of the firm's and the speculator's information, a higher σ and θ increase the firm's direct information acquisition incentives, as well as the indirect incentives in anticipation of a more informative stock price. As both effects go in the same direction, the public firm chooses a higher information acquisition intensity for higher σ and θ.

5.2.2 Gross Firm Values

In the complement information case, the public firm has the opportunity to learn from internal sources about the fundamental B, as well as from the market about A. Naturally, the private firm's shares are not traded in the market, leaving it with only the internal information source. Lemma 9 shows that the public firm acquires also more information about the fundamental B than the private firm in equilibrium. As a consequence, the public firm's expected value, gross of information acquisition costs, is always higher.

Lemma 10 (Gross Firm Value Difference – Complement Information) *The public firm has a (weakly) higher expected gross value in equilibrium.*

5.2.3 Net Firm Values

The public firm's value dominance carries over to the net firm values. As in the substitute information case, the public firm could choose the same degree of internal information acquisition as the private firm and additionally observe the informative stock price. Consequently, the public firm's net firm value is higher is equilibrium.

Lemma 11 (Net Firm Value Difference – Complement Information) *Net of internal information costs, the public firm has a (weakly) higher expected value in equilibrium.*

5.3 Real Efficiency

By acquiring more information about B, the public manager incentivizes the speculator to also choose a higher information acquisition intensity about A. While this leads to a higher firm value of the public firm, we will again investigate the efficiency implications by incorporating the speculator's information acquisition costs.

The definition of real efficiency is as before:

$$\mathcal{E}_{pri} = \mathbb{E}_{pri}[V] \tag{14}$$

$$\mathcal{E}_{pub} = \mathbb{E}_{pub}\left[V - \frac{1}{2}\kappa x^2\right] = \mathbb{E}_{pub}[V] - \frac{1}{2}\kappa\left(\frac{\gamma}{2}\left(x_H^2 + x_L^2\right) + (1-\gamma)\, x_\emptyset^2\right) \tag{15}$$

The private and public firms' expected values follow from (12). The trader's information acquisition intensity and costs depend on whether the firm's announcement revealed the fundamental B. Indeed, with probability $\frac{\gamma}{2}$, the fundamental B is revealed to be H (L), leading the trader to choose the intensity x_H (x_L). With a probability $1-\gamma$, there is no further information about B, leading to a choice of $x_\emptyset$.

The next lemma highlights the social planner's choices maximizing real efficiency for the private and public firm.

Lemma 12 (Social Planner's Choices – Complement Information) *A social planner would choose*

$$\begin{aligned}
\gamma_{pri}^{SP} &= \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2\right) = \gamma_{pri}^*,\\
\gamma_{pub}^{SP} &= \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^2}{4}\left(\frac{x_H^{SP}H^2 + x_L^{SP}L^2}{2} - x_\emptyset^{SP}\theta^2\right) - \frac{\kappa}{2}\left(\frac{\left(x_H^{SP}\right)^2 + \left(x_L^{SP}\right)^2}{2} - \left(x_\emptyset^{SP}\right)^2\right)\right)\\
&= \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^6\left(\sigma^2 + 6\theta^2\right)}{32\kappa}\right) < \gamma_{pub}^*,\\
x_H^{SP} &= \frac{H^2\sigma^2}{4\kappa} < x_H^*,\ x_L^{SP} = \frac{L^2\sigma^2}{4\kappa} < x_L^*,\ x_\emptyset^{SP} = \frac{\theta^2\sigma^2}{4\kappa} < x_\emptyset^*.
\end{aligned}$$

As in the substitute information case, the private firm's real efficiency coincides with its expected net firm value, implying that the social planner and the private manager choose the same intensity of information acquisition.

The public firm invests too much in information acquisition from the planner's perspective. In particular, the public manager ignores the consequence of her information acquisition on the speculator's information costs. If the firm invests more in information acquisition, it is more likely it learns the fundamental and announces it publicly. Thus, the trader will be more likely to choose x_H and x_L, and less likely to choose $x_\emptyset$ as her information acquisition intensity. As the trader faces a convex cost function, more information acquisition of the firm increases the expected costs incurred by the trader. The firm does not take this increase in the trader's costs into considerations and chooses to acquire more information than the social planner, $\gamma^*_{pub} < \gamma^{SP}_{pub}$.

The trader's over-investment in information acquisition follows the same logic as in the substitute information case. The marginal value from trading on private information is higher than the marginal value from the information revealed through the stock price, generating higher information acquisition incentives for the trader than is socially desirable. Intuitively, the trader's over-investment holds independently of the information revealed about the fundamental B.

The next proposition highlights that these inefficient information acquisition choices lead to lower efficiency for the public firm, just as in the substitute information case.

Proposition 2 (Real Efficiency – Complement Information) *In equilibrium, real efficiency is (weakly) higher if the firm is private:* $\mathcal{E}^*_{pri} \geq \mathcal{E}^*_{pub}$.

In the complement case, both firms are able to internally generate information about B, while only the public firm has access to information about the fundamental A through the stock price. Despite this advantage of costless learning about A, the public firm is associated with lower efficiency.

As in the substitute information case, the inefficiency of the public firm is driven by two mutually reinforcing channels. In anticipation of an informative stock price about A, the public firm invests intensively in information acquisition about fundamental B as the speculator's and the firm's information are strategic complements. While beneficial for the public firm, it does not take into account the trader's elevated information acquisition costs. As a consequence, the firm invests too much in information acquisition from an efficiency perspective. More information about B, in turn, increases the incentives of the speculator to acquire more information about A himself. The speculator's incentives to acquire information are too high from an efficiency perspective to start with, as he is maximizing his trading profits, but not real efficiency. The firm's strong information acquisition efforts incentivize the speculator to generate even more information. In turn, the trader's excessive information acquisition results in an even higher willingness of the public firm to learn from the market, elevating its information generation incentives further.

This vicious cycle of information acquisition incentives results in the firm and the speculator to acquire too much information in equilibrium, generating substantial information acquisition costs. Consequently, the public firm is associated with lower real efficiency than the private firm.

6 Conclusion

We investigate efficiency differences between private and public firms' information generation strategies, emphasizing public firms' unique ability to learn additional information from financial markets through the feedback effect.

Our analysis allows us to compare internal information acquisition levels between private and public firms, depending on the information structure, i.e. whether the firm's and traders' information are substitutes or complements. We find that a public firm invests less (more) in internal information acquisition relative to an otherwise identical private firm if the firm's and traders' information are substitutes (complements). In either information structure, the public firm generates a higher expected value than the private firm.

The main point of our paper is that while the private firm acts efficiently, the public firm features two sources of inefficiency. First, the public firm relies too much on market prices. Second, the financial market participants' incentives to acquire information are too strong. Importantly, these two sources of inefficiency are mutually reinforcing, generating lower efficiency levels for the public than for the private firm

The real efficiency implications of the public firm's knowledge generation sheds a new light on financial market regulation. We show that even though more informative prices generate value for firms through market feedback, they may be detrimental for real efficiency. This is due to the distortions in the allocation of information acquisition efforts between firms and the financial market.

References

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? *Journal of Financial Economics*, 122(3):625–654, 2016.

Tor-Erik Bakke and Toni M Whited. Which firms follow the market? an analysis of corporate investment decisions. *The Review of Financial Studies*, 23(5):1941–1980, 2010.

Snehal Banerjee, Jesse Davis, and Naveen Gondhi. Incentivizing effort and informing investment: the dual role of stock prices. *Available at SSRN 3955129*, 2021.

Snehal Banerjee, Bradyn Breon-Drish, and Kevin Smith. Feedback effects and systematic risk exposures. *Available at SSRN*, 2023.

Benjamin Bennett, René Stulz, and Zexi Wang. Does the stock market make firms more productive? *Journal of Financial Economics*, 136(2):281–306, 2020.

Shai Bernstein. Does going public affect innovation? *The Journal of finance*, 70(4):1365–1403, 2015.

Sudipto Bhattacharya and Jay R Ritter. Innovation and communication: Signalling with partial disclosure. *The Review of Economic Studies*, 50(2):331–346, 1983.

Andrew Bird, Stephen A Karolyi, Thomas G Ruchti, and Phong Truong. More is less: Publicizing information and market feedback. *Review of Finance*, 25(3):745–775, 2021.

Iván Blanco and David Wehrheim. The bright side of financial derivatives: Options trading and firm innovation. *Journal of Financial Economics*, 125(1):99–119, 2017.

Philip Bond and Itay Goldstein. Government intervention and information aggregation by prices. *The Journal of Finance*, 70(6):2777–2812, 2015.

Philip Bond, Itay Goldstein, and Edward Simpson Prescott. Market-based corrective actions. *The Review of Financial Studies*, 23(2):781–820, 2010.

Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *Annu. Rev. Financ. Econ.*, 4(1):339–360, 2012.

Arnoud W. A. Boot and Anjan V. Thakor. The Many Faces of Information Disclosure. *The Review of Financial Studies*, 14(4):1021–1057, 2001.

Gilles Chemla and Katrin Tinn. Learning through crowdfunding. *Management Science*, 66 (5):1783–1801, 2020.

Qi Chen, Itay Goldstein, and Wei Jiang. Price informativeness and investment sensitivity to stock price. *The Review of Financial Studies*, 20(3):619–650, 2007.

Jesse Davis and Naveen Gondhi. Learning in financial markets: Implications for debt-equity conflicts. *The Review of Financial Studies*, 37(5):1584–1639, 2024.

Craig Doidge, G Andrew Karolyi, and René M Stulz. The us listing gap. *Journal of Financial Economics*, 123(3):464–487, 2017.

James Dow and Rohit Rahi. Informed trading, investment, and welfare. *The Journal of Business*, 76(3):439–454, 2003.

Alex Edmans, Itay Goldstein, and Wei Jiang. Feedback effects, asymmetric trading, and the limits to arbitrage. *American Economic Review*, 105:3766–3797, 2015.

Alex Edmans, Sudarshan Jayaraman, and Jan Schneemeier. The source of information in prices and investment-price sensitivity. *Journal of Financial Economics*, 126(1):74–96, 2017.

Naomi Feldman, Laura Kawano, Elena Patel, Nirupama Rao, Michael Stevens, and Jesse Edgerton. Investment differences between public and private firms: Evidence from us tax returns. *Journal of Public Economics*, 196:104370, 2021.

Thierry Foucault and Laurent Frésard. Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *The Review of Financial Studies*, 25(11):3305–3350, 2012.

Thierry Foucault and Laurent Frésard. Learning from peers' stock prices and corporate investment. *Journal of Financial Economics*, 111(3):554–577, 2014.

Zackery D. Fox, Jaewoo Kim, and Bryce Schonberger. Investment opportunities, market feedback, and voluntary disclosure: Evidence from the shale oil revolution. *Available at SSRN 3782638*, 2021.

Kenneth R French. Presidential address: The cost of active investing. *The Journal of Finance*, 63(4):1537–1573, 2008.

Richard Friberg, Itay Goldstein, and Kristine W Hankins. Corporate responses to stock price fragility. *Journal of Financial Economics*, 153:103795, 2024.

Pingyang Gao and Pierre Jinghong Liang. Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research*, 51(5):1133–1158, 2013.

Simon Gervais and Günter Strobl. Money management and real investment. *Available at SSRN 4325133*, 2023.

Itay Goldstein. Information in financial markets and its real effects. *Review of Finance*, 27 (1):1–32, 2023.

Itay Goldstein and Alexander Guembel. Manipulation and the Allocation Role of Prices. *Review of Economic Studies*, 75:133–164, 2008.

Itay Goldstein and Liyan Yang. Information diversity and complementarities in trading and information acquisition. *The Journal of Finance*, 70(4):1723–1765, 2015.

Itay Goldstein and Liyan Yang. Information disclosure in financial markets. *Annual Review of Financial Economics*, 9:101–125, 2017.

Itay Goldstein and Liyan Yang. Good disclosure, bad disclosure. *Journal of Financial Economics*, 131(1):118–138, 2019.

Itay Goldstein, Jan Schneemeier, and Liyan Yang. Market feedback: Who learns what? *Unpublished working paper*, 10(2):8, 2020.

Itay Goldstein, Shijie Yang, and Luo Zuo. The real effects of modern information technologies: Evidence from the edgar implementation. *Journal of Accounting Research*, 61(5): 1699–1733, 2023.

Itay Goldstein, Alexandr Kopytov, Lin Shen, and Hoatin Xiang. On esg investing: Heterogeneous preferences, information, and asset prices. *Available at SSRN*, 2024.

Erik Hapnes. Endogenous information acquisition with feedback effects. *Informational frictions in financial markets, EPFL Ph.D. Thesis*, 2021.

Sudarshan Jayaraman and Joanna Wu. Should i stay or should i grow? using voluntary disclosure to elicit market feedback. *The Review of Financial Studies*, 33(8):3854–3888, 2020.

Sudarshan Jayaraman and Joanna Shuang Wu. Is silence golden? real effects of mandatory disclosure. *The Review of Financial Studies*, 32(6):2225–2259, 2019.

Albert S. Kyle. Continuous Auctions and Insider Trading. *Econometrica*, 53(6):1315–1335, 1985.

Matthias Lassak. Voluntary disclosure, price informativeness, and efficient investment. *Available at SSRN 3556575*, 2023.

Josh Lerner, Morten Sorensen, and Per Strömberg. Private equity and long-run investment: The case of innovation. *The Journal of Finance*, 66(2):445–477, 2011.

Tse-Chun Lin, Qi Liu, and Bo Sun. Contractual managerial incentives with stock price feedback. *American Economic Review*, 109(7):2446–68, 2019.

Yuanzhi Luo. Do insiders learn from outsiders? evidence from mergers and acquisitions. *The Journal of Finance*, 60(4):1951–1982, 2005.

Caio Machado and Ana Elisa Pereira. Competing for stock market feedback. *Available at SSRN 3632063*, 2020.

Caio Machado and Ana Elisa Pereira. Optimal capital structure with stock market feedback. *Working Paper*, 2021.

Evgeny Petrov and Stefan Schantl. Conversations between managers and investors. *Available at SSRN*, 2023.

Jan Schneemeier. Voluntary precision disclosure and endogenous market feedback. *Management Science*, 69(9):5618–5637, 2023.

David Weild and Edward Kim. A wake-up call for america. *Grant Thornton LLP White Paper*, 2009.

Yan Xiong and Liyan Yang. Disclosure, competition, and learning from asset prices. *Journal of Economic Theory*, 197:105331, 10 2021.

A Proofs

Proof of Lemma 1. The fundamental A has been perfectly revealed if the public belief is $\mu_A \in \{0,1\}$, eliminating the speculator's potential to generate trading gains with his signal α. Thus, the trader is indifferent in trading and chooses $w^* = 0$, by assumption. Given a belief of $\mu_A \in \{0,1\}$, the scale investment policy and the equilibrium stock price follow as prescribed.

Suppose the public belief is $\mu_A = \frac{1}{2}$. As a first step, we take the speculator's trading behavior as given and show its optimality afterwards. Thus, we conjecture that the speculator buys one unit of the asset if $\alpha = H$, sells one unit if $\alpha = L$, and does not trade if $\alpha = \emptyset$. The resulting total order flow of $z = 2$ ($z = -2$) reveals $\alpha = H$ ($\alpha = L$), to the market maker. It follows from the conjectured scale investment policy that the equilibrium stock price is $P_H = \frac{1}{2}(\theta + \sigma)^2\theta^2$ ($P_L = \frac{1}{2}(\theta - \sigma)^2\theta^2$). In contrast, an orderflow of $z \in \{-1, 0, 1\}$ is not informative for the market maker, resulting in an expected firm value as a function of the belief prior to the trading stage μ_A. Indeed, an announced intended investment scale of $\tilde{K}(a = H) = (\theta+\sigma)\theta$ ($\tilde{K}(a = L) = (\theta-\sigma)\theta$) reveals the fundamental A, while $\tilde{K}(a = \emptyset) = \theta^2$, indicates that the manager received an uninformative signal. Thus, even if the orderflow z is uninformative, the market maker sets the stock price as P_H (P_L) for a belief of $\mu_A = 1$ ($\mu_A = 0$). In contrast, if the orderflow and scale announcement are uninformative, i.e., $z \in \{-1, 0, 1\}$ and $\tilde{K}(a = \emptyset)$, the market maker sets the stock price based on the prior belief $\mu_A = 1/2$.

Finally, we show that the speculator's trading behavior is indeed optimal. First, suppose the trader has received the signal $\alpha = H$. Anticipating the market maker's pricing strategy and the firm's investment policy, the expected profits from buying, selling, and no trading are:

$$
\begin{aligned}
\mathbb{E}[\pi(buy)|\alpha = H] =& \frac{1}{2}\left(\mathbb{E}[A|\alpha = H]\theta K^*(\mu_A = 1/2) - 1/2K^*(\mu_A = 1/2)^2 - P_\emptyset\right) \\
=& \frac{1}{2}\left(\theta K^*(\mu_A = 1/2)\sigma\right) = \frac{1}{2}\theta^3\sigma, \qquad (16)\\
\mathbb{E}[\pi(sell)|\alpha = H] =& \frac{1}{2}\left(P_L - \left(\mathbb{E}[A|\alpha = H]\theta K^*(\mu_A = 0) - 1/2K^*(\mu_A = 0)^2\right)\right) \\
&+ \frac{1}{2}\left(P_\emptyset - \left(\mathbb{E}[A|\alpha = H]\theta K^*(\mu_A = 1/2) - 1/2K^*(\mu_A = 1/2)^2\right)\right) \\
=& \frac{1}{2}\theta\left(K^*(\mu_A = 0)(-2\sigma) + K^*(\mu_A = 1/2)(-\sigma)\right) < 0, \\
\mathbb{E}[\pi(no\ trade)|\alpha = H] =& 0,
\end{aligned}
$$

highlighting the optimality of buying based on $\alpha = H$. Similarly, the trader's expected profits from selling, buying, and no trade based on a signal $\alpha = L$ are:

$$
\begin{aligned}
\mathbb{E}[\pi(sell)|\alpha = L] =& \frac{1}{2}\left(P_\emptyset - \left(\mathbb{E}[A|\alpha = L]\theta K^*(\mu_A = 1/2) - 1/2K^*(\mu_A = 1/2)^2\right)\right) \\
=& \frac{1}{2}\left(\theta K^*(\mu_A = 1/2)\sigma\right) = \frac{1}{2}\theta^3\sigma, \qquad (17)\\
\mathbb{E}[\pi(buy)|\alpha = L] =& \frac{1}{2}\left(\mathbb{E}[A|\alpha = L]\theta K^*(\mu_A = 1) - 1/2K^*(\mu_A = 1)^2 - P_H\right) \\
&+ \frac{1}{2}\left(\mathbb{E}[A|\alpha = L]\theta K^*(\mu_A = 1/2) - 1/2K^*(\mu_A = 1/2)^2 - P_\emptyset\right) \\
=& \frac{1}{2}\theta\left(K^*(\mu_A = 1)(-2\sigma) + K^*(\mu_A = 1/2)(-\sigma)\right) < 0, \\
\mathbb{E}[\pi(no\ trade)|\alpha = L] =& 0,
\end{aligned}
$$

showing the optimality of selling based on a signal $\alpha = L$. Finally, it is optimal for the trader not to trade based on $\alpha = \emptyset$ as

$$\begin{aligned}
\mathbb{E}[\pi(no\ trade)|\alpha = \emptyset] =& 0, \\
\mathbb{E}[\pi(buy)|\alpha = \emptyset] =& \frac{1}{2}\left(\mathbb{E}[A|\alpha = \emptyset]\theta K^*(\mu_A = 1) - 1/2K^*(\mu_A = 1)^2 - P_H\right) \\
=& \frac{1}{2}\theta K^*(\mu_A = 1)\,(-\sigma) < 0, \\
\mathbb{E}[\pi(sell)|\alpha = \emptyset] =& \frac{1}{2}\left(P_L - \left(\mathbb{E}[A|\alpha = \emptyset]\theta K^*(\mu_A = 0) - 1/2K^*(\mu_A = 0)^2\right)\right) \\
=& \frac{1}{2}\theta K^*(\mu_A = 0)\,(-\sigma) < 0.
\end{aligned}$$

■

Proof of Lemma 2. When making her information acquisition choice, the trader takes into account his equilibrium trading positions as outlined by Lemma 1. Naturally, whenever the public belief is $\mu_A \in \{0, 1\}$, the trader can not generate trading profits, resulting in $x^* = 0$. For a belief of $\mu_A = \frac{1}{2}$, the trader's expected profit, net of information acquisition costs, at the information acquisition stage is:

$$x\left(\frac{1}{2}\mathbb{E}[\pi(buy)|\alpha = H] + \frac{1}{2}\mathbb{E}[\pi(sell)|\alpha = L]\right) - \frac{1}{2}\kappa x^2,$$

resulting in $x^* = \frac{1}{2\kappa}\left(\mathbb{E}[\pi(buy)|\alpha = H] + \mathbb{E}[\pi(sell)|\alpha = L]\right) = \frac{1}{2\kappa}\theta^3\sigma$, with $\mathbb{E}[\pi(buy)|\alpha = H]$ and $\mathbb{E}[\pi(sell)|\alpha = L]$ given by (16) and (17), respectively. Assumption 1 ensures that $x^* \in (0, 1]$. ■

Proof of Lemma 3. γ^*_{pub} follows directly from the public manager maximizing (4) and inserting the trader's $x^*_\emptyset$ from Lemma 2. For the private firm's γ^*_{pri}, set $x_\emptyset = 0$. Assumption 2 ensures that $\gamma^*_{pri}, \gamma^*_{pub} \in (0, 1]$. ■

Proof of Corollary 1. The comparative statics follow from Lemma 3 and are $\frac{d\gamma^*_{pri}}{dc} = -\frac{\theta^2\sigma^2}{2c^2} < 0$ and $\frac{d\gamma^*_{pub}}{dc} = -\frac{\theta^2\sigma^2(2-x)}{4c^2} < 0$ for c. A change of σ implies $\frac{d\gamma^*_{pri}}{d\sigma} = \frac{\theta^2\sigma}{c} > 0$ and $\frac{d\gamma^*_{pub}}{d\sigma} = \frac{\theta^2\sigma}{c}\left(1 - \frac{3\theta^3\sigma}{8\kappa}\right)$, which is positive due to $\kappa \geq \underline{\kappa}^{sub} = \frac{\theta^3\sigma}{2}$ according to Assumption 2. A change of θ implies $\frac{d\gamma^*_{pri}}{d\theta} = \frac{\theta\sigma^2}{c} > 0$ and $\frac{d\gamma^*_{pub}}{d\theta} = \frac{\theta\sigma^2}{c}\left(1 - \frac{5\theta^3\sigma}{8\kappa}\right)$. $\frac{d\gamma^*_{pub}}{d\theta} > 0$ for $\kappa > \frac{5}{4}\underline{\kappa}^{sub} = \frac{5\theta^3\sigma}{8}$

and $\frac{d\gamma^*_{pub}}{d\theta} < 0$ for $\kappa < \frac{5}{4}\underline{\kappa}^{sub}$. Finally, $\frac{d\gamma^*_{pub}}{d\kappa} = -\frac{\theta^2\sigma^2}{4c}\frac{dx^*_\emptyset}{d\theta} > 0$ as $\frac{dx_\emptyset}{d\kappa} = -\frac{\theta^3\sigma}{2\kappa^2} < 0$. ■

Proof of Lemma 4. The private firm's expected gross value is higher if

$$\gamma^*_{pub} + (1-\gamma^*_{pub})\frac{x_\emptyset}{2} < \gamma^*_{pri}.$$

Using $\gamma^*_{pub} = \gamma^*_{pri}\left(1 - \frac{x_\emptyset}{2}\right)$ results in

$$\begin{aligned}\frac{1}{2} &< \gamma^*_{pri}\left(1 - \frac{x_\emptyset}{4}\right)\\ \Leftrightarrow c &< \frac{\theta^2\sigma^2}{2}\left(2 - \frac{\theta^3\sigma}{4\kappa}\right) = \underline{c}^{sub}\left(2 - \frac{\theta^3\sigma}{4\kappa}\right).\end{aligned}$$

■

Proof of Lemma 5. The proof follows directly from the text's arguments. Suppose the public manager chooses $\gamma^*_{pub} = \gamma^*_{pri}$, resulting in $\mathbb{E}_{pub}\left[V;\gamma^*_{pri}\right] = \mathbb{E}_{pri}\left[V;\gamma^*_{pri}\right] + \frac{x_\emptyset}{4}\theta^2\sigma^2$ which is greater than (equal to) the private firm's value for $x_\emptyset > 0$ ($x_\emptyset = 0$). If the public manager chooses $\gamma^*_{pub} \neq \gamma^*_{pri}$, it has to be associated with $\mathbb{E}_{pub}\left[V;\gamma^*_{pub}\right] > \mathbb{E}_{pub}\left[V;\gamma^*_{pri}\right]$, otherwise γ^*_{pub} would not be optimal. In sum, the public firm's expected net value is at least as large as the private firms expected net value. ■

Proof of Lemma 6. The social planner's choices of γ and $x_\emptyset$ follow directly from maximizing $\mathcal{E}_{pri}$ in (8) and $\mathcal{E}_{pub}$ (9) for the private and public firm, respectively. ■

Proof of Proposition 1. Using the net expected firm values from (6) and (7) and deducting the trader's expected information acquisition cost, results in the efficiency difference:

$$
\begin{aligned}
\mathcal{E}_{pub}^* - \mathcal{E}_{pri}^* &= \frac{x_\emptyset^*}{4}\theta^2\sigma^2 + \frac{1}{2}c\left((\gamma_{pub}^*)^2 - (\gamma_{pri}^*)^2\right) - (1-\gamma_{pub}^*)\frac{1}{2}\kappa\left(x_\emptyset^*\right)^2 \\
&= x_\emptyset^*\left[\frac{1}{4}\theta^2\sigma^2 - \frac{1}{2}c(\gamma_{pri}^*)^2\left(1-\frac{x_\emptyset^*}{4}\right) - \left(1-\gamma_{pri}^*\left(1-\frac{x_\emptyset^*}{2}\right)\right)\frac{1}{2}\kappa x_\emptyset^*\right] \\
&= x_\emptyset^*\left[\frac{1}{2}c\gamma_{pri}^* - \frac{1}{2}c(\gamma_{pri}^*)^2\left(1-\frac{x_\emptyset^*}{4}\right) - \left(1-\gamma_{pri}^*\left(1-\frac{x_\emptyset^*}{2}\right)\right)\frac{1}{4}\theta^3\sigma\right] \\
&= x_\emptyset^*\left[\frac{1}{2}c\gamma_{pri}^* - \frac{1}{2}c(\gamma_{pri}^*)^2\left(1-\frac{x_\emptyset^*}{4}\right) - \left(1-\gamma_{pri}^*\left(1-\frac{x_\emptyset^*}{2}\right)\right)\frac{\theta}{2\sigma}c\gamma_{pri}^*\right] \\
&= x_\emptyset^*\left[\frac{1}{2}c\gamma_{pri}^*\left(1-\frac{\theta}{\sigma}\right) + \frac{1}{2}c(\gamma_{pri}^*)^2\left(\frac{\theta}{\sigma}\left(1-\frac{x_\emptyset^*}{2}\right) - 1 + \frac{x_\emptyset^*}{4}\right)\right] \\
&= x_\emptyset^*\left[\frac{1}{2}c\gamma_{pri}^*\left(1-\gamma_{pri}^*\right)\left(1-\frac{\theta}{\sigma}\right) + \frac{1}{2}c(\gamma_{pri}^*)^2\frac{x_\emptyset^*}{2}\left(\frac{1}{2}-\frac{\theta}{\sigma}\right)\right],
\end{aligned}
$$

where we have used $\gamma_{pub}^* = \gamma_{pri}^*\left(1-\frac{x_\emptyset^*}{2}\right)$, $\gamma_{pri}^* = \frac{\theta^2\sigma^2}{2c}$, and, $x_\emptyset^* = \frac{\theta^3\sigma}{2\kappa}$. Our assumption $\sigma < \theta$ implies that both $1-\frac{\theta}{\sigma}$ and $\frac{1}{2}-\frac{\theta}{\sigma}$ are negative, proving the proposition. ■

Proof of Lemma 7. The proof of Lemma 7 follows similar steps as the proof of Lemma 1. The market maker's and firm's updating about A follows the same logic as before. In contrast, the announcement $\tilde{K}(b)$ makes the firm's information about B public and thus allows the trader to make trading profits with his information about A. The trading strategy follows the trader's signal. For instance, suppose it is public knowledge that $B = H$ and the trader received the signal $\alpha = H$. Anticipating the market maker's pricing strategy and the firm's investment policy, the expected profits from buying, selling, and no trading are:

$$
\begin{aligned}
\mathbb{E}[\pi(buy)|\alpha = H, B = H] =& \frac{1}{2}\left(H^2K^*(\mu_A = 1/2, B = H) - 1/2K^*(\mu_A = 1/2, B = H)^2 - P_H\right) \\
=& \frac{1}{2}\left(HK^*(\mu_A = 1/2, B = H)\sigma\right) = \frac{1}{2}H^2\theta\sigma, \\
\mathbb{E}[\pi(sell)|\alpha = H, B = H] =& \frac{1}{2}\left(P_{HL} - \left(H^2K^*(\mu_A = 0, B = H) - 1/2K^*(\mu_A = 0, B = H)^2\right)\right) \\
&+ \frac{1}{2}\left(P_H - \left(H^2K^*(\mu_A = 1/2, B = H) - 1/2K^*(\mu_A = 1/2, B = H)^2\right)\right) \\
=& \frac{1}{2}H\left(K^*(\mu_A = 0, B = H)(-2\sigma) + K^*(\mu_A = 1/2, B = H)(-\sigma)\right) < 0, \\
\mathbb{E}[\pi(no\ trade)|\alpha = H] =& 0,
\end{aligned}
$$

highlighting the optimality of buying based on $\alpha = H$ and $B = H$. Using similar calculations, it is straightforward to show that it is optimal for the trader to sell based $\alpha = L$ and not to trade upon $\alpha = \varnothing$ for all possible revelation of B and is thus omitted. In equilibrium, the trader's expected profits are:

$$\mathbb{E}[\pi(buy)|\alpha = H, B = H] = \mathbb{E}[\pi(sell)|\alpha = L, B = H] = \frac{1}{2}H^2\theta\sigma,$$
$$\mathbb{E}[\pi(buy)|\alpha = H, B = L] = \mathbb{E}[\pi(sell)|\alpha = L, B = L] = \frac{1}{2}L^2\theta\sigma,$$
$$\mathbb{E}[\pi(buy)|\alpha = H] = \mathbb{E}[\pi(sell)|\alpha = L] = \frac{1}{2}\theta^3\sigma.$$

■

Proof of Lemma 8. When making her information acquisition choice, the trader takes into account his equilibrium trading positions as outlined by Lemma 7. Importantly, the potential profits depend on the public's belief about fundamental B. If it is public knowledge that $B = H$, the trader's expected profits, net of information acquisition costs, at the information acquisition stage are:

$$x_H \left(\frac{1}{2}\mathbb{E}[\pi(buy)|\alpha = H, B = H] + \frac{1}{2}\mathbb{E}[\pi(sell)|\alpha = L, B = H] \right) - \frac{1}{2}\kappa x_H^2,$$

resulting in $x_H^* = \frac{1}{2\kappa}\left(\mathbb{E}[\pi(buy)|\alpha = H, B = H] + \mathbb{E}[\pi(sell)|\alpha = L, B = H]\right) = \frac{1}{2\kappa}H^2\theta\sigma$, with $\mathbb{E}[\pi(buy)|\alpha = H, B = H]$ and $\mathbb{E}[\pi(sell)|\alpha = L, B = H]$ given in the proof of Lemma 7. Following similar steps it is easy to show the values of x_L^* and $x_\varnothing^*$. As x_H^* is the highest among the three information acquisition intensities, Assumption 3 ensures that $x_H^* \in (0, 1]$ and thus $x_L^*, x_\varnothing^* \in (0, 1]$. ■

Proof of Lemma 9. γ_{pub}^* follows directly from the public manager maximizing (12) and inserting the trader's x_H^*, x_L^*, and $x_\emptyset^*$ from Lemma 8. For the private firm's γ_{pri}^*, set $x_\emptyset = 0$. Assumption 4 ensures that $\gamma_{pri}^*, \gamma_{pub}^* \in (0, 1]$. ■

Proof of Corollary 2. The comparative statics for the private firm are proved in the proof of Corollary 1. The comparative statics for the public firm follow from Lemma 9 and are $\frac{d\gamma^*_{pub}}{dc} = -\frac{\gamma^*_{pub}}{c} < 0$, $\frac{d\gamma^*_{pub}}{d\sigma} = \frac{1}{2c}\left[2\theta^2\sigma + \frac{7\sigma^6\theta+30\sigma^4\theta^3}{4\kappa}\right] > 0$, $\frac{d\gamma^*_{pub}}{d\theta} = \frac{1}{2c}\left[2\theta\sigma^2 + \frac{\sigma^7\theta+18\sigma^5\theta^2}{4\kappa}\right] > 0$, and $\frac{d\gamma^*_{pub}}{d\kappa} = \gamma^*_{pri}\left(-\frac{\sigma^3(\sigma^2+6\theta^2)}{4\theta\kappa^2}\right) < 0$. ■

Proof of Lemma 10. The proof follows directly from the arguments in the text and $\gamma^*_{pri} < \gamma^*_{pub}$ from Lemma 9. ■

Proof of Lemma 11. The proof follows directly from the arguments in the text. ■

Proof of Lemma 12. Using the public firm's net expected value from (12), we can calculate real efficiency for the public firm as:

$$\begin{aligned}\mathcal{E}_{pub} =& \mathbb{E}_{pub}\left[V - \frac{1}{2}\kappa x^2\right]\\ =& \frac{1}{2}\theta^4 + \frac{\gamma\sigma^2}{2}\left[\theta^2 + \frac{x_H}{4}H^2 + \frac{x_L}{4}L^2 - \frac{x_\emptyset}{2}\theta^2\right] + \frac{x_\emptyset}{4}\theta^2\sigma^2 - \frac{1}{2}c\gamma^2 - \mathbb{E}_{pub}\left[\frac{1}{2}\kappa x^2\right],\end{aligned}$$

with the ex ante expected information costs for the speculator are

$$\begin{aligned}\mathbb{E}_{pub}\left[\frac{1}{2}\kappa x^2\right] &= \frac{1}{2}\kappa\left(\frac{\gamma}{2}\left((x_H)^2 + (x_L)^2\right) + (1-\gamma)(x_\emptyset)^2\right)\\ &= \frac{1}{2}\kappa\left((x_\emptyset)^2 + \gamma\left(\frac{1}{2}\left((x_H)^2 + (x_L)^2\right) - (x_\emptyset)^2\right)\right).\end{aligned}$$

Maximizing real efficiency for the public firm generates four first-order conditions:

$$\begin{aligned}foc_\gamma &: 0 = \frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^2}{4}\left(\frac{x_H^{SP}H^2 + x_L^{SP}L^2}{2} - x_\emptyset^{SP}\theta^2\right) - \frac{\kappa}{2}\left(\frac{\left(x_H^{SP}\right)^2 + \left(x_L^{SP}\right)^2}{2} - \left(x_\emptyset^{SP}\right)^2\right) - \gamma^{SP}c,\\ foc_{x_H} &: 0 = \frac{\sigma^2}{4}H^2 - \kappa x_H^{SP},\\ foc_{x_L} &: 0 = \frac{\sigma^2}{4}L^2 - \kappa x_L^{SP},\\ foc_{x_\emptyset} &: 0 = \frac{\sigma^2}{4}\theta^2 - \kappa x_\emptyset^{SP},\end{aligned}$$

resulting in $x_H^{SP} = \frac{\sigma^2}{4\kappa}H^2$, $x_L^{SP} = \frac{\sigma^2}{4\kappa}L^2$, and $x_\emptyset^{SP} = \frac{\sigma^2}{4\kappa}\theta^2$. Inserting these choices into foc_γ

yields $\gamma^{SP} = \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^6(\sigma^2+6\theta^2)}{32\kappa}\right)$. Note that:

$$\begin{aligned}
&\gamma^{SP} < \gamma^* \\
\Leftrightarrow &\frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^6\left(\sigma^2+6\theta^2\right)}{32\kappa}\right) < \frac{1}{c}\left(\frac{1}{2}\sigma^2\theta^2 + \frac{\sigma^5\theta\left(\sigma^2+6\theta^2\right)}{8\kappa}\right) \\
\Leftrightarrow &\frac{\sigma}{32} > \frac{\theta}{8},
\end{aligned}$$

which is true as $\theta > \sigma$. ■

Proof of Proposition 2. Similar as in the proof of Lemma 6, we use the public firm's net expected value from (12) to calculate real efficiency for the public firm in equilibrium as:

$$\begin{aligned}
\mathcal{E}^*_{pub} =&\mathbb{E}_{pub}\left[V - \frac{1}{2}\kappa x^2\right] \\
=&\frac{1}{2}\theta^4 + \frac{\gamma^*_{pub}\sigma^2}{2}\left[\theta^2 + \frac{x^*_H}{4}H^2 + \frac{x^*_L}{4}L^2 - \frac{x^*_\emptyset}{2}\theta^2\right] + \frac{x^*_\emptyset}{4}\theta^2\sigma^2 - \frac{1}{2}c\left(\gamma^*_{pub}\right)^2 - \mathbb{E}_{pub}\left[\frac{1}{2}\kappa x^2\right] \\
=&\frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pub}\right)^2 + \frac{x^*_\emptyset}{4}\theta^2\sigma^2 - \mathbb{E}_{pub}\left[\frac{1}{2}\kappa x^2\right],
\end{aligned}$$

where we used $\gamma^*_{pub} = \frac{\theta^2\sigma^2}{2c} + \sigma^2\frac{\frac{1}{2}(x^*_H H^2 + x^*_L L^2) - x^*_\emptyset\theta^2}{4c}$. In equilibrium, the ex ante expected information costs for the speculator are:

$$\begin{aligned}
\mathbb{E}_{pub}\left[\frac{1}{2}\kappa x^2\right] &= \frac{1}{2}\kappa\left(\left(x^*_\emptyset\right)^2 + \gamma^*_{pub}\left(\frac{1}{2}\left(\left(x^*_H\right)^2 + \left(x^*_L\right)^2\right) - \left(x^*_\emptyset\right)^2\right)\right) \\
&= \frac{1}{2}\kappa\left(\frac{\theta^6\sigma^2}{4\kappa^2} + \gamma^*_{pub}\left(\frac{1}{2}\left(\frac{\theta^2H^4\sigma^2}{4\kappa^2} + \frac{\theta^2L^4\sigma^2}{4\kappa^2}\right) - \frac{\theta^6\sigma^2}{4\kappa^2}\right)\right) \\
&= \frac{\sigma^2\theta^2}{8\kappa}\left(\theta^4 + \gamma^*_{pub}\left(\frac{1}{2}\left(H^4 + L^4\right) - \theta^4\right)\right) \\
&= \frac{\sigma^2\theta^6}{8\kappa} + \frac{\sigma^4\theta^2\gamma^*_{pub}}{8\kappa}\left(\sigma^2 + 6\theta^2\right),
\end{aligned}$$

where we used the equilibrium information acquisition intensities of the speculator x^*_H, x^*_L, and $x^*_\emptyset$, highlighted in Lemma 8.

Next, we insert the speculators expected cost of information acquisition back in the real efficiency calculation for the public firm:

$$\begin{aligned}\mathcal{E}^*_{pub} &= \frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pub}\right)^2 + \frac{x^*_\emptyset}{4}\theta^2\sigma^2 - \left(\frac{\sigma^2\theta^6}{8\kappa} + \frac{\sigma^4\theta^2\gamma^*_{pub}}{8\kappa}\left(\sigma^2+6\theta^2\right)\right)\\ &= \frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pri}(1+\epsilon)\right)^2 + \gamma^*_{pri}\frac{\theta^3\sigma c}{4\kappa} - \left(\frac{\theta^4 c\gamma^*_{pri}}{4\kappa} + \gamma^*_{pri}(1+\epsilon)\frac{\epsilon\sigma\theta^3}{2}\right)\\ &= \frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pri}\right)^2(1+\epsilon))^2 - \gamma^*_{pri}\left(\frac{\theta^3(\theta-\sigma)c}{4\kappa} + (1+\epsilon)\frac{\epsilon\sigma\theta^3}{2}\right),\end{aligned}$$

where we used $\gamma^*_{pub} = \gamma^*_{pri}\left(1+\frac{\sigma^3\left(\sigma^2+6\theta^2\right)}{4\kappa\theta}\right) = \gamma^*_{pri}(1+\epsilon)$, $\gamma^*_{pri} = \frac{\theta^2\sigma^2}{2c}$, and $x^*_\emptyset = \frac{\theta^3\sigma}{2\kappa}$.

Finally, let's calculate the efficiency difference between the public and the private firm in equilibrium:

$$\begin{aligned}\mathcal{E}^*_{pub} - \mathcal{E}^*_{pri} &= \frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pri}\right)^2(1+\epsilon))^2 - \gamma^*_{pri}\left(\frac{\theta^3(\theta-\sigma)c}{4\kappa} + (1+\epsilon)\frac{\epsilon\sigma\theta^3}{2}\right) - \left(\frac{1}{2}\theta^4 + \frac{1}{2}c\left(\gamma^*_{pri}\right)^2\right)\\ &= \frac{1}{2}c\left(\gamma^*_{pri}\right)^2(2+\epsilon)\epsilon - \gamma^*_{pri}\left(\frac{\theta^3(\theta-\sigma)c}{4\kappa} + (1+\epsilon)\frac{\epsilon\sigma\theta^3}{2}\right).\end{aligned}$$

Suppose, contrary to the proposition's claim, that $\mathcal{E}^*_{pub} - \mathcal{E}^*_{pri} > 0$, which is equivalent to:

$$\begin{aligned}&0 < c\gamma^*_{pri}(2+\epsilon)\epsilon - \frac{\theta^3(\theta-\sigma)c}{2\kappa} - (1+\epsilon)\epsilon\sigma\theta^3\\ \Leftrightarrow &0 < c\frac{\theta^2\sigma^2}{2c}(2+\epsilon)\epsilon - \frac{\theta^3(\theta-\sigma)c}{2\kappa} - (1+\epsilon)\epsilon\sigma\theta^3\\ \Leftrightarrow &0 < \frac{\sigma^2}{2}(2+\epsilon)\epsilon - \frac{\theta(\theta-\sigma)c}{2\kappa} - (1+\epsilon)\epsilon\sigma\theta\\ \Leftrightarrow &c < \frac{2\kappa\epsilon\sigma}{\theta(\theta-\sigma)}\left[\frac{\sigma}{2}(2+\epsilon) - (1+\epsilon)\theta\right].\end{aligned}$$

Note that $\frac{\sigma}{2}(2+\epsilon) - (1+\epsilon)\theta = (\sigma-\theta) + \epsilon\left(\frac{\sigma}{2}-\theta\right) < 0$, as $\sigma < \theta$ and $\epsilon > 0$. Thus, for $\mathcal{E}^*_{pub} - \mathcal{E}^*_{pri} > 0$, $c < 0$ which is a contradiction. ■